Model validation for control and controller validation in a prediction error identification framework - Part II : illustrations

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Abstract

In this paper, we illustrate our new results on model validation for control and controller validation in a prediction error identification framework, developed in a companion paper (Gevers *et al.*, 2002), through two realistic simulation examples, covering widely different control design applications. The first is the control of a flexible mechanical system (the Landau benchmark example) with a tracking objective, the second is the control of a ferrosilicon production process with a disturbance rejection objective.

Key words: System identification, identification for robust control, model validation, controller validation

1 Introduction

In the companion paper (Gevers *et al.*, 2002) we have developed a model validation procedure that consists of a prediction error identification experiment with a full order model. This procedure delivers an uncertainty set in transfer function space that we have characterized and baptized generic Prediction Error (PE) uncertainty set. It is defined as a ratio of linear combinations of known transfer functions, with the coefficient vector constrained to lie in an ellipsoid. We have derived two sets of results for such PE uncertainty sets:

- *Controller validation* results in the form of necessary and sufficient conditions for a specific controller to stabilize - or to achieve a given level of performance with - all systems in such PE uncertainty set;
- *Model validation for control* results in the form of a measure of size of such model uncertainty sets that is connected to the size of a set of robustly stabilizing controllers.

In this paper, we illustrate these technical results with two realistic identification and control design applications. These simulation examples have been chosen to illustrate two typical but very different control design problems. The first one is the widely publicized Landau benchmark transmission system (Landau *et al.*, 1995*b*): a tracking problem with a step disturbance rejection objective in an essentially noise-free environment. The second is a typical industrial application: a ferrosilicon production process described in (Ingason and Jonsson, 1998), in which the main objective is stochastic disturbance rejection.

For each of these two applications, we apply our methodology. A PE identification experiment is performed on the true system leading to a model G_{mod} [1] and an uncertainty region $\mathcal{D}$ containing the true input-output transfer function G_0 at a certain probability level. The worst-case ν-gap is then used to assess the quality of the pair $\{G_{mod},\ \mathcal{D}\}$ for robustly stable control design, i.e. one checks whether the worst-case ν-gap $\delta_{WC}(G_{mod}, \mathcal{D})$ is much smaller than the optimal stability margin $b_{opt}(G_{mod})$ of the model G_{mod}. If that is the case, the model G_{mod} is used to design a controller satisfying the performance specifications with this nominal model. The controller validation results are then used to verify if these specifications are also satisfied with all systems in $\mathcal{D}$, and therefore also with the true system G_0.

In the first illustration, we choose the identified model as the model G_{mod} used for control design. In the second illustration, we consider a case where the model G_{mod} used for control design is given a priori. We illustrate the role played by the experimental conditions in our methodology by comparing,

[1] The model G_{mod} can also be given a priori.

for each application, the results delivered by an open loop PE identification experiment with the results delivered by a closed loop experiment.

2 The flexible transmission system

2.1 Problem setting

We consider as *unknown true system* the half-load model of the flexible transmission system used as a benchmark in a special issue of the European Journal of Control (Landau *et al.*, 1995*b*).

$$\begin{aligned} G_0(z) &= z^{-3}\frac{0.10276 + 0.18123z^{-1}}{1 - 1.99185z^{-1} + 2.20265z^{-2} - 1.84083z^{-3} + 0.89413z^{-4}} \\ &\triangleq z^{-3}\frac{B(z)}{A(z)}. \end{aligned} \tag{1}$$

The sampling period is $0.05s$. The output of the system is subject to step disturbances filtered through $H_0(z) = \frac{1}{A(z)}$. This means that the plant can be seen as a nonstandard ARX system described by

$$A(z)y(t) = z^{-3}B(z)u(t) + p(t) \tag{2}$$

where $u(t)$ is the input of the plant, $y(t)$ its output and $p(t)$ is sequence of zero mean step disturbances, modeled as a square wave signal with random transitions. A standard ARX description of such system with step disturbances is given by

$$A(z)\Delta(z)y(t) = z^{-3}B(z)\Delta(z)u(t) + e(t), \tag{3}$$

where $\Delta(z) = 1 - z^{-1}$ and $e(t)$ is a sequence of Gaussian white noise with zero mean and appropriate variance. The effect of the filter $\Delta(z)$ is to put an integrator in the controller such as to reject the step disturbances. The standard prediction error identification algorithm for ARX models can be used to identify the parameters of this system, provided the data are prefiltered by $\Delta(z)$.

Our objective is to apply our PE validation methodology to the flexible transmission system G_0, considered as an unknown system. In order to illustrate the role played by the experimental conditions, we shall compare two validation experiments, one in open loop, one in closed loop. In both cases, we shall estimate a model G_{mod} and an uncertainty set containing the true system G_0 at a probability level of 95%, compute a nominal controller C from the identified model G_{mod} that satisfies some prior specifications with this model, and apply our validation tools to check whether this controller also satisfies the specifications with the "unknown" G_0. The main specifications we shall deal with here are (Landau *et al.*, 1995*b*):

- stability of the loop $[C\ G_0]$
- a maximum value of less than 6 dB for the sensitivity function $T_{22}(G_0, C) = 1/(1 + G_0C)$.

2.2 Open-loop validation experiment

The input signal $u_{ol}(t)$ applied to the stable true system G_0 for open-loop validation is chosen as a PRBS with variance $\sigma^2_{u_{ol}} = 0.1$ and a flat spectrum. The output step disturbances $p(t)$ are simulated as a zero mean random binary sequence with variance $\sigma^2_p = 0.01$ and cut-off frequency at 0.05 times the Nyquist frequency; that is, the mean length of the steps of $p(t)$ is about twenty times longer than that of the steps of $u_{ol}(t)$, while the amplitude of the steps $p(t)$ is $\sqrt{10}$ times smaller than those of $u_{ol}(t)$. The spectra and a realization of $u_{ol}(t)$ and $p(t)$ are shown in Figure 1. The disturbance signal $p(t)$ is

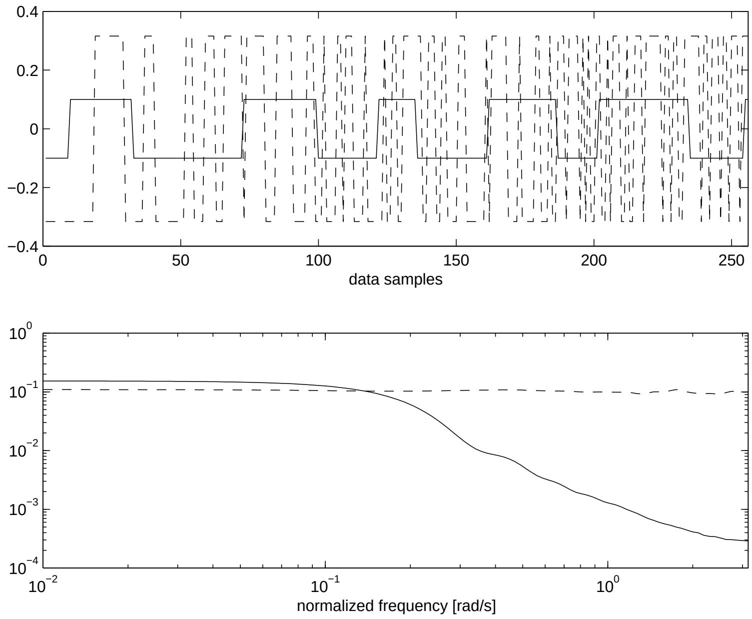

Fig. 1. Open-loop validation. Top: $u_{ol}(t)$ (−−) and $p(t)$ (—). Bottom: $\phi_{u_{ol}}(\omega)$ (−−) and $\phi_p(\omega)$ (—)

filtered by $1/A_o(z)$ and added to the output of the system. 256 data are measured, and the identification is performed with the same ARX(4,2,3) structure as G_0 after prefiltering these data by $\Delta(z)$. The numerical values attached to this open-loop validation experiment are displayed in Table 1, where $\frac{\int_0^\pi \phi_y^p(\omega)d\omega}{\int_0^\pi \phi_y^u(\omega)d\omega}$ represents the output noise-to-signal ratio ($\phi_y^u(\omega) = \left|G_0\left(e^{j\omega}\right)\right|^2 \sigma^2_{u_{ol}}$ is the spectrum of the part of the output due to the input, and $\phi_y^p(\omega) = \left|H_0\left(e^{j\omega}\right)\right|^2 \phi_p(\omega)$

is the spectrum of the part of the output due to the disturbances).

$\sigma^2_{u_{ol}}$	σ^2_p	$\sigma^2_{y_{ol}}$	$\frac{\int_0^\pi \phi_y^p(\omega)d\omega}{\int_0^\pi \phi_y^u(\omega)d\omega}$
0.1	0.01	0.8932	1.3102

Table 1
Open-loop validation

Using these settings, the identified model $G^{ol}_{mod} = G(z, \hat{\delta}_{ol})$ is given by:

$$G^{ol}_{mod} = G(z, \hat{\delta}_{ol}) = z^{-3} \frac{0.1052 + 0.1774z^{-1}}{1 - 1.997z^{-1} + 2.23z^{-2} - 1.876z^{-3} + 0.9039z^{-4}}. \quad (4)$$

The parameter vector $\hat{\delta}_{ol}$ is the vector made up of the two numerator coefficients followed by the four denominator coefficients. The covariance matrix of this estimated parameter vector is

$$P^{ol}_{\delta} = 0.001 \times \begin{pmatrix} 0.2034 & -0.2970 & 0.2411 & -0.1150 & -0.0139 & -0.0027 \\ -0.2970 & 0.5735 & -0.5136 & 0.2397 & 0.0119 & -0.0064 \\ 0.2411 & -0.5136 & 0.5725 & -0.2962 & -0.0130 & 0.0008 \\ -0.1150 & 0.2397 & -0.2962 & 0.2013 & 0.0094 & 0.0020 \\ -0.0139 & 0.0119 & -0.0130 & 0.0094 & 0.0392 & 0.0126 \\ -0.0027 & -0.0064 & 0.0008 & 0.0020 & 0.0126 & 0.0391 \end{pmatrix}.$$

The 95% uncertainty region $\mathcal{D}_{ol}$ around G^{ol}_{mod} can then be expressed as follows:

$$\mathcal{D}_{ol} = \{G(z, \delta) \mid G(z, \delta) = \frac{Z_N \delta}{1 + Z_D \delta} \; with \; \delta \in U_{ol}\} \quad (5)$$

$$U_{ol} = \{\delta \in \mathbf{R}^{6 \times 1} \mid (\delta - \hat{\delta}_{ol})^T (P^{ol}_{\delta})^{-1} (\delta - \hat{\delta}_{ol}) < 12.6\}, \quad (6)$$

where

$$Z_N(z) = \begin{pmatrix} 0 \; 0 \; 0 \; 0 \; z^{-3} \; z^{-4} \end{pmatrix}$$

$$Z_D(z) = \begin{pmatrix} z^{-1} \; z^{-2} \; z^{-3} \; z^{-4} \; 0 \; 0 \end{pmatrix}.$$

The size χ of the ellipsoid U_{ol} is equal to 12.6 since $Pr(\chi^2(6) < 12.6) = 0.95$. This uncertainty region $\mathcal{D}_{ol}$ does actually contain the true system since we have

$$\left(\delta_0 - \hat{\delta}_{ol}\right)^T (P^{ol}_{\delta})^{-1} \left(\delta_0 - \hat{\delta}_{ol}\right) = 5.7555 < 12.6,$$

where $\delta_0 = \begin{pmatrix} -1.99185 \; 2.20265 \; -1.84083 \; 0.89413 \; 0.10276 \; 0.18123 \end{pmatrix}^T$ denotes the parameter vector of the true system:

$$G_0 = \frac{Z_N \delta_0}{1 + Z_D \delta_0}. \tag{7}$$

2.3 Closed-loop validation experiment

In order to perform a validation experiment in closed loop, we need to connect a controller K in feedback with G_0. The controller K chosen here is the controller obtained by Landau et al. using a combined pole placement/sensitivity function shaping method (Landau *et al.*, 1995*a*). Its feedback part is described by

$$K(z) = \frac{0.401602 - 1.079378z^{-1} + 0.284895z^{-2} + 1.358224z^{-3}}{1 - 1.031142z^{-1} - 0.995182z^{-2} + 0.752086z^{-3}} \\ \frac{-0.986549z^{-4} - 0.271961z^{-5} + 0.306937z^{-6}}{+0.710744z^{-4} - 0.242297z^{-5} - 0.194209z^{-6}}. \tag{8}$$

It also has a feedforward part that we shall not consider here.

The closed-loop system $[K \; G_0]$ is excited by means of a reference signal $r(t)$ injected at the input of G_0, while the disturbance $p(t)$ is the same as in the previous subsection. In order to establish a fair comparison with the results obtained in open-loop validation, $r(t)$ is a PRBS with a variance $\sigma_r^2 = 0.5541$ that is chosen such that the total output variance is the same in closed loop as in open loop: $\sigma_{y_{cl}}^2 = \sigma_{y_{ol}}^2 = 0.8932$. Other choices could have been made, but from an industrial user's point of view, it is usually the total output variance that matters. Other numerical values attached to this closed-loop validation experiment are displayed in Table 2. $\phi_y^r(\omega)$ and $\phi_y^p(\omega)$ are the part of the output spectrum due to the reference and the disturbance, respectively; thus $\frac{\int_0^\pi \phi_y^p(\omega)d\omega}{\int_0^\pi \phi_y^r(\omega)d\omega}$ represents the output noise-to-signal ratio. Observe that in closed loop identification, the disturbance contribution in the input signal does not contribute to the estimation of the plant model G_0 (Gevers *et al.*, 2001).

σ_r^2	$\sigma_{u_{cl}}^2$	σ_p^2	$\sigma_{y_{cl}}^2$	$\frac{\int_0^\pi \phi_y^p(\omega)d\omega}{\int_0^\pi \phi_y^r(\omega)d\omega}$
0.5541	0.6475	0.01	0.8932	0.2389

Table 2
Closed-loop validation

The controller K has unstable poles and nonminimum phase zeros. Therefore, the indirect closed-loop identification method *cannot* be used for validation, as it would deliver a model G^{cl}_{mod} that would be destabilized by K (see (Codrons *et al.*, 1999), (Codrons, 2000)). We therefore use a direct approach to perform the closed-loop identification. Once again, 256 data samples $\{y_{cl}(t),\, u_{cl}(t) : t = 1...256\}$ are measured, and a model G^{cl}_{mod} with the same ARX(4,2,3) structure as G_0 is identified after prefiltering these data by $\Delta(z)$.

The model identified under those closed loop experimental conditions is:

$$G^{cl}_{mod} = G(z, \hat{\delta}_{cl}) = z^{-3} \frac{0.1016 + 0.1782z^{-1}}{1 - 1.986z^{-1} + 2.187z^{-2} - 1.824z^{-3} + 0.8897z^{-4}}.$$

The estimated covariance matrix of the identified parameter vector is:

$$P^{cl}_{\delta} = 10^{-3} \times \begin{pmatrix} 0.0840 & -0.1166 & 0.1024 & -0.0532 & -0.0062 & -0.0027 \\ -0.1166 & 0.2145 & -0.1966 & 0.1009 & 0.0057 & 0.0008 \\ 0.1024 & -0.1966 & 0.2184 & -0.1197 & -0.0074 & -0.0041 \\ -0.0532 & 0.1009 & -0.1197 & 0.0853 & 0.0063 & 0.0037 \\ -0.0062 & 0.0057 & -0.0074 & 0.0063 & 0.0064 & 0.0021 \\ -0.0027 & 0.0008 & -0.0041 & 0.0037 & 0.0021 & 0.0061 \end{pmatrix}.$$

The 95% uncertainty region $\mathcal{D}_{cl}$ around $G^{cl}_{mod} = G(z, \hat{\delta}_{cl})$ can then be expressed as follows:

$$\mathcal{D}_{cl} = \{G(z,\delta) \mid G(z,\delta) = \frac{Z_N \delta}{1 + Z_D \delta} \; with \; \delta \in U_{cl}\}, \tag{9}$$

$$U_{cl} = \{\delta \in \mathbf{R}^{6\times 1} \mid (\delta - \hat{\delta}_{cl})^T (P^{cl}_{\delta})^{-1} (\delta - \hat{\delta}_{cl}) < 12.6\}, \tag{10}$$

where Z_N and Z_D are defined in (5). This uncertainty region $\mathcal{D}_{cl}$ contains the true system since we have

$$\left(\delta_0 - \hat{\delta}_{cl}\right)^T (P^{cl}_{\delta})^{-1} \left(\delta_0 - \hat{\delta}_{cl}\right) = 4.7050 < 12.6$$

where δ_0 denotes the parameter vector of the true system.

2.4 Robust stability measure of $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$

In the previous section, we have performed two different validation experiments leading to two uncertainty regions ($\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$) with different nominal models (G^{ol}_{mod} and G^{cl}_{mod}). Each validation experiment has delivered a possible pair "model for control design-uncertainty region" i.e. $\{G^{ol}_{mod}\ \mathcal{D}_{ol}\}$ and $\{G^{cl}_{mod}\ \mathcal{D}_{cl}\}$. Let us first assess the quality of both pairs with respect to robustly stable control design. For this purpose, the results of Sections 5-6 in (Gevers *et al.*, 2002) are used in order to verify whether all models in $\mathcal{D}_{ol}$ (resp. $\mathcal{D}_{cl}$) are stabilized by a large set of controllers designed with the identified model G^{ol}_{mod} (resp. G^{cl}_{mod}). This is done by computing the worst case ν-gap $\delta_{WC}(G^{ol}_{mod}, \mathcal{D}_{ol})$ (resp. $\delta_{WC}(G^{cl}_{mod}, \mathcal{D}_{cl})$) and comparing it with the optimal stability margin $b_{opt}(G^{ol}_{mod})$ (resp. $b_{opt}(G^{cl}_{mod})$). The optimal stability margin can be computed using expression (19) in (Gevers *et al.*, 2002) and the worst case ν-gap can be derived from the LMI-based computation of the worst case chordal distances at each frequency. The worst case chordal distances $\kappa_{WC}(G^{ol}_{mod}(e^{j\omega}), \mathcal{D}_{ol})$ and $\kappa_{WC}(G^{cl}_{mod}(e^{j\omega}), \mathcal{D}_{cl})$ are represented in Figure 2 where they are compared with the actual chordal distance $\kappa(G^{cl}_{mod}(e^{j\omega}), G_0(e^{j\omega}))$ between the model G^{cl}_{mod} identified in closed-loop and the true system G_0. We have not represented $\kappa(G^{ol}_{mod}(e^{j\omega}), G_0(e^{j\omega}))$ in order to keep the figure sufficiently readable.

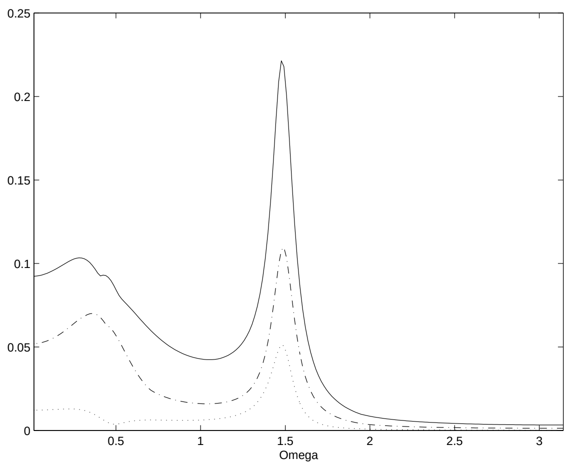

Fig. 2. $\kappa_{WC}(G^{ol}_{mod}(e^{j\omega}), \mathcal{D}_{ol})$ (solid), $\kappa_{WC}(G^{cl}_{mod}(e^{j\omega}), \mathcal{D}_{cl})$ (dashdot) and $\kappa(G^{cl}_{mod}(e^{j\omega}), G_0(e^{j\omega}))$ (dotted) at each frequency

Using Lemma 1 and expression (19) of (Gevers *et al.*, 2002), we obtain the following values for $\delta_{WC}(G^{ol}_{mod}, \mathcal{D}_{ol})$, $\delta_{WC}(G^{cl}_{mod}, \mathcal{D}_{cl})$, $b_{opt}(G^{ol}_{mod})$ and $b_{opt}(G^{cl}_{mod})$:

$$\delta_{WC}(G^{ol}_{mod}, \mathcal{D}_{ol}) = \max_{\omega} \kappa_{WC}(G^{ol}_{mod}(e^{j\omega}), \mathcal{D}_{ol}) = 0.2214 \qquad b_{opt}(G^{ol}_{mod}) = 0.4685 \tag{11}$$

$$\delta_{WC}(G^{cl}_{mod}, \mathcal{D}_{cl}) = \max_{\omega} \kappa_{WC}(G^{cl}_{mod}(e^{j\omega}), \mathcal{D}_{cl}) = 0.1097 \qquad b_{opt}(G^{cl}_{mod}) = 0.4650 \tag{12}$$

As $\delta_{WC}(G^{cl}_{mod}, \mathcal{D}_{cl})$ is much smaller than $b_{opt}(G^{cl}_{mod})$, we conclude that the set $\mathcal{C}_{\delta}(G^{cl}_{mod}, \mathcal{D}_{cl})$ of G^{cl}_{mod}-based controllers that are guaranteed by the ν-gap theory to robustly stabilize $\mathcal{D}_{cl}$, is relatively large: see sections 5.3 and 6.2 of (Gevers *et al.*, 2002). The difference between $\delta_{WC}(G^{ol}_{mod}, \mathcal{D}_{ol})$ and $b_{opt}(G^{ol}_{mod})$

is much smaller. Therefore, there is a strong incentive to give preference to the pair $\{G^{cl}_{mod}\ \mathcal{D}_{cl}\}$ for robust control design. Nevertheless, for the sake of illustration and comparison, we will also keep this pair $\{G^{ol}_{mod}\ \mathcal{D}_{ol}\}$ for further analysis.

2.5 Control design based on the identified model

The identified model G^{ol}_{mod} (resp. G^{cl}_{mod}) would normally be used in order to design a controller C^{ol} (resp. C^{cl}) such that the nominal closed loop system satisfies the specifications presented at the end of Section 2.1. However, for comparison purposes, we will consider here the same controller C for both models. This controller is the robust controller for the Landau benchmark that was obtained by Nordin and Gutman using Quantitative Feedback Theory (QFT) design (Nordin and Gutman, 1995):

$$\begin{aligned} C(z) = & \frac{0.0355 + 0.0181z^{-1}}{1 - z^{-1}} \times \frac{18.8379 - 43.4538z^{-1} + 26.4126z^{-2}}{1 + 0.6489z^{-1} + 0.1478z^{-2}} \\ & \times \frac{0.5626 - 0.7492z^{-1} + 0.3248z^{-2}}{1 - 1.4998z^{-1} + 0.6379z^{-2}} \times \frac{1.0461 + 0.5633z^{-2}}{1 + 0.4564z^{-1} + 0.1530z^{-2}} \\ & \times \frac{1.3571 - 1.0741z^{-1} + 0.4702z^{-2}}{1 - 0.6308z^{-1} + 0.3840z^{-2}}. \end{aligned}$$

This controller has not been designed from either G^{ol}_{mod} or G^{cl}_{mod}, but it satisfies all specifications with both models.

We now verify whether this controller satisfies these specifications with all plants in $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$, respectively (and therefore also with the true system G_0). Let us begin by the validation of C for stability.

2.6 Controller validation for stability

Following the procedure of Section 3 in (Gevers *et al.*, 2002), we build the dynamic vectors $M_{\mathcal{D}_{ol}}(e^{j\omega})$ and $M_{\mathcal{D}_{cl}}(e^{j\omega})$ corresponding to the candidate controller C and the uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$, respectively, and we compute their stability radius at each frequency according to Theorem 1 in (Gevers *et al.*, 2002). These stability radii are represented in Figure 3.

The maximum values of the stability radii are, respectively:

$$\max_{\omega} \mu(M_{\mathcal{D}_{ol}}(e^{j\omega})) = 0.3244$$

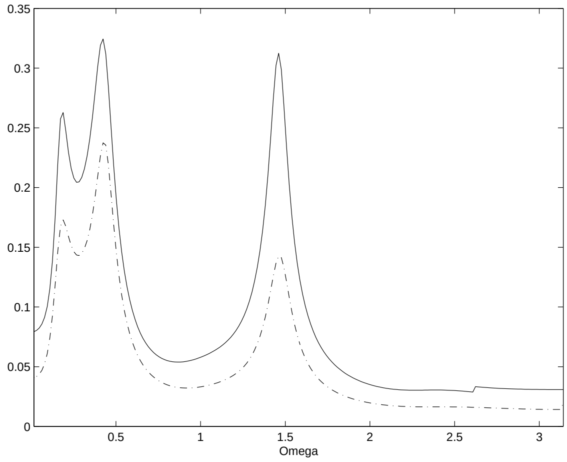

Fig. 3. $\mu\left(M_{\mathcal{D}_{ol}}(e^{j\omega})\right)$ (solid) and $\mu\left(M_{\mathcal{D}_{cl}}(e^{j\omega})\right)$ (dashdot) at each frequency

$$\max_{\omega}\ \mu(M_{\mathcal{D}_{cl}}(e^{j\omega})) = 0.2375$$

Since this maximum value is smaller than one in both cases, we may conclude that the controller C stabilizes all plants in both uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$. Consequently, we can also guarantee that the "to-be-validated" controller $C(z)$ stabilizes the true flexible transmission system G_0 with probability 95%. The first specification presented at the end of Section 2.1 (i.e. the stability of the achieved loop $[C\ G_0]$) is thus satisfied.

2.7 Controller validation for performance

The second requirement presented at the end of Section 2.1 was that the designed controller should ensure *a maximum value of less than* 6 *dB for the sensitivity function*. Since the true system is assumed unknown, we verify whether the controller C achieves these requirements with all systems in $\mathcal{D}_{ol}$ and/or $\mathcal{D}_{cl}$. For this purpose, we choose the following worst case performance criterion: the largest modulus of the sensitivity function T_{22} over all models in $\mathcal{D}$, denoted by $t_{\mathcal{D}}(\omega, T_{22})$. This worst case performance criterion can be computed using the LMI procedure presented in Theorem 2 of (Gevers *et al.*, 2002) using the following weights: $W_l = W_r = diag(0,1)$. Using this worst case performance criterion, the controller C is termed validated for performance if

$$\max_{\omega} t_{\mathcal{D}}(\omega, T_{22}) < 6\ dB.$$

We compute this criterion for the uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$ delivered by our two validation experiments. Figure 4 presents $t_{\mathcal{D}_{ol}}(\omega, T_{22})$, $t_{\mathcal{D}_{cl}}(\omega, T_{22})$, and compares them with the actual sensitivity $|T_{22}(G_0, C)|$. We observe that

$$\max_{\omega} t_{\mathcal{D}_{ol}}(\omega, T_{22}) = 5.97\ dB, \quad \max_{\omega} t_{\mathcal{D}_{cl}}(\omega, T_{22}) = 5.00\ dB < 6\ dB.$$

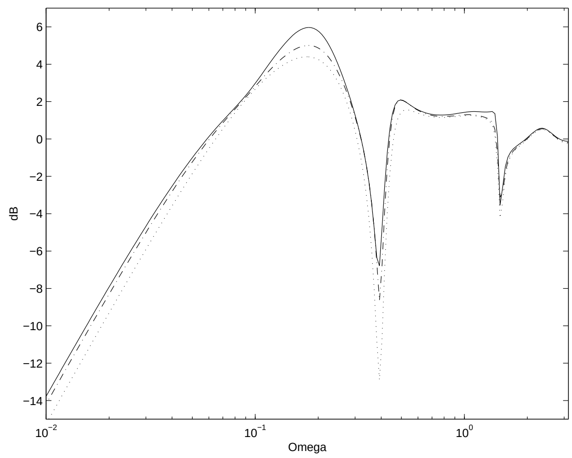

Fig. 4. $t_{\mathcal{D}_{ol}}(\omega, T_{22})$ (solid), $t_{\mathcal{D}_{cl}}(\omega, T_{22})$ (dashdot), and $|T_{22}(G_0, C)|$ (dotted) at each frequency

This means that the open-loop validation procedure nearly leads to rejection of the controller, while the closed-loop validation procedure leads to its acceptance.

With the controller validation procedures for stability and for performance, we have thus been able to establish that the "model-based" controller C achieves the specifications presented at the end of Section 2.1 with the true system G_0 since it achieves these specifications with all systems in $\mathcal{D}_{cl}$. Furthermore, we have also shown that, whereas the controller C is clearly validated with the uncertainty set $\mathcal{D}_{cl}$ delivered by a closed loop PE identification experiment, it is nearly rejected when we apply our controller validation procedure with the uncertainty set $\mathcal{D}_{ol}$ delivered by open loop identification. This fact illustrates the important role played by the experimental conditions in our validation procedure.

3 Ferrosilicon production process

The first illustration was representative of a mechanical engineering control problem, in which there was no stochastic noise, and where the control objective was one of tracking and of rejection of step disturbances. In order to illustrate the generality of our validation theory, we now present an application that is representative of industrial process control applications, in which the control objective is one of reducing the effects of stochastic disturbances. In this second illustration, we will assume that the model G_{mod} for control design has been given a priori.

The plant model and the controllers used in this simulation example are taken from a paper by Ingason and Jonsson (Ingason and Jonsson, 1998). Ferrosilicon is a two-phase mixture of the chemical compound $FeSi_2$ and the element silicon. The balance between silicon and iron is regulated around 76% of the total weight in silicon, 22% in iron and 2% in aluminium by adjusting the input of raw materials to the furnace. Those are charged batchwise to the top of the furnace, each batch consisting of a fixed amount of quartz (SiO_2) and a variable quantity of coal/coke (C) and iron oxide (Fe_2O_3). The quantity of coal/coke which is burned in the furnace does not influence the silicon ratio in the mixture, hence the control input is the amount of iron oxide.

The authors of (Ingason and Jonsson, 1998) have obtained the following ARX model for the process:

$$y(t) + ay(t-1) = bu(t-1) + d + e(t) \tag{13}$$

where the sampling period is one day, $y(t)$ is the percentage of silicon in the mixture that must be regulated around 76%, $u(t)$ is the quantity of iron oxide in the raw materials (expressed in kilogrammes), d is a constant and $e(t)$ is a stochastic disturbance. The nominal values of the parameters and their standard deviations are:

$$\begin{aligned} a &= -0.44,\ b = -0.0028,\ d = 46.1, \\ \sigma_a &= 0.07, \quad \sigma_b = 0.001, \quad \sigma_d = 5.6. \end{aligned} \tag{14}$$

Here, for the sake of illustrating our theory, we make the assumption that the true system is [2]

$$G_0(z) = \frac{-0.0032z^{-1}}{1-0.34z^{-1}} = \frac{b_0 z^{-1}}{1+a_0 z^{-1}},$$
$$H_0(z) = \frac{1}{1-0.34z^{-1}} = \frac{1}{1+a_0 z^{-1}}, \qquad d_0 = 44.$$

The nominal model chosen for control design is the one obtained by Ingason and Jonsson (Ingason and Jonsson, 1998):

$$G_{mod}(z) = \frac{-0.0028z^{-1}}{1-0.44z^{-1}} = \frac{bz^{-1}}{1+az^{-1}},$$

$$H_{mod}(z) = \frac{1}{1-0.44z^{-1}} = \frac{1}{1+az^{-1}}, \qquad d = 46.1,$$

[2] Since we have no access to the real plant, we have randomly selected one system in the two-standard-deviation confidence interval around the nominal model and used it as a surrogate true system.

This model G_{mod} was used by the authors of (Ingason and Jonsson, 1998) to compute a GPC controller. The control law that minimizes the cost function

$$J_u = E\left[\sum_{j=1}^{2}(y(t+j)-r(t+j))^2 + \sum_{j=1}^{2}\lambda\left(\Delta u(t+j-1)\right)^2\right] \tag{15}$$

with $\Delta(z) = 1 - z^{-1}$, is given by

$$u(t) = \begin{bmatrix}1 & 0\end{bmatrix}\left(H^T H + F^T \Lambda F\right)^{-1}\left(H^T\left(w(t)-v(t)\right) - F^T \Lambda g(t)\right) \tag{16}$$

where

$$H = \begin{bmatrix} b & 0 \\ -ab & b \end{bmatrix}, \tag{17}$$

$$F = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \tag{18}$$

$$v(t) = \begin{bmatrix} -ay(t)+d \\ a^2y(t)-ad+d \end{bmatrix}, \tag{19}$$

$$w(t) = \begin{bmatrix} r(t) & r(t+1) \end{bmatrix}^T, \tag{20}$$

$$g(t) = \begin{bmatrix} u(t-1) & 0 \end{bmatrix}^T, \tag{21}$$

$$\Lambda = \lambda I. \tag{22}$$

λ is a tuning parameter. The resulting controller, $C_\lambda(z)$, is made up of three parts:

$$u(t) = C_\lambda(z)\begin{pmatrix} r(t) \\ -y(t) \\ d \end{pmatrix} = \begin{pmatrix} C_\lambda^r(z) & C_\lambda^y(z) & C_\lambda^d(z) \end{pmatrix}\begin{pmatrix} r(t) \\ -y(t) \\ d \end{pmatrix} \tag{23}$$

where

$$C_\lambda^r(z) = \frac{b^3+2b\lambda-ab\lambda}{(b^4+3b^2\lambda+a^2b^2\lambda+\lambda^2-2ab^2\lambda)-(b^2\lambda+\lambda^2)z^{-1}},$$

$$C_\lambda^y(z) = -\frac{ab^3+ab\lambda-a^2b\lambda+a^3b\lambda}{(b^4+3b^2\lambda+a^2b^2\lambda+\lambda^2-2ab^2\lambda)-(b^2\lambda+\lambda^2)z^{-1}},$$

$$C_\lambda^d(z) = -\frac{b^3+b\lambda+b\lambda(1-a)^2}{(b^4+3b^2\lambda+a^2b^2\lambda+\lambda^2-2ab^2\lambda)-(b^2\lambda+\lambda^2)z^{-1}}$$

The controller $C_\lambda^d(z)$ aims at rejecting the constant disturbance d. The feedback controller $C_\lambda^y(z)$ is the only part that has an impact on closed loop stability. The reference signal $r(t)$ is generally constant and given by $r(t) = 76$.

Our objective is to use the validation tools developed in (Gevers *et al.*, 2002) to check whether the controller $C_\lambda(z)$, with $\lambda = 0.0007$, that is based on the model G_{mod}, can be applied *with confidence* to the true system G_0, that is to say with the assurance that the closed loop $[C_{\lambda=0.0007}\ G_0]$ will satisfy the following specifications [3] :

- stability of the loop $[C_{\lambda=0.0007}\ G_0]$
- rejection of the stochastic noise $v(t) = H_0 e(t)$.

To check this, we have used the surrogate true plant model (G_0, H_0) as a simulator on which validation experiments have been performed. As with the first illustration, we have performed two validation experiments: one in open loop and one in closed loop.

3.2 Open-loop validation experiment

The "true plant" model $(G_0,\ H_0)$ was excited with $u(t)$ chosen as a PRBS with variance $\sigma^2_{u_{OL}} = 20$, which is the maximum input variance admissible for this process (Ingason and Jonsson, 1998). The noise $e(t)$ was chosen as a Gaussian white noise sequence with variance $\sigma_e^2 = 0.078$, which corresponds to the noise acting on the real process, as shown by experiments made by the authors of (Ingason and Jonsson, 1998). The variance of the output was then $\sigma^2_{y_{ol}} = 0.0884$. Recall that the validation experiment, i.e. the construction of a validated uncertainty set $\mathcal{D}_{ol}$, consists of performing a PE identification using a full order model structure. Therefore, 300 input-output data samples were collected, corresponding approximately to one year of measurements. These data were used to identify an ARX model with exact structure

$$G(z,\ \delta_{ol}) = \frac{\delta_2 z^{-1}}{1+\delta_1 z^{-1}}, \qquad H(z,\ \delta_{ol}) = \frac{1}{1+\delta_1 z^{-1}}. \tag{24}$$

Recall that this model is only a vehicle for the construction of an uncertainty region $\mathcal{D}_{ol}$ since the model G_{mod} used for control design has been given a priori.

[3] The controller of course achieves these specifications with the nominal model G_{mod}.

We found

$$\hat{\delta}_{ol} = \begin{pmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \end{pmatrix} = \begin{pmatrix} -0.3763 \\ -0.0073 \end{pmatrix}, \qquad P_\delta^{ol} = \begin{pmatrix} 2.8131 \times 10^{-3} & -1.2784 \times 10^{-5} \\ -1.2784 \times 10^{-5} & 1.4887 \times 10^{-5} \end{pmatrix}, \tag{25}$$

The 95% uncertainty region $\mathcal{D}_{ol}$ around $G(z, \hat{\delta}_{ol})$ is then given by

$$\mathcal{D}_{ol} = \{G(z,\delta) \mid G(z,\delta) = \frac{Z_N\delta}{1+Z_D\delta} \; with \; \delta \in U_{ol}\}$$

$$U_{ol} = \{\delta \in \mathbf{R}^{2\times 1} \mid (\delta - \hat{\delta}_{ol})^T (P_\delta^{ol})^{-1} (\delta - \hat{\delta}_{ol}) < 5.99\},$$

where

$$Z_N(z) = \begin{pmatrix} 0 & z^{-1} \end{pmatrix} \quad and \quad Z_D(z) = \begin{pmatrix} z^{-1} & 0 \end{pmatrix}.$$

The size χ of the ellipsoid U_{ol} is equal to 5.99 since $Pr(\chi^2(2) < 5.99) = 0.95$. The validated uncertainty region $\mathcal{D}_{ol}$ contains both the "unknown" surrogate true system G_0 and the model G_{mod} used for controller design.

3.3 Closed-loop validation experiment

The closed-loop validation was performed with a sub-optimal GPC controller obtained by setting $\lambda = 0.001$ in (23). We added a PRBS signal to the constant reference $r(t) = 76$, with variance $\sigma_r^2 = 0.014$ so as to obtain the same input variance as in the open loop experiment, i.e. $\sigma_{u_{cl}}^2 = 20$. The white noise $e(t)$ had the same properties as in open-loop validation. With these settings, the output variance was $\sigma_{y_{cl}}^2 = 0.0880$, very close to that of the open-loop experiment. Again, 300 input-output data samples were collected and used to identify an ARX model with the same structure as in open-loop validation (24), using a direct prediction error method. We found

$$\hat{\delta}_{cl} = \begin{pmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \end{pmatrix} = \begin{pmatrix} -0.3575 \\ -0.0067 \end{pmatrix}, \qquad P_\delta^{cl} = \begin{pmatrix} 2.8323 \times 10^{-3} & -8.7845 \times 10^{-6} \\ -8.7845 \times 10^{-6} & 6.2416 \times 10^{-6} \end{pmatrix}. \tag{26}$$

We then designed a 95% uncertainty region $\mathcal{D}_{cl}$ around $G(z, \hat{\delta}_{cl})$ defined by:

$$\mathcal{D}_{cl} = \{G(z,\delta) \mid G(z,\delta) = \frac{Z_N\delta}{1+Z_D\delta} \; with \; \delta \in U_{cl}\},$$

$$U_{cl} = \{\delta \in \mathbf{R}^{2\times 1} \mid (\delta - \hat{\delta}_{cl})^T (P_\delta^{cl})^{-1} (\delta - \hat{\delta}_{cl}) < 5.99\},$$

with the same Z_N and Z_D as in $\mathcal{D}_{ol}$. As with $\mathcal{D}_{ol}$, this uncertainty region $\mathcal{D}_{cl}$ contains both the true system G_0 and the model G_{mod}.

3.4 Comparison of $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$

The worst case ν-gap is now used to compare the two uncertainty sets deduced from the two validation experiments. For this purpose, we first compute the worst case chordal distances at each frequency for $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$ using the LMI tools developed in Section 5 of the companion paper (Gevers *et al.*, 2002). According to Lemma 1 of that paper, and since G_{mod} lies in both uncertainty sets, we can derive the worst case Vinnicombe distances from the worst chordal distances as follows:

$$\delta_{WC}(G_{mod}, \mathcal{D}_{ol}) = \max_{\omega} \kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}_{ol}) = 0.0225$$

$$\delta_{WC}(G_{mod}, \mathcal{D}_{cl}) = \max_{\omega} \kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}_{cl}) = 0.0156$$

Observe that the worst case gap is again smaller for the set validated under closed loop experimental conditions than it is for the set validated in open loop. Since the optimal stability margin $b_{opt}(G_{mod})$ is equal to 0.99, the sets $\mathcal{C}_\delta(G_{mod}, \mathcal{D}_{ol})$ and $\mathcal{C}_\delta(G_{mod}, \mathcal{D}_{cl})$ of controllers that robustly stabilize $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$, respectively, are both large. Indeed, the worst case ν-gaps $\delta_{WC}(G_{mod}, \mathcal{D}_{ol})$ and $\delta_{WC}(G_{mod}, \mathcal{D}_{cl})$ are very small with respect to $b_{opt}(G_{mod})$. Consequently, both uncertainty sets are well tuned for robustly stable controller design based on G_{mod}. We can therefore keep both uncertainty sets for further analysis and controller valiation procedures.

3.5 Controller validation for stability

We now examine whether the controller $C_{\lambda=0.0007}$ stabilizes all models in $\mathcal{D}_{ol}$ and/or $\mathcal{D}_{cl}$, using the robust stability analysis tools developed for such PE uncertainty sets in (Gevers *et al.*, 2002).

3.5.1 Test based on sufficient condition

We first consider the sufficient robust stability condition based on the worst case ν-gap , in order to show that this condition can be conservative with respect to the necessary and sufficient condition developed in Theorem 1 of (Gevers *et al.*, 2002).

The nominal stability margin achieved by the controller $C_{\lambda=0.0007}$ with the nominal model G_{mod} is very small: $b_{G_{mod}C^y_{\lambda=0.0007}} = 0.0169$. We conclude that the controller $C_{\lambda=0.0007}$ lies in $\mathcal{C}_\delta(G_{mod}, \mathcal{D}_{cl})$ but not in $\mathcal{C}_\delta(G_{mod}, \mathcal{D}_{ol})$ since

$$\delta_{WC}\left(G_{mod}, \mathcal{D}_{ol}\right) > b_{G_{mod}C_{\lambda=0.0007}} = 0.0169 > \delta_{WC}\left(G_{mod}, \mathcal{D}_{cl}\right). \qquad (27)$$

Therefore, from this sufficiency test, we can conclude that $C_{\lambda=0.0007}$ stabilizes all plants in the set $\mathcal{D}_{cl}$, but we cannot conclude that it stabilizes all plants in $\mathcal{D}_{ol}$. To ascertain this, we need to consider the necessary and sufficient condition for robust stability.

3.5.2 Test based on necessary and sufficient condition

We first check whether $C_{\lambda=0.0007}$ stabilizes the nominal models $G(z, \hat{\delta}_{ol})$ and $G(z, \hat{\delta}_{cl})$. Since this is indeed the case, we build the dynamic vectors $M_{\mathcal{D}_{ol}}(e^{j\omega})$ and $M_{\mathcal{D}_{cl}}(e^{j\omega})$ corresponding to the candidate controller $C_{\lambda=0.0007}$, and we compute their stability radii according to Theorem 1 of (Gevers *et al.*, 2002). Their maximum values are, respectively,

$$\max_{\omega} \mu\left(M_{\mathcal{D}_{ol}}\left(e^{j\omega}\right)\right) = 0.6572 < 1, \quad \max_{\omega} \mu\left(M_{\mathcal{D}_{cl}}\left(e^{j\omega}\right)\right) = 0.2111 < 1, \quad (28)$$

Since these two values are smaller than one, $C_{\lambda=0.0007}$ stabilizes all systems in both uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$. This is remarkable, given that $C_{\lambda=0.0007}$ has a very small nominal stability margin with G_{mod}. This *quantitative* result confirms our earlier *qualitative* observation that both uncertainty sets are well tuned for robustly stable controller design based on G_{mod}, even though that qualitative observation is based on a sufficient condition that would have invalidated the particular controller $C_{\lambda=0.0007}$ when $\mathcal{D}_{ol}$ is considered (see (27)).

We also observe that, just as with the first application, the stability radius is much smaller for the set $\mathcal{D}_{cl}$ obtained by closed loop validation than for the set $\mathcal{D}_{ol}$ obtained by open loop validation. Finally we conclude from these stability tests that the "to-be-validated" controller $C_{\lambda=0.0007}$ is guaranteed to stabilize the surrogate G_0 of the true ferrosilicon production process.

3.6 Controller validation for performance

The second specification presented at the end of Section 3.1 is to reject the noise $v(t) = H_0(z)e(t)$, which is essentially located at low frequencies ($H_0(e^{j\omega})$ is a first order low-pass filter; see Figure 5). A performance specification in the frequency domain is therefore that the sensitivity function $T_{22}(G_0, C^y_{\lambda=0.0007}(z)) = 1/(1 + G_0 C^y_{\lambda=0.0007}(z))$ be low at low frequencies in order to attenuate $v(t)$. We thus define the worst-case performance criterion as

$$t_{\mathcal{D}}(\omega, T_{22}) = \max_{G(e^{j\omega},\delta)\in\mathcal{D}} \left| \frac{1}{1 + G(z,\delta)C_{\lambda=0.0007}(z)} \right| \quad (29)$$

This worst case performance criterion can be computed using the LMI procedure presented in Theorem 2 of (Gevers *et al.*, 2002). We will call the controller $C_{\lambda=0.0007}(z)$ validated if $t_{\mathcal{D}}(\omega, T_{22})$ is high-pass with $\max_\omega t_{\mathcal{D}}(\omega, T_{22}) < 1\ dB$.

The Bode diagrams of the worst-case and achieved sensitivity functions are depicted in Figure 5. Clearly, the controller is validated by the closed-loop val-

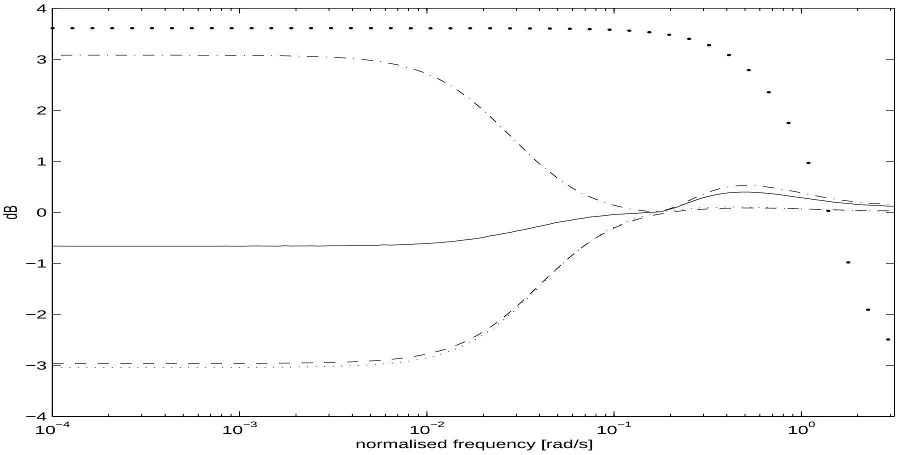

Fig. 5. Open-loop and closed-loop controller validation for performance: $t_{\mathcal{D}_{ol}}(\omega, T_{22})$ $(-\cdot)$, $t_{\mathcal{D}_{cl}}(\omega, T_{22})$ (—), $|T_{22}(G_0, C_{\lambda=0.0007})|$ $(--)$, $|T_{22}(G_{mod}, C_{\lambda=0.0007})|$ $(\cdots)$ and $|H_0|$ $(\cdot)$

idation experiment yielding $\mathcal{D}_{cl}$ but not by the open-loop experiment yielding $\mathcal{D}_{ol}$.

The main conclusion we can derive from this performance test is that the controller $C_{\lambda=0.0007}$ will achieve the desired performance (i.e. sufficiently decrease the output variance) when applied to G_0. We have indeed proved that, for the uncertainty set $\mathcal{D}_{cl}$ containing G_0, the worst case modulus of the sensitivity function is a high pass filter with a reasonably small reasonance peak allowing rejection of the noise $v(t)$. This application shows once again the important role played by the experimental conditions. Indeed, the controller C designed from G_{mod} is validated with the uncertainty set $\mathcal{D}_{cl}$ delivered by a closed loop PE identification experiment, but is invalidated with the uncertainty set $\mathcal{D}_{ol}$ delivered by an open loop PE identification experiment.

Remark. Even though the uncertainty region $\mathcal{D}_{ol}$ is well tuned with respect to robustly stable controller design with G_{mod} (i.e. it has a large set of stabilizing controllers), our analysis shows that the worst case performance achieved by the controller $C_{\lambda=0.0007}$ with all plants in $\mathcal{D}_{ol}$ is really bad. This is a consequence of the fact that the worst case ν-gap is only an indicator of robust stability and not an indicator of robust performance. This observation has recently led us to extend our results to an indicator of robust performance for the uncertainty set $\mathcal{D}$ delivered by PE identification (Bombois *et al.*, 2002).

4 Conclusions

Using two control design applications that are representative of a noise-free mechanical tracking problem and of an industrial problem with a noise rejection objective, respectively, we have illustrated the various model and controller validation results developed in the companion paper (Gevers *et al.*, 2002). In doing so, we have not only illustrated the relevance and practical usefulness of our prediction error framework for model and controller validation, but we have also highlighted the important role of the experimental conditions under which the validation experiments are performed.

Acknowledgements

The authors acknowledge the Belgian Programme on Inter-university Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. B.D.O. Anderson also acknowledges funding of this research by the US Army Research Office, Far East, the Office of Naval Research, Washington. The scientific responsibility rests with its authors.

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