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IDENTIFICATION FOR CONTROL

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Abstract. We present a conceptual characterization of the various ways of addressing the problem of identification for control. This leads us to distinguish between a dual control approach, an optimal experiment design approach and a robust control approach. The connections and distinctions between these three viewpoints are discussed, and recent results for each approach are briefly presented.

Key Words. Identification; identification for control; optimal experiment design.

1. INTRODUCTION

Few people would object to the assertion that control engineers have been among the key players in the development of identification theory. And yet, it is fair to say that for much of the seventies and eighties - the golden age of the development of identification theory as a part of science - only scant attention has been given to the control objective in the design of identification methods.

The reason for this is that for a long time identification has been viewed as a search for 'the true system' rather than as a methodology whose aim is to produce an 'approximation of reality'. Thus, the focus of attention during most of the seventies was to produce identification methods that could be shown to converge to the 'true system' under the assumption that this 'true system' was in the model set. In the early eighties, the effort shifted from the concept of identifying the true system to that of 'approximating' the true system and of characterizing this approximation in terms of bias and variance of the transfer function estimates. The first results date back to the mid-eighties.

Once it is recognized that identification is essentially a methodology for the construction of approximate models, it makes sense to attempt tuning the identification towards the objective for which the model is to be used. Thus, the concept of *identification design* becomes central, namely the choices to be made for the experimental conditions, the model set, the data filters, the criterion, etc... One of the main contributions of L. Ljung's book (Ljung, 1987) was to introduce this engineering view of identification and to lay down some foundations for the formal design of goal-oriented identification. However, the contributions to control-oriented identification design were still very modest in 1987.

At the same time, it is widely recognized by process control engineers that model-based control design often achieves remarkably good performance on fairly complex processes with models that can at best be considered as very crude approximations of the physical system. This implies that, for control design purposes, it often suffices to work with models that may have large errors, provided they possess with great accuracy some features that are essential for the design of high performance controllers.

To make our message less opaque, let us focus on linear time-invariant systems. It is then reasonable to expect that the achievement of good closed loop performance probably allows for large model errors in some frequency bands but requires high precision in other frequency bands. The research on *Identification for Control* aims at replacing such loose statements by analysis, and at producing identification design guidelines for the case where the goal of the identification is to design model-based controllers.

The study of the interactions between identification and control design can be performed at various levels of generality and idealisation. First one should examine whether a model is really necessary for control design, or whether one cannot obtain better performance by the direct tuning of controller parameters towards the minimization of a closed loop performance criterion. We shall come back to this question in the concluding section of this paper. Assuming now that one takes

a model-based approach to controller design, then the problem of identification for control can be addressed at the following levels.

1. At the most general (or ideal) level, one can pose the problem as an optimal control problem in which the learning of the unknown parameters is included in the control problem formulation. Thus, at each sampling time, the control variable is computed that minimizes a performance criterion which explicitly incorporates the information about parameter uncertainty. Whether these parameters are model parameters or controller parameters does not really matter: the optimal controller has the dual - and conflicting - task of learning the unknown parameters and tracking (or regulating) the output variables. This is the area of *dual control*, in which identification (or, more precisely, parameter estimation) and control design are posed as a *combined problem*. It is an immensely difficult subject for which few results are available. We will explain the basic concepts of dual control and present some recent results in Section 3. We should add that all available results are limited to the case where full-order models are used, i.e. where 'the system is in the model set'.
2. At the next level of idealisation, one can consider the situation where the identification is designed in such a way that the ensuing model-based controller performs as close as possible to the 'ideal controller' on the actual closed loop system. By 'ideal controller' is meant the controller that would result from the model-based control design criterion if the true system were known. Thus, one compares the performance on the actual plant of the identified controller and of the ideal controller. This problem formulation is often referred to as *optimal experiment design*. It addresses the problem of *identification for control* rather than the more ideal combined identification and control design problem addressed by dual control. A main drawback of this approach is that, as with all optimal experiment design results, the solution depends on the unknown system. However, it can deliver qualitative guidelines, and, more recently, it has been the basis of a feasible suboptimal scheme for certain control designs. We will present some recent results in this direction in Section 4. All available results on optimal experiment design are again limited to the case where the 'system is in the model set'.
3. In the previous approach, the identification design criterion is based on a comparison between the *optimal closed loop system* (= true system with its optimal controller) and the *achieved closed loop system* (= true system with the 'identified model'-based controller). What makes this design unrealistic is that the optimal loop depends on the unknown system. The next and more realistic approach, is to compare the *achieved closed loop system* with the *designed or nominal closed loop sytem* (= identified model with its model-based controller). This approach is in line with robust control design thinking. Of course, there is no guarantee with this approach that the performance achieved by the model-based designed controller will be anywhere near the optimal performance. On the other hand, if this nominal performance is judged satisfactory, and if the achieved performance can be made close to the nominal performance, then the achieved performance should be at least satisfactory. We shall see later that in this third approach the key feature is to match the identification to the control design criterion in such a way that the control design and the identification design aim at the same global objective, and that the achieved and design loops become 'close' in a sense that will have to be made precise.[1] In this approach, it is not necessary to restrict oneself to the case where 'the system is in the model set'. In fact, most of the available results focus more on the *bias error* (due to the fact that the system is not in the model set) than on the *variance error* (due to the fact that the data are noisy).

In this paper we have chosen to take a rather conceptual view on the problem of *identification for control*. Hence our attempt to present the fundamental approaches to this problem in terms of their conceptual differences as explained above: what are the global criteria that these various apporaches attempt to minimize? There are various other (perhaps more technical) subdivisions that one could have taken, along the following lines.

- What kind of noise and model uncertainty description (hard bounded, stochastic, etc) ?
- What kind of norm for the identification criterion (L_2, l_1, H_∞, or no norm at all as in set membership identification) ?
- What kind of control criterion (LQG, H_∞, l_1, pole placement) ?

Each of the three conceptual approaches to *identification for control* enunciated above can be phrased in many different variants, depending on the uncertainty models and the norms used. One reason for adopting the conceptual subdivision is to bring some fresh way of looking at the prob-

[1] Or, as one would say in French, 'that will have to be precised'.

lem. The other is that the more technical comparisons have already been handled in several surveys, tutorials and plenaries in the last few years: see (Gevers, 1991; Gevers, 1993; Bitmead, 1993; Kosut, 1993; Van den Hof and Schrama, 1994; Mäkilä et al. 1994).

Thus, we shall not attempt to be exhaustive in this paper, or to survey the various approaches to identification for control. We therefore make no apologies to the hundreds of authors in this lively field whom we have omitted to quote. Instead, we shall try to highlight some of the key issues and present some recent results in each of the three approaches mentioned above. We shall also, all through the paper, develop our ideas in the context of stochastic Least Squares prediction error identification.

In Section 2 we first recall the fundamentals of Least Squares prediction error identification. In Section 3 we present the concepts of dual control, as well as some recent results, illustrating the superiority of dual control over the separate designs of identification followed by Certainty Equivalence control. Section 4 presents the key ideas of optimal experiment design for control. In Section 5 we examine how the identification criterion should be determined by the control design criterion so that the achieved and design loops become 'close', and we present some thoughts on direct optimal controller parameter tuning.

2. IDENTIFICATION IN OPEN AND CLOSED LOOP

We consider that the task is to design a controller for some "true" linear time-invariant scalar system described by

$$\mathcal{S} : y_t = P(q)u_t + H(q)e_t \tag{1}$$

where $P(q)$ and $H(q)$ are scalar rational transfer function operators, with $H(q)$ normalized such that $\lim_{|q|\to\infty} H(q) = 1$. Here q^{-1} is the delay operator ($q^{-1}u_t = u_{t-1}$), u_t is the control input signal, y_t is the observed output signal, e_t is white noise of zero mean and variance σ^2, and $v_t \stackrel{\Delta}{=} H(q)e_t$ is the noise acting on y_t.

A controller is to be designed on the basis of a model of the plant identified using a finite set of N input and output data $\{y_t, u_t, t = 1, 2, \ldots, N\}$ collected on the plant. A parametrized model set $\mathcal{M} = \{M(\theta) : \theta \in D_\theta \subset \mathbb{R}^d\}$ is used, where D_θ is a set of admissible values and $M(\theta)$ is described by:

$$M(\theta) : y_t = P(q,\theta)u_t + H(q,\theta)e_t. \tag{2}$$

If there exists a $\theta_0 \in D_\theta$ such that $P(q) = P(q,\theta_0)$, $H(q) = H(q,\theta_0)$, then we say that 'the system is in the model set': $\mathcal{S} \in \mathcal{M}$.

The data collection can be done in open loop or in closed loop. In the case of closed loop identification, we denote by $C_{id}(q)$ the controller that operates during identification:

$$u_t = C_{id}(q)[r_t - y_t], \tag{3}$$

where r_t is the reference excitation signal used during identification.

The Least Squares prediction error method applied to N input-output data delivers an estimate $\hat{\theta}_N$ of θ:

$$\hat{\theta}_N = \arg\min_{\theta\in D_\theta} V_N(\theta) \tag{4}$$

where

$$V_N(\theta) = \sum_{t=1}^{N} [\epsilon_t^f(\theta)]^2 \tag{5}$$

$$\epsilon_t^f(\theta) = D(q)\epsilon_t(\theta) = \frac{D(q)}{H(q,\theta)}[y_t - P(q,\theta)u_t]. \tag{6}$$

Here $\epsilon_t(\theta) \stackrel{\Delta}{=} y_t - \hat{y}_{t|t-1}(\theta)$ is the one-step-ahead prediction error for the model $M(\theta)$, while $D(q)$ is a data filter whose role is to shape the least squares fit towards the objective of the identification. In turn, this produces a model:

$$\hat{P}_N = P(e^{j\omega}, \hat{\theta}_N), \quad \hat{H}_N = H(e^{j\omega}, \hat{\theta}_N). \tag{7}$$

We shall sometimes use the vector notation

$$\hat{T}_N = [\hat{P}_N \quad \hat{H}_N]^T \text{ and } T = [P \quad H]^T. \tag{8}$$

Under reasonable conditions on the data and the model structure (Ljung, 1987), $\hat{\theta}_N$ converges as $N \to \infty$ to

$$\theta^* = \arg\min_{\theta\in D_\theta} \bar{V}(\theta) \tag{9}$$

where

$$\bar{V}(\theta) = \lim_{N\to\infty} EV_N(\theta) = E[\epsilon_t^f(\theta)]^2. \tag{10}$$

In identification for control, closed loop identification is most often used, as will be shown later. Therefore we now present the key formulas that characterize the bias and variance of models identified in closed loop.

Consider that the data are collected in closed loop with a controller C_{id} operating in the loop, as expressed by (3), and assume that a direct prediction error identification method is used. Substituting

the expression (3) for u_t and (1) for y_t in (6) and using Parseval's theorem yields[2]:

$$\bar{V}(\theta) = \frac{1}{2\pi}\int_{\pi}^{\pi}\left\{\left|\frac{(P-P(\theta))}{1+PC_{id}}\right|^2|C_{id}|^2\Phi_r + \left|\frac{1+P(\theta)C_{id}}{1+PC_{id}}\right|^2\Phi_v\right\}\frac{|D|^2}{|H(\theta)|^2}d\omega \quad (11)$$

Since $\hat{\theta}_N$ converges to θ^* defined by (9), these integral expressions give an implicit characterization of the model to which $P(e^{j\omega},\hat{\theta}_N), H(e^{j\omega},\hat{\theta}_N)$ will converge if the number of data tends to infinity. In other words, they give an implicit characterization of the asymptotic *bias error* obtained by closed loop identification. It follows in particular from (11) that, even if the input-output model set $\{P(q,\theta)\}$ is able to represent the true $P(q)$, closed loop identification using a direct prediction error method leads to a biased estimate of $P(q)$ if the noise model is incorrect, that is if $H(q) \notin \{H(q,\theta)\}$. This may be a drawback. One way to cope with this noise-induced bias in closed loop identification is to use one of the indirect schemes that have been specifically designed for this purpose. Their key feature is that they use more information than just the input and output data. A good survey of these methods can be found in van den Hof and Schrama (1994).

The expression (11) for the bias error is implicit. Approximate, but explicit expressions are also available for the noise-induced error, also called *variance error*. Recall (8) and denote

$$T^* = [P(e^{j\omega},\theta^*)\ \ H(e^{j\omega},\theta^*)]^T$$
$$\Delta T_N \triangleq \hat{T}_N - T^* \quad (12)$$

Then, for models of sufficiently high order, the covariance of ΔT_N at a frequency ω is approximately given by (Ljung, 1987):

$$E[\Delta T_N\ \ \Delta T_N^*] \cong \frac{n}{N}\phi_v\begin{bmatrix}\phi_u & \phi_{ue}^*\\ \phi_{eu}^* & \sigma^2\end{bmatrix}^{-1}. \quad (13)$$

Here n is the model order, N the number of data, and σ^2 the white noise variance of the true system (1), while ϕ_u is the spectrum of the input applied during identification and ϕ_{ue} the cross-spectrum between this input and the white noise source. The formula applies to both open loop identification ($\phi_{ue}=0$) and closed loop identification ($\phi_{ue}\neq 0$).

[2] For reasons of space, we have deleted the ω-dependence in all arguments.

3. THE DUAL CONTROL APPROACH

In this section we present the key ideas of *dual control*, as well as some recent developments and an illustrative example. Dual control can be seen as one way (perhaps even the optimal way) of solving the joint identification and control design problem when the system is in the model set. Thus, all the ideas and results of this section are limited to the case where $\mathcal{S}\in\mathcal{M}$. Our presentation borrows heavily from the recent PhD thesis of C. Kulcsár (1995).

The concept of dual control was introduced by Fel'dbaum (1960) who understood that, when one wants to minimize a control performance criterion for a system with unknown parameters, the control has the dual role of maintaining the state (or output) close to its desired value while at the same time learning the unknown parameters. These two roles are conflicting: learning the unknown parameters requires a sufficiently rich input, while regulating a system with poorly known parameters requires a more cautious control action than if the parameters were known. The formulation of the solution of dual optimal control problems owes a large part to the theory of dynamic programming developed by Bellman (1957, 1961).

Thus, consider that the true system $\mathcal{S}$ is given by (1) and that $\mathcal{S}\in\mathcal{M}\triangleq\{M(\theta)\}$ (see (2)) for some θ_0. We now consider that the parameter vector θ has a prior probability distribution, say $\Pi(\theta)$, and that the task is to design an optimal control sequence $U_0^N\triangleq\{u_0,u_1,\cdots,u_{N-1}\}$ that minimizes the following cost:

$$J_{0,N}(U_0^{N-1}) = E[\sum_{t=0}^{N-1} c_t(u_t,\theta)\mid I^0], \quad (14)$$

where

$$c_t(u_t,\theta) = (y_{t+1}-y_{t+1}^*)^2+\lambda_t u_t^2, \quad (15)$$

with λ_t nonnegative scalars, and where I^0 contains all prior information about the system, i.e. the noise distribution and the prior distribution on θ. The minimum of (14) is obtained by solving the following succession of nested optimization problems:

$$J_{0,N}^* = \min_{u_0} E[c_0(u_0,\theta)+\min_{u_1}E[c_1(u_1,\theta)+\ldots + \min_{u_{N-1}}E[c_{N-1}(u_{N-1},\theta)\mid I^{N-1}]\ldots\mid I^1\mid I^0]$$

Defining

$$J_{t,N}(u_t) = E[c_t(u_t,\theta)+J_{t+1,N}^*\mid I^t] \quad (16)$$

and

$$J^*_{t,N} = \min_{u_t} J_{t,N}(u_t), \tag{17}$$

one can then proceed by backwards induction and obtain the following set of *dynamic programming equations*:

$$\begin{aligned} J^*_{N-1,N} &= \min_{u_{N-1}} E[c_{N-1}(u_{N-1},\theta) \mid I^{N-1}] \\ J^*_{N-2,N} &= \min_{u_{N-2}} E[c_{N-2}(u_{N-2},\theta) \\ &\qquad + J^*_{N-1,N} \mid I^{N-2}] \\ &\vdots \\ J^*_{0,N} &= \min_{u_0} E[c_0(u_0,\theta) + J^*_{1,N} \mid I^0] \end{aligned} \tag{18}$$

Notice that the 'cost-to-go', $J_{t,N}(u_t)$, can be written as

$$\begin{aligned} J_{t,N}(u_t) = \int & \left[c_t(u_t,\theta) + J^*_{t+1,N}\right] \\ & \times \Pi(y \mid \theta, u_t)\, \Pi(\theta \mid I^t)\, dy\, d\theta \end{aligned} \tag{19}$$

If the parameters were known, the only stochastic variable over which the expectations of future costs would have to be taken would be the noise process e_t. However, in general the expectation needs to be taken also over the random parameter vector θ. Observe that the distribution of the random vector θ influences the distribution of all future input and output signals, while it is itself influenced by these signals through the parameter estimation procedure. This puts the exact solution to the dual optimal control problem beyond the reach of present-day computer technology except in the simplest of cases. Even a one parameter problem can take hours of CPU time: see (Åström and Wittenmark, 1989).

For this reason, a lot of effort has been spent on computing suboptimal solutions. One particularly simple suboptimal solution is to assume, at time t, that $\hat{\theta}_t$ is exact, i.e. to replace the probability distribution of θ in (19) by a probability density centered at $\hat{\theta}_t$ and with zero variance. This idea is called *Certainty Equivalence* (CE) control. It does not take into account the effect of the control on the precision of future estimates of θ. Another suboptimal policy, already suggested by Bellman, is to replace, at time t, the minimization of $J_{t,N}(u_t)$, in which the future controls $u_{t+1}, \cdots, u_{N-1}$ are optimal, by the following problem:

$$\min_{u_t,\ldots,u_{N-1}} E\left[\sum_{k=t}^{N-1} c_k(u_k,\theta) \mid I^t\right] \tag{20}$$

This policy is called *Open-Loop-Feedback-Optimal* (OLFO). Note that the key difference with the dual control policy (which is feedback-optimal) is that the controls are computed without taking account of the future measurements.

An interesting progress has been accomplished in the PhD thesis of C. Kulcsár (1995). For Finite Impulse Response (FIR) models, and assuming a Gaussian distribution for all random variables, she observed that the expected values of all future covariances of θ depend on the future only through their dependence on the future control signals. She then proposed a suboptimal solution in which, at time t, the mean of the future $\{\hat{\theta}_k, k \geq t\}$ is frozen at $\hat{\theta}_t$, while the expected values of the covariances $\{\Sigma_k, k \geq t\}$ are calculated exactly. These future covariances depend on all future controls $\{u_t, \cdots, u_{N-1}\}$, that need to be optimized. This is now a more tractable problem, but for large N, the optimization of such cost-to-go is still a formidable task. An additional simplification is to perform the optimization only over the first $m+1$ values $u_t, \cdots, u_{t+m}$, assuming that the remaining controls will be determined using an open loop, say Certainty Equivalence, policy. This suboptimal problem can now be solved using CPU times that are entirely reasonable. C. Kulcsár showed that the same technique works for ARX models, with an additional approximation.

We illlutrate with an example from Kulcsár (1995) the advantages of this suboptimal policy, which is a true feedback policy that takes account of the future measurement policy, over the classical CE and OLFO policies.

Example 1
Let the system and the model set be described by the FIR model

$$y_t = \theta_1 u_{t-1} + \theta_2 u_{t-2} + \theta_3 u_{t-3} + \epsilon_t. \tag{21}$$

Let ϵ_t be zero mean white Gaussian noise with variance $\sigma^2 = 0.004$, and let the true parameter vector be $\theta_0 = (15, 1, -0.2)^T$. Consider the cost function (14)-(15) with $N = 39$ and $\lambda_t = 0.001\ \forall t$. Consider a piecewise constant reference trajectory:

$$\begin{aligned} y^*_t &= 0, \quad t = 1, \cdots, 10; \quad t = 21, \cdots, 30; \\ y^*_t &= 27, \quad t = 11, \cdots, 20; \quad t = 31, \cdots, 39. \end{aligned}$$

The prior distribution of θ is $\mathcal{N}(\hat{\theta}_0, \Sigma_0)$ with $\hat{\theta}_0 = (10, 2, 0)^T$ and $\Sigma_0 = diag(2, 0.3, 0.09)$.

Figures 1 and 2 represent, respectively, the output signals and the control signals for three different stragies: LQ with known θ_0 (full line), Certainty Equivalence ($\cdots$), and the suboptimal dual policy just described, with $m = 0$ (- -). This dual

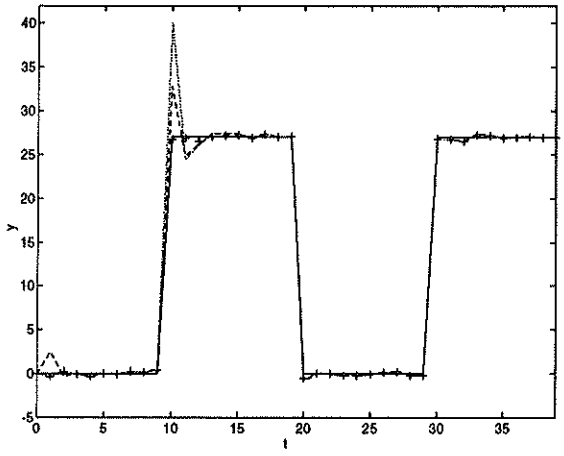

Fig. 1. Output sequence with full measurements. LQ with known θ_0: ++ ; EFC: $\cdots$; Dual: - -

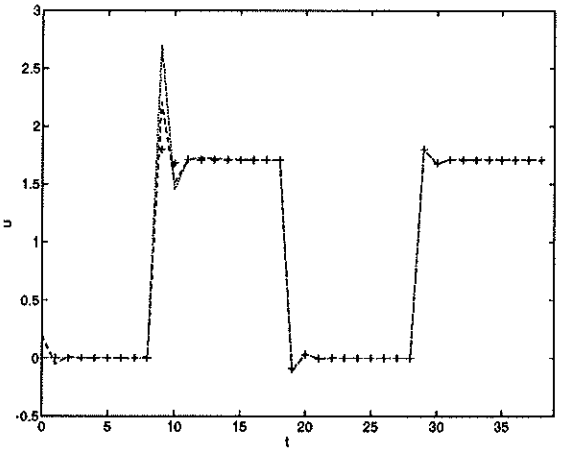

Fig. 2. Input sequence with full measurements. LQ with known θ_0: ++ ; EFC: $\cdots$; Dual: - -

policy takes account of the fact that future measurements will be made, but at time t only u_t is optimized in feedback. Note that it spends energy in the initial stages, where CE does not, in order to learn the unknown parameters in anticipation of the jump at $t = 11$ for which knowledge of θ will be useful.

The first column of Table 1 gives a comparison of the average costs obtained by these three strategies, as well as the OLFO strategy. These averages have been computed from 100 Monte Carlo simulations.

Average cost	Case I	Case II
LQ(known θ_0)	1.62	1.62
CE	191.61	1049.08
OLFO	166.23	912.96
Dual control	24.78	50.96

Table1: Case I: all measures available; Case II: no measures between t=5 and t=25.

We now consider the case where no measurements can be taken between $t = 5$ and $t = 25$. For the CE policy, nothing changes until $t = 4$ since each measurement is considered as the last one. However, the dual control strategy, even with a horizon $m = 0$ for the feedback computation of the control, takes account of the future measurement policy. Knowing that the parameters will not be updated between $t = 5$ and $t = 25$, it anticipates and learns them before $t = 5$. This is clear from Figures 3 and 4 which show, respectively, the outputs and inputs for the same three strategies. Notice that, because the dual control policy spends some initial control energy on learning the parameters, it is much better able than the CE policy to respond to the change in the reference signal at time $t = 11$. For this second case, the respective costs for these three strategies, as well as the OLFO strategy, have again been estimated by Monte Carlo simulations. The results are presented in the second column of Table 1. These simulation results confirm the clear superiority of a dual control policy, even a suboptimal one, over the more classical Certainty Equivalence policy that prevails in most of the *identification for control* literature.

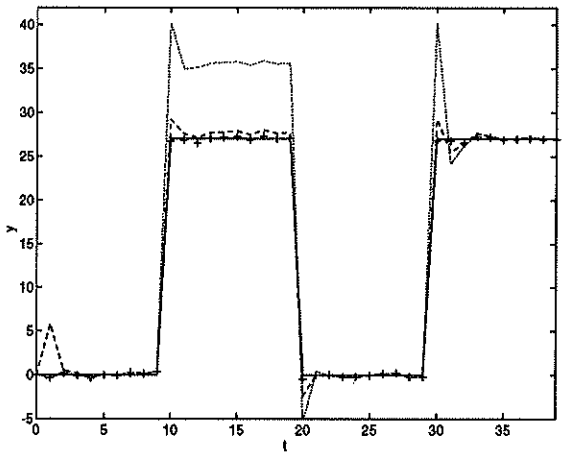

Fig. 3. Output sequence; no measures between $t = 5$ and $t = 25$. LQ with known θ_0: ++ ; EFC: $\cdots$; Dual: - -

4. OPTIMAL IDENTIFICATION DESIGN FOR CONTROL

We now abandon the dual control paradigm that attempts to solve the identification and control problem as a combined optimal control problem, in favour of a more tractable *optimal identification design problem for certainty equivalence control.* The results of this section are again limited to the case where the true system is in the model set: $\mathcal{S} \in \mathcal{M}$. They are based on (Hjalmarsson et al., 1994a).

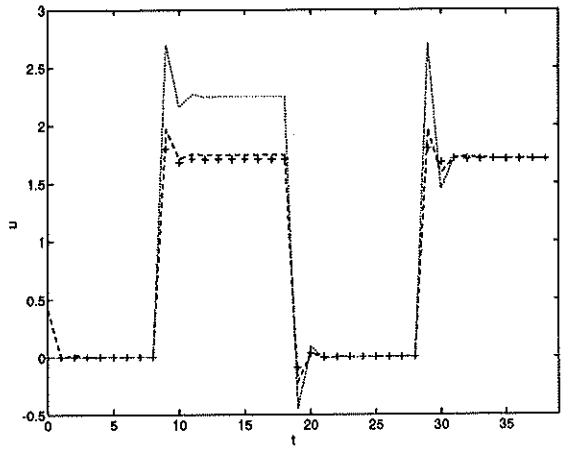

Fig. 4. Input sequence; no measures between $t = 5$ and $t = 25$. LQ with known θ_0: ++ ; EFC: $\cdots$; Dual: - -

The optimal identification design is based on the minimization of a quality criterion that compares the *optimal closed loop system* with the *actual closed loop system*: see Figures 5 and 6.

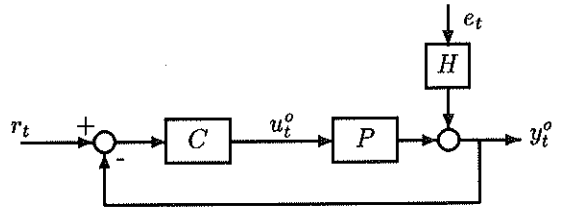

Fig. 5. Optimal closed loop system

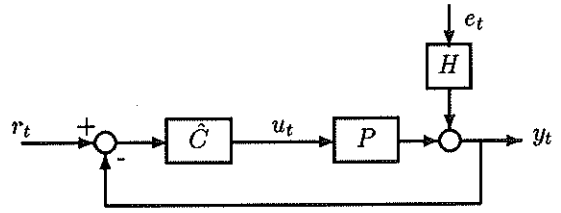

Fig. 6. Actual closed loop system

These two loops are assumed to be driven by the same external signals, the reference signal r_t with spectrum $\phi_r(\omega)$ and the white noise e_t with variance σ^2. Their outputs are denoted y_t^o and y_t, their inputs u_t^o and u_t, respectively. In the optimal closed loop sytem of Figure 5, the controller C is computed from the true plant $[P, H]$ using the control design criterion that the user has chosen: this control design criterion can be viewed as a mapping from $[P, H]$ to a controller: $C = c(P, H)$. In the actual closed loop system of Figure 6, the controller $\hat{C}$ is the Certainty Equivalence controller that results from a model $[\hat{P}_N, \hat{H}_N]$ identified using N data: $\hat{C} = \hat{C}_N \triangleq c(\hat{P}_N, \hat{H}_N)$. The control design criterion can be any one- or two-degree-of-freedom control design criterion. Here we shall, for simplicity, specialize our analysis to the case of a one-degree-of-freedom controller.

One measure of the degradation that results from using the 'estimated' controller $\hat{C}_N$ on the plant instead of the optimal C, is the variance of the error between the outputs of the ideal and the actual closed loop systems. We shall use this degradation measure as our *identification design criterion:*

$$J_V = E[y_t^o - y_t]^2. \tag{22}$$

The use of this measure as an identification design criterion was first proposed in Gevers and Ljung (1986). The results of this section can be extended to error measures that contain $E[u_t^o - u_t]^2$ or to any frequency weighted measure of $E[C(e^{i\omega}) - \hat{C}_N(e^{i\omega})]^2$.

From Figures 5 and 6 we note that (dropping the operators q):

$$y_t^o = \frac{PC}{1+PC} r_t + \frac{H}{1+PC} e_t \tag{23}$$

$$y_t = \frac{P\hat{C}}{1+P\hat{C}_N} r_t + \frac{H}{1+P\hat{C}_N} e_t \tag{24}$$

By a Taylor series expansion and assuming $\Delta C_N \triangleq \hat{C}_N - C$ to be small, we can write:

$$\begin{aligned} y_t^o - y_t &\cong \frac{P}{(1+PC)^2} \Delta C_N \\ &\quad \times [Pr_t + He_t] \\ &= \frac{P}{1+PC} \Delta C_N y_t^o \end{aligned} \tag{25}$$

Therefore, using Parseval's relation,

$$J_V \cong \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{|P|^2 \phi_{y^o}}{|1+PC|^2} E|\Delta C_N|^2 d\omega \tag{26}$$

Here, the expected value is taken with respect to the probability distribution of the noise during the identification experiment, which produces the random variable $\hat{C}_N = c(\hat{P}_N, \hat{H}_N)$. With ΔC_N small, assume again that we can write :

$$\Delta C_N = [F_1 \;\; F_2] \begin{bmatrix} \Delta P_N \\ \Delta H_N \end{bmatrix}, \tag{27}$$

where $\Delta P_N \triangleq P - \hat{P}_N$, $\Delta H_N \triangleq H - \hat{H}_N$, $F_1 = \frac{\partial C}{\partial P}$ and $F_2 = \frac{\partial C}{\partial H}$. Using the vector notation $\Delta T_N \triangleq \hat{T}_N - T$ introduced earlier, we can then write :

$$\begin{aligned} E|\Delta C_N(e^{j\omega})|^2 &= [F_1 \;\; F_2] \; E \; [\Delta T_N \cdot \Delta T_N^*] \\ &\quad \times [F_1 \;\; F_2]^*, \end{aligned} \tag{28}$$

where the superscript $*$ denotes conjugate transpose. We note that all the expressions are frequency functions. For brevity of notation, the argument ω has been dropped in this and subsequent expressions. Inserting the approximate covariance formula (13) into (28) yields, after some

manipulations, the following expression for the variance of the controller error at a frequency ω:

$$E|\Delta C_N(e^{j\omega})|^2 \cong \frac{n}{N}|H|^2|F_2|^2 \times \left(1+\frac{|\sigma^2\frac{F_1}{F_2}-\phi_{ue}|^2}{\sigma^2\phi_u-|\phi_{ue}|^2}\right).$$

This controller variance is minimized, at every frequency, by a choice of experimental conditions such that

$$\phi_{ue}^{opt}(e^{i\omega}) = \frac{F_1(e^{i\omega})}{F_2(e^{i\omega})}\sigma^2. \tag{29}$$

During closed loop identification, we have:

$$u_t = \frac{1}{1+PC_{id}}r_t - \frac{HC_{id}}{1+PC_{id}}e_t, \tag{30}$$

where $C_{id}(q)$ is the controller that operates during data collection: see (3). Therefore,

$$\phi_{ue} = \frac{-HC_{id}}{1+PC_{id}}\sigma^2. \tag{31}$$

Equating (29) and (31) we conclude that the variance of the controller error is minimized at every frequency if the identification is performed *in closed loop* with an operating controller:

$$C_{id}^{opt}(q) = -\frac{F_1(q)}{F_1(q)P(q)+F_2(q)H(q)}. \tag{32}$$

This optimal choice of course also minimizes the closed loop degradation criterion J_V, since it minimizes the integrand of (26) at every ω.

COMMENTS

- As is typical of optimal experiment design results, this optimal design depends on the unknown system $[P, H]$ and is therefore not feasible. It depends on the control criterion through the sensitivities F_1 and F_2.
- It has been shown in (Hjalmarsson et al., 1994a) that, for Model Reference Control design, $C_{id}^{opt}(q) = C(q)$. The same result had been shown for Minimum Variance Control in (Gevers and Ljung, 1986). Thus, for these control design criteria the optimal experiment design is to perform closed loop identification with the ideal controller $C(q)$ operating in the loop.
- If identification is performed under the ideal closed loop condition with $C_{id} = C_{id}^{opt}(q)$, the control error variance becomes:

$$E|\Delta C_N(e^{j\omega})|^2_{idcl} \cong \frac{n}{N}|H|^2|F_2|^2. \tag{33}$$

With open loop identification, we get

$$E|\Delta C_N(e^{j\omega})|^2_{ol} \cong \frac{n}{N}|H|^2|F_2|^2 \times \left(1+\sigma^2\frac{|F_1|^2}{\phi_u|F_2|^2}\right)$$

This result shows that, *whatever the control design criterion*, the variance of $C-\hat{C}_N$ is minimized by performing the identification in closed loop with the feedback controller (32). Under those ideal closed loop experimental conditions, at every frequency $E|\Delta C_N(e^{i\omega})|^2$ is smaller than the corresponding variance under open loop identification.

One merit of this result, and the earlier result of (Gevers and Ljung, 1986) for minimum variance control design, is that they give a proof, albeit under a severe restrictive assumption, that if the identification is performed for the purpose of control design, then closed loop identification is optimal. The restrictive assumption is that the system is in the model set. As far as we know, all other results that show that identification for control should be performed in closed loop are based on showing that a θ-dependent control criterion can be approximated by an identification criterion; in addition most of these results are based on minimizing upper bounds on closed loop performance measures rather than the measure itself: see our next section.

Even though this result is of obvious theoretical interest, it might not appear very useful given that the optimal identification design depends on the unknown system. However, it is shown in (Hjalmarsson et al., 1994a) that, when $C_{id}^{opt}(q) = C(q)$ (as is the case for Minimum Variance Control and Model Reference Control), an iterative identification and control design leads to a better controller than open loop identification. By iterative design is meant that identification is performed first in open loop for a fraction of the total data collection interval; the model estimated at the end of that interval is used to design a certainty equivalence controller, which is applied to the plant; the identification is continued in closed loop during a second time interval with this controller operating on the plant; at the end of this second interval a new certainty equivalence controller is computed from the present model and applied to the plant again; etc.

We now present a simulation example, from (Hjalmarsson et al., 1994a), to illustrate the benefits that can be gained by applying this iterative scheme over straight open loop identification. The example implements only one iteration: identification is performed in open loop during the first part of the data collection interval, and in closed loop during the second part.

Example 2
Consider a 'true system' with the following ARX structure:

$$(1 - 1.5q^{-1} + 0.7q^{-2})y_t = q^{-1}(1 + 0.5q^{-1})u_t + e_t,$$

with e_t white noise of unit variance, and a Minimum Variance (MV) control design objective. The optimal MV controller is

$$u_t = -\frac{1.5 - 0.7q^{-1}}{1 + 0.5q^{-1}} y_t + r_t.$$

Now consider that the system is unknown (but its structure is known), and that $N = 1000$ data points are allowed to be collected to estimate the model parameters, and, from these, the controller parameters. We compare the performance degradation J_V obtained in the following three experiments.

The optimal, but unfeasible, identification design is to perform the identification in closed loop with this optimal controller in the loop. The objective of our simulation is to compare the performance degradation J_V that results from this unfeasible optimal design with that obtained by open loop identification, and by the feasible iterative scheme. Thus, we compare the following three experiments.

1. *The optimal unfeasible design.* The identification is performed in closed loop using 1000 data generated with the ideal MV controller operating in the loop, and with $\phi_r = 1$.
2. *Open loop identification* using 1000 data generated with $\phi_u = 1$.
3. *The iterative design.* The identification is performed in open loop first using $(1-\alpha)1000$ data (with $0 < \alpha < 1$) generated with $\phi_u = 1$; at the end of that first interval, the corresponding certainty equivalence MV controller $\hat{C}_{(1-\alpha)1000}$ is applied to the plant, and the remaining $\alpha 1000$ data are collected on the closed loop system with $\phi_r = 1$. The final parameter estimate $\hat{\theta}_{1000}$ is computed using all the 1000 data thus collected.[3] This has been done for all values of α between 0 and 1.

In each case, the certainty equivalence MV controller $\hat{C}_{1000}$ is computed from the model obtained at the end of the 1000 data and the corresponding performance degradation J_V vis-à-vis the optimal controller achieved on the 'true system' is computed. Experimental estimates of J_V have been computed using 200 Monte Carlo simulations for each experiment. The results are shown in the top part of Figure 7.

[3] If a model other than ARX is used (such as OE, or BJ), then the estimate $\hat{\theta}_{(1-\alpha)1000}$ obtained at the end of the open loop phase is used as initial estimate for the closed loop phase.

The bottom line of this top figure represents the performance degradation $J_V^1 = 0.0015$ that results from identifying the system using 1000 data collected under the ideal experimental conditions. The top line represents the performance degradation $J_V^2 = 0.0187$ that results from identifying the system using 1000 data collected in open loop. We observe that open loop identification performs more than 10 times worse than closed loop identification with the ideal controller. The broken line represents the performance degradation J_V^3 that results from using open loop followed by closed loop identification, for different values of the fraction α of 'closed loop identification time'. We observe that for most values of α, this feasible two-stage scheme performs almost as well as the unfeasible ideal experiment design. Except for α close to 0 (mostly open loop identification) or close to 1 (mostly closed loop identification), the curve is fairly insensitive to the choice of α; thus, it appears that it would not make much sense to optimize the design parameter α.

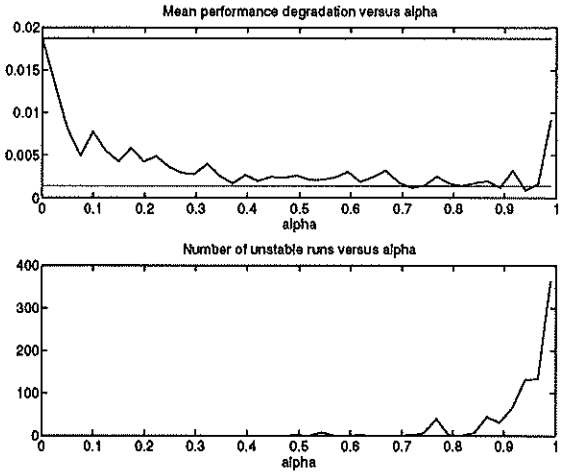

Fig. 7. Mean performance degradation and number of unstable closed loops versus α

If the open loop identification time is too short (α too large), then the model obtained at the end of open loop identification may be so poor that the corresponding controller destabilizes the true system. This is indeed what happens. The lower part of Figure 7 represents the total number of runs that produced unstable closed loops as a function of α, until 200 stable runs were produced. The averages in the top part of the figure are computed on the stable runs only.

5. MATCHING IDENTIFICATION AND CONTROL CRITERION

The main drawback of the *optimal identification design for control*, as we have noted, is that the design depends upon the unknown system, because the design criterion J_V is based on a comparison

between the *optimal* and the *achieved* closed loop systems of Figures 5 and 6.

An alternative, but suboptimal, formulation of the identification criterion is based on a comparison between the *achieved* closed loop system of Figure 6 and the *designed or nominal* closed loop system of Figure 8.

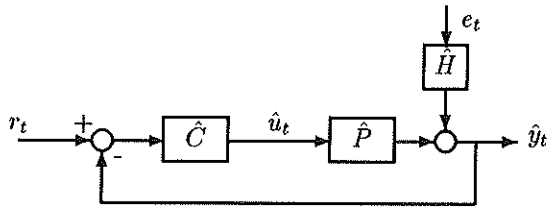

Fig. 8. Designed (or nominal) closed loop system

Comparing these two loops is classical in robust control theory; thus, we could call this third approach to identification design for control the *robust control paradigm*. However, there is an important distinction with the classical robust control thinking. In classical robust control theory, a nominal model, say $\hat{P}$, is given; a controller, say $\hat{C}$, is then called *robust* if the performance achieved by this controller on the actual plant P is not too different from the *nominal performance*, that is the performance designed on the nominal model $\hat{P}$. A key difference (and complication) in our problem formulation with respect to classical robust control theory is that now the nominal model $\hat{P}$ is also an object to be designed. Thus, in this robust control paradigm to identification for control one must attempt to perform both the identification design and the control design in such a way that the nominal performance is high and that the two loops of Figures 6 and 8 are 'close to one another' in a sense to be defined.

If nothing is fixed, then such problem formulation might not make much sense. To simplify matters somewhat, we shall therefore make the following assumptions.

- A control criterion has been selected, i.e. the designer has selected a mechanism that maps a model into a controller (e.g. LQG, H_∞, etc...)
- At some stage of the design, a model structure has been chosen, typically of lower complexity than the 'true system'. Choosing a model order is a way of imposing a controller complexity via the control design mapping. The model order may have to be increased at a later stage of the design if the controller is found to be inappropriate.

With these assumptions we claim that every control criterion induces an identification criterion that 'matches' that control criterion. In addition these identification criteria take the form of *closed loop identification* criteria. The idea that an identification criterion can be made to match a control criterion was initially advanced for LQG control by Zang et al. (1991) and for H_∞ control by Schrama (1992). It was then applied to pole placement control by Åström (1993) and Åström and Nilsson (1994), where a convergence (and divergence!) analysis of the resulting iterative scheme is given for some simple examples.

To explain the matching of the control and identification criteria, we take the simplest control design problem, namely the pole placement control problem without disturbances analysed by Åström. Thus, consider the two loops of Figures 6 and 8, and consider that the control design problem is to design $\hat{C}$ such that the designed closed loop transfer function from r_t to $\hat{y}_t$ is a given reference model. This is identical to computing $\hat{C}$ from the identified model $\hat{P}$ such that

$$\frac{1}{1+\hat{P}(q)\hat{C}(q)} = S(q), \tag{34}$$

where $S(q)$ is some admissible reference model.

Now, staring at Figures 6 and 8, and remembering that there are no disturbances, one observes that:

$$\begin{aligned} y_t &= \frac{P\hat{C}}{1+P\hat{C}} r_t \quad \text{and} \quad u_t = \frac{\hat{C}}{1+P\hat{C}} r_t, \\ \hat{y}_t &= \frac{\hat{P}\hat{C}}{1+\hat{P}\hat{C}} r_t \end{aligned} \tag{35}$$

It then follows that the 'control performance error'[4], defined as the error between the actual and the designed outputs, is given by:

$$\begin{aligned} y_t - \hat{y}_t &= S\left[\frac{P\hat{C}}{1+P\hat{C}} r_t - \frac{\hat{P}\hat{C}}{1+P\hat{C}} r_t\right] \\ &= S[y_t - \hat{P}u_t]. \end{aligned} \tag{36}$$

Equation (36) can be seen as an equality between a control performance error on the left hand side (LHS) and a filtered identification error on the right hand side (RHS). Indeed, the RHS is a filtered (by $S(q)$) version of the output error $y_t - P(q,\theta)u_t$, where u_t and y_t are collected on the actual closed loop system of Figure 6 with $\hat{C}$ operating. Thus, it appears that the control performance error can be minimized by performing *identification in closed loop with a data filter* $S(q)$. However, life is more subtle and complicated. Indeed, in the RHS of (36) the controller $\hat{C}$ and the model $\hat{P}$ are both functions of the model parameter vector θ. Hence, a more suggestive way to

[4] It is called that way by Åström (1993) because, if the closed loop transfer function of the actual system was equal to the reference model $S(q)$, this error would be zero.

write (36) is as follows:

$$y_t - \hat{y}_t = S[y_t(\theta) - P(q,\theta)u_t(\theta)]. \quad (37)$$

Consider now optimization-based control design criteria, such as e.g. LQG, or H_∞. For such criteria one cannot equate a 'control performance error' to an 'identification error' as above. However, the matching of the identification criterion to the control criterion is obtained via a double triangle inequality, in which the achieved performance (of the system of Figure 6) is shown to be very close to the designed performance (of the system of Figure 8) provided a robustness criterion can be made very small. This robustness criterion expresses the mismatch between the two closed loop systems in a measure determined by the control design criterion: see (Schrama, 1992) or (Gevers, 1993). In the simplest cases this robustness criterion is a norm of the following error:

$$y_t - \hat{y}_t = S(q,\theta)[y_t(\theta) - P(q,\theta)u_t(\theta)], \quad (38)$$

where the data filter $S(q,\theta)$ is proportional to the sensitivity function of the design loop (compare with (34)) and is now also θ-dependent.

Even though the errors (37) or (38) look like closed loop prediction errors, they cannot be minimized by standard identification techniques, because θ appears everywhere and not just in $P(\theta)$. As a consequence, the approach suggested in all known 'identification for control' schemes is to perform identification and control design steps in an iterative way, whereby the i-th identification step is performed on filtered closed loop data collected on the actual closed loop system with the $(i-1)$-th controller operating in the loop. This corresponds to an i-th identification step in which the following filtered prediction error is minimized with respect to θ:

$$\begin{aligned} y_t - \hat{y}_t &= S(q,\hat{\theta}_{i-1}) \\ &\quad \times[y_t(\hat{\theta}_{i-1}) - P(q,\theta)u_t(\hat{\theta}_{i-1})]. \end{aligned} \quad (39)$$

We refer the reader to (Gevers, 1993) and to (van den Hof and Schrama, 1994) for details and for a survey on such iterative schemes.

An interesting question is whether these iterative identification and control schemes converge to the minimum of the achieved cost over the set $\mathcal{C} \triangleq \{\hat{C}(P(\theta)) \quad \forall\theta \in D_\theta\}$ of all certainty equivalence controllers. For the example given above, this corresponds to asking whether by successively minimizing over θ the mean square of the prediction errors defined by (39) one will converge to the minimum of

$$J(\theta) \triangleq E\{S(q,\theta)[y_t(\theta) - P(q,\theta)u_t(\theta)]\}^2. \quad (40)$$

This question has been thoroughly analyzed in (Hjalmarsson et al., 1995), where it has been shown that the answer is in general negative: the iterative identification and control schemes do not generically converge to the minimum of the achieved cost. In fact, it has been shown in (De Bruyne and Gevers, 1994) that the optimal controller within this reduced order controller set $\mathcal{C}$ is not always the Certainty Equivalence controller of a model $\hat{P}$ that can be obtained as a result of an identification experiment on the true system, whether the data are collected in open loop or in closed loop.

The last observation raises the question whether one could not minimize the criterion (40) directly by some optimization method, without resorting to identification. In fact, minimizing (40) over all possible θ corresponds to a direct minimization over the controller set $\mathcal{C}$, since every model parameter vector θ defines a controller parameter vector, say $\rho = \rho(\theta)$, via the mapping $C(\rho) = C(P(\theta))$. In particular, returning to the model reference problem above, we note that $y_t - \hat{y}_t$ can also be written as:

$$\begin{aligned} y_t - \hat{y}_t &= \left[\frac{P\hat{C}}{1+P\hat{C}}r_t - Tr_t\right] \\ &= [y_t(\rho) - Tr_t]. \end{aligned} \quad (41)$$

where $T = 1 - S$ and is a fixed design quantity. Thus, minimizing $J(\theta)$ is equivalent to minimizing the control criterion $E[y_t(\rho) - Tr_t]^2$ with respect to the controller parameters ρ. The direct minimization of a control criterion with respect to the parameters of a restricted complexity controller has been a longstanding pursuit of control engineers. Recent work of Hjalmarsson et al. (1994b) has shown that this is indeed possible using an iterative scheme that avoids any identification step.

6. CONCLUSIONS

It should be clear from this paper that the subject area of identification for control is still full of challenging open problems. In an attempt to clarify the issues, we have proposed to distinguish between three approaches, going from the idealised 'dual control approach' through the 'optimal design approach' to the more realistic 'robust control approach'. This last approach leads to the much publicised iterative identification and control design schemes.

Our discussion of the last section need not imply that such iterative schemes are doomed for failure, because the suggestion has never been to iterate indefinitely, but to improve the performance of an existing controller by performing a few iterations

of identification and control using available closed loop data. However, it certainly should send a warning about the use (or misuse) of the robust control paradigm. The danger of abandoning any comparison with the optimal loop of Figure 5 is that in the end all one is able to ascertain is that the actual and the designed closed loop systems perform much the same way. However, this could just as well mean 'equally well' as 'equally poorly'. Indeed, nothing tells us that the designed performance will be anywhere near what could actually be achieved on the real system. In conclusion, one needs to question the use of such indirect schemes in the face of the new possibility of optimally tuning the parameters of a reduced order controller.

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