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# Iterative Feedback Tuning: theory and applications in chemical process control*

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**Abstract**

In 1994 H. Hjalmarsson, S. Gunnarsson and M. Gevers developed an iterative controller parameter tuning scheme, which was inspired by iterative identification and control schemes. It was entirely driven by closed loop data obtained on the actual closed loop system operating under a sequence of controllers.

The simplicity of the scheme made it an obvious candidate for experimentation and industrial application. In the two years since the publication of the method, it has been widely experimented with on laboratory experiments and on industrial processes. Here we briefly present the method and we report on its application to the optimal tuning of industrial PID controllers at the Solvay S.A. company.

**Keywords:** Control Design, Optimization, PID Tuning, Process Control

## 1 Introduction

Many control objectives can be expressed in terms of a criterion function, $LQG$ and $H_\infty$ control being the standard examples. Generally, explicit solutions to such optimization problems require full knowledge of the plant and disturbances, and complete freedom in the complexity of the controller. In practice, the plant and the disturbances are seldom known, and it is often desired to achieve the best possible performance with a controller of prescribed complexity. For example, one may want to tune the parameters of a PID controller in order to extract the best possible performance from such simple controller.

The optimization of such control performance criterion typically requires iterative gradient-based minimization procedures. The major stumbling block for the solution of this optimal control problem is the computation of the gradient of the criterion function with respect to the controller parameters: it is a fairly complicated function of the plant and disturbance dynamics. When these are unknown, it is not clear how this gradient can be computed.

Within the framework of restricted complexity controllers, previous attempts at achieving the minimum of a control performance criterion have relied on the availability of the plant and

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disturbance model, or on the estimation of a full order model of these quantities. Alternatively, reduced order controllers can be obtained from a full-order controller followed by a controller reduction step [1].

In the context of controllers of simple structure for unknown systems, such as PID controllers, some schemes have been proposed for the direct tuning of the controller parameters. These schemes are based on achieving certain properties for the closed loop system that are found to be desirable in general. These properties can then be translated into constraints on the Nyquist plot (or the Ziegler-Nichols plot) of the controlled system. We refer the reader to [2] for a representative of this family of methods.

Recently, so called iterative identification and control design schemes have been proposed in order to address the problem of the model-based design of controller parameters for restricted complexity controllers, see e.g. [22], [21] and [15]. These schemes iteratively perform plant model identification and model-based controller update, with the successive controllers being applied to the actual plant. Behind these schemes is the notion that closed loop experiments with the presently available controller should generate data that are "informative" for the identification of a model suited for a new and improved control design, and that controllers based on models that are better and better tuned towards the control objective should achieve increasingly higher performance on the actual system. See [8], [9] and [6] for a presentation of these ideas.

So far, there are very few hard results to support these expectations, except for the ideal (but unrealistic) situation where full-order models (and hence full-order controllers) are used: it has been shown in [11] that, for that situation, closed loop identification with a specific controller in the loop yields an estimated controller that achieves the best possible performance on the actual system. In addition, an iterative identification and control design scheme has been proposed that approaches these ideal experimental conditions.

In the case of low-order controllers, there are reported successes, including experimental and industrial ones, of the above-mentioned iterative identification-based controller design schemes [19], but there are also examples where these schemes are known to diverge. Most importantly, with the exception of some examples analyzed in [3], there is no analysis of the performance properties of the closed loop systems to which such schemes converge in the cases where they do so. In [14] it was shown that such iterative identification-based control design schemes do not converge to a controller that minimizes the control performance criterion, except possibly for full order models and controllers.

In the combined identification/control design schemes, the model is only used as a vehicle towards the achievement of the minimization of a control performance objective. An obvious alternative is to directly optimize the control performance criterion over the controller parameters. However, as stated above, earlier attempts at minimizing the control performance criterion by direct controller parameter tuning had stumbled against the difficulty of computing the gradient of this cost criterion with respect to the controller parameters.

The contribution of [12] was to show that an unbiased estimate of this gradient can be computed from signals obtained from closed loop experiments with the present controller operating on the actual system. For a controller of given (typically low-order) structure, the minimization of the criterion is then performed iteratively by a Gauss-Newton based scheme. At each step of the iterative design, three experiments are to be performed, two of which consist of collecting data under normal operating conditions, while the third one requires feeding back, at the reference input, the output measured during normal operation. Hence the acronym Iterative Feedback Tuning (IFT) given to this scheme. No identification procedure is involved. A closely related idea of using covariance estimates of signals obtained on the closed loop system to adjust the controller parameters in the gradient direction was used in an adaptive control context by Narendra and coworkers some 30 years ago: see [18] and [17]. In all other optimization-based approaches that have appeared in an adaptive control context, the gradient of the criterion was obtained through the estimation of a full-order model of the plant.

As in any numerical optimization routine, a variable step size can be used. This allows one to control the rate of change between the new controller and the previous one. This is an important aspect from an engineering perspective. Furthermore, a variable step size is the key to establishing convergence of the algorithm under noisy conditions. With a step size tending to zero appropriately fast, ideas from stochastic averaging can be used to show that, under the condition that the signals remain bounded, the achieved performance will converge to a (local) minimum of the criterion function as the number of data tends to infinity. This appears to be the first time that convergence to a local minimum of the design criterion has been established for an iterative restricted complexity controller scheme.

The optimal IFT scheme of [12] was initially derived in 1994 and presented at the IEEE CDC 1994. Given the simplicity of the scheme, it became clear (and not just to the authors) that this new scheme had wide-ranging potential, from the optimal tuning of simple PID controllers to the systematic design of controllers of increasing complexity that have to meet some prespecified specifications. In particular, the IFT method is appealing to process control engineers because, under this scheme, the controller parameters can be successively improved without ever opening the loop. In addition, the idea of improving the performance of an already operating controller, on the basis of closed loop data, corresponds to a natural way of thinking.

Since 1994, much experience has been gained with the IFT scheme:

- It has been shown to outperform comparable identification-based schemes in simulation examples: see [12].
- It has been successfully applied to the flexible transmission benchmark problem posed by I.D. Landau for ECC95, where it achieved the performance specifications with the simplest controller structure: see [13].
- It has been tested in real time on the flexible arm of the Laboratoire d'Automatique de Grenoble [5].
- It has been shown to handle time varying, and in particular periodically time-varying, systems [10].
- It has been applied by the chemical multinational Solvay S.A. to the tuning of PID controllers for a large number of control loops: temperature control in furnaces, in distillation columns, flow control in evaporators etc. The performance improvements achieved by applying the IFT scheme to the existing PID loops have been rather striking (see further in this paper).

Our objective in this paper is to first present the IFT scheme, and to then review performances achieved by the scheme at the S.A. Solvay, where it was used for the optimal tuning of PID controllers on a number of control loops. We shall leave aside the connections with identification-based schemes and all other technicalities that might be of interest to theoretically inclined researchers, but that would otherwise distract the reader from the essential ideas of the scheme and its potential applications.

The paper is organized as follows. In Section 2 we present the design criterion and in Section 3 we show how this criterion can be minimized using experimental data. Sections 4, 5, and 6 deal, respectively, with implementation issues, the major design choices and some practical engineering aspects. Section 7 presents the application of the method to the tuning of PID controllers on several chemical processes at S.A. Solvay. Some conclusions are offered in Section 8.

## 2 The control design criterion

We consider an unknown true system described by the discrete time model

$$y_t = G_0 u_t + v_t \tag{1}$$

where $G_0$ is a linear time-invariant operator, $\{v_t\}$ is an unmeasurable (process) disturbance and $t$ represents the discrete time instants. We shall consider here, for future analysis purposes, that $\{v_t\}$ is a weakly stationary (see e.g. [16]) random process, but other disturbance assumptions can also be made.

We consider that this system is to be controlled by a two degrees of freedom controller:

$$u_t = C_r(\rho) r_t - C_y(\rho) y_t \tag{2}$$

where $C_r(\rho)$ and $C_y(\rho)$ are linear time-invariant transfer functions parametrized by some parameter vector $\rho$, and $\{r_t\}$ is an external deterministic reference signal, independent of $\{v_t\}$. It is possible for $C_r(\rho)$ and $C_y(\rho)$ to have common parameters.

Whenever signals are obtained from the closed loop system with the controller $C(\rho) \triangleq \{C_r(\rho), C_y(\rho)\}$ operating, we will indicate this by using the $\rho$-argument; on the other hand, to ease the notation we will from now on omit the time argument of the signals. Thus, $y(\rho)$, $u(\rho)$ will denote, respectively, the output and the control input of the system (1) in feedback with the controller (2).

Let $y^d$ be a desired output response to a reference signal $r$ for the closed loop system. This response may possibly be defined as the output of a reference model $T_d$, i.e.

$$y^d = T_d r, \tag{3}$$

but for the control design method that will be developed later the knowledge of the signal $y^d$ is sufficient. The error between the achieved and the desired response is

$$\tilde{y}(\rho) \triangleq y(\rho) - y^d = \left( \frac{C_r(\rho) G_0}{1 + C_y(\rho) G_0} r - y^d \right) + \frac{1}{1 + C_y(\rho) G_0} v. \tag{4}$$

When a reference model (3) has been defined this error can also be written as

$$\tilde{y}(\rho) = \left( \frac{C_r(\rho) G_0}{1 + C_y(\rho) G_0} - T_d \right) r + \frac{1}{1 + C_y(\rho) G_0} v. \tag{5}$$

This error consists of a contribution due to incorrect tracking of the reference signal $r$ and an error due to the disturbance.

For a controller of some fixed structure parametrized by $\rho$, it is natural to formulate the control design objective as a minimization of some norm of $\tilde{y}(\rho)$ over the controller parameter vector $\rho$. In the IFT design scheme the following quadratic criterion is adopted:

$$J_N(\rho) = \frac{1}{2N} E\left[\sum_{t=1}^{N} (L_y \tilde{y}_t(\rho))^2 + \lambda \sum_{t=1}^{N} (L_u u_t(\rho))^2\right]. \tag{6}$$

where $E[\cdot]$ denotes expectation w.r.t. the weakly stationary disturbance $v$. The optimal controller parameter $\rho$ is defined by

$$\rho^* = \arg\min_{\rho} J(\rho), \tag{7}$$

The objective of the criterion (6) is to tune the process response to a desired deterministic response of finite length $N$ in a mean square sense. The first term in (6) is the frequency weighted (by a filter $L_y$) error between the desired response and the achieved response. The second term is the penalty on the control effort which is frequency weighted by a filter $L_u$. The filters $L_y$ and $L_u$ can of course be set to 1, but they give added flexibility to the design. As formulated, this is a model reference problem with an additional penalty on the control effort. With $T_d \equiv 1$ this becomes an LQG problem with tracking.

With $T_0(\rho)$ and $S_0(\rho)$ denoting the achieved closed loop response and output sensitivity function with the controller $\{C_r(\rho), C_y(\rho)\}$, i.e.

$$T_0(\rho) = \frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} \tag{8}$$

$$S_0(\rho) = \frac{1}{1 + C_y(\rho)G_0} \tag{9}$$

and given the independence of $r$ and $v$, $J(\rho)$ can be written as

$$J(\rho) = \frac{1}{2N} \sum_{t=1}^{N} \{L_y(y_t^d - T_0(\rho)r_t)\}^2 + \frac{1}{2} E\left[\{L_y S_0(\rho)v\}^2\right] + \lambda \frac{1}{2N} E\left[\sum_{t=1}^{N} (L_u u_t(\rho))^2\right]. \tag{10}$$

The first term is the tracking error, the second term is the disturbance contribution, and the last term is the penalty on the control effort.

In the case where a reference model $y^d = T_d r$ is used, the problem setting has close connections with model reference adaptive control (MRAC): see e.g. [4]. MRAC is based on the minimization of a criterion of the same type as (6) with respect to the controller parameters. To carry out the minimization it is necessary to have an expression for the gradient of this criterion with respect to the controller parameters. As will be seen below this gradient depends on the transfer function of the unknown closed loop plant. The MRAC solution to this minimization problem is then to, essentially, replace the true closed loop plant by the reference model in the gradient computation. The novel contribution of the IFT approach [12] was to show that, in contrast to the MRAC approach, the gradient can be obtained entirely from input-output data collected on the actual closed loop system, by performing one special experiment on that system. Thus, no approximations are required here to generate the gradient.

## 3 Criterion minimization

We now address the minimization of $J(\rho)$ given by (6) with respect to the controller parameter vector $\rho$ for a controller of prespecified structure. We shall see later how the method can be adapted to handle controllers of increasing complexity. To facilitate the notation we shall in this section assume that $L_y = L_u = 1$. In Section 4 we show how the frequency filters can be incorporated. It is evident from (4) that $J(\rho)$ depends in a fairly complicated way on $\rho$, on the unknown system $G_0$ and on the unknown spectrum of $\{v\}$.

To obtain the minimum of $J(\rho)$ we would like to find a solution for $\rho$ to the equation

$$0 = \frac{\partial J}{\partial \rho}(\rho) = \frac{1}{N} E\left[\sum_{t=1}^{N} \tilde{y}_t(\rho)\frac{\partial \tilde{y}_t}{\partial \rho}(\rho) + \lambda \sum_{t=1}^{N} u_t(\rho)\frac{\partial u_t}{\partial \rho}(\rho)\right]. \tag{11}$$

If the gradient $\frac{\partial J}{\partial \rho}$ could be computed, then the solution of (11) would be obtained by the following iterative algorithm:

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1}\frac{\partial J}{\partial \rho}(\rho_i). \tag{12}$$

Here $R_i$ is some appropriate positive definite matrix, typically a Gauss-Newton approximation of the Hessian of $J$, while $\gamma_i$ is a positive real scalar that determines the step size. The sequence $\{\gamma_i\}$ must obey some constraints for the algorithm to converge to a local minimum of the cost function $J(\rho)$: see [12].

As stated, this problem is intractable since it involves expectations that are unknown. However, such problem can be solved by using a stochastic approximation algorithm of the form (12) such as suggested by Robbins and Monro [20], provided the gradient $\frac{\partial J}{\partial \rho}(\rho_i)$ evaluated at the current controller can be replaced by an unbiased estimate. In order to solve this problem, one thus needs to generate the following quantities:

1. the signals $\tilde{y}(\rho_i)$ and $u(\rho_i)$;
2. the gradients $\frac{\partial \tilde{y}}{\partial \rho}(\rho_i)$ and $\frac{\partial u}{\partial \rho}(\rho_i)$;
3. unbiased estimates of the products $\tilde{y}(\rho_i)\frac{\partial \tilde{y}}{\partial \rho}(\rho_i)$ and $u(\rho_i)\frac{\partial u}{\partial \rho}(\rho_i)$.

The computation of the last two quantities has always been the key stumbling block in solving this direct optimal controller parameter tuning problem. The main contribution of [12] was to show that these quantities can indeed be obtained by performing experiments on the closed loop system formed by the actual system in feedback with the controller $\{C_r(\rho_i), C_y(\rho_i)\}$. We now explain how this can be done.

## Output related signals

From (4) it is clear that $\tilde{y}(\rho_i)$ is obtained by taking the difference between the achieved response from the system operating with the controller $C(\rho_i)$ and the desired response. As for $\frac{\partial \tilde{y}}{\partial \rho}(\rho)$, we first note $\frac{\partial \tilde{y}}{\partial \rho}(\rho) = \frac{\partial y}{\partial \rho}(\rho)$. We then have the following expression

$$\begin{aligned}\frac{\partial y}{\partial \rho}(\rho) &= \frac{G_0}{1+C_y(\rho)G_0}\frac{\partial C_r}{\partial \rho}(\rho)r - \frac{C_r(\rho)G_0^2}{(1+C_y(\rho)G_0)^2}\frac{\partial C_y}{\partial \rho}(\rho)r - \frac{G_0}{(1+C_y(\rho)G_0)^2}\frac{\partial C_y}{\partial \rho}(\rho)v \\ &= \frac{1}{C_r(\rho)}\frac{\partial C_r}{\partial \rho}(\rho)T_0(\rho)r - \frac{1}{C_r(\rho)}\frac{\partial C_y}{\partial \rho}(\rho)\left([T_0]^2(\rho)r + T_0(\rho)S_0(\rho)v\right).\end{aligned} \tag{13}$$

In this expression the quantities $C_r(\rho)$, $\frac{\partial C_r}{\partial \rho}(\rho)$ and $\frac{\partial C_y}{\partial \rho}(\rho)$ are known functions of $\rho$ which depend on the parametrization of the (restricted complexity) controller, while the quantities

$T_0(\rho)$ and $S_0(\rho)$ depend on the unknown system and are thus not computable. Therefore, unless an accurate model of the system is assumed to be available (assumption which we shall not make), the signal $\frac{\partial y}{\partial \rho}(\rho)$ can only be obtained by running experiments on the actual closed loop system.

Now observe that the last two terms in (13) involve a double filtering of the signals $r$ and $v$ through the closed loop system. More precisely, notice that

$$[T_0]^2 r + T_0 S_0 v = T_0 y.$$

Therefore, (13) can be rewritten as

$$\begin{aligned}\frac{\partial y}{\partial \rho}(\rho) &= \frac{1}{C_r(\rho)}\left[\frac{\partial C_r}{\partial \rho}(\rho)T_0(\rho)r - \frac{\partial C_y}{\partial \rho}(\rho)T_0(\rho)y\right] \\ &= \frac{1}{C_r(\rho)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho) - \frac{\partial C_y}{\partial \rho}(\rho)\right)T_0(\rho)r + \frac{\partial C_y}{\partial \rho}(\rho)T_0(\rho)(r-y)\right].\end{aligned} \tag{14}$$

The last term in (14) can be obtained by substracting the output signal from one experiment on the closed loop system from the reference, and by using this error signal as reference signal in a new experiment. This observation leads us to suggest the following procedure.

In each iteration $i$ of the controller tuning algorithm, we will use three experiments, each of duration $N$, with the fixed controller $C(\rho_i) \triangleq \{C_r(\rho_i), C_y(\rho_i)\}$ operating on the actual plant. Two of these experiments (the first and third) just consist in collecting data under normal operating conditions: the second is a real (i.e. *special*) experiment. We denote $N$-length reference signals by $\{r_i^j\}$, $j = 1, 2, 3$, and the corresponding output signals by $\{y^j(\rho_i)(\}$, $j = 1, 2, 3$.[1] Thus we have

$$r_i^1 = r, \qquad y^1(\rho_i) = T_0(\rho_i)r + S_0(\rho_i)v_i^1, \tag{15}$$

$$\begin{aligned} r_i^2 = r - y^1(\rho_i), \qquad y^2(\rho_i) &= T_0(\rho_i)r - [T_0(\rho_i)]^2 r - T_0(\rho_i)S_0(\rho_i)v_i^1 + S_0(\rho_i)v_i^2. \\ &= T_0(\rho_i)(r - y^1(\rho_i)) + S_0(\rho_i)v_i^2, \end{aligned} \tag{16}$$

$$r_i^3 = r, \qquad y^3(\rho_i) = T_0(\rho_i)r + S_0(\rho_i)v_i^3. \tag{17}$$

Here $v_i^j$ denotes the disturbance acting on the system during experiment $j$ at iteration $i$. These disturbances can be assumed to be mutually independent since they come from different experiments, provided the length $N$ of one experiment is large compared to the correlation time of the disturbances. These experiments yield an exact realization of $\tilde{y}(\rho_i)$:

$$\tilde{y}(\rho_i) = y^1(\rho_i) - y^d, \tag{18}$$

while

$$\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) \triangleq \frac{1}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right)y^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i)y^2(\rho_i)\right] \tag{19}$$

is a perturbed version (by the disturbances $v_i^2$ and $v_i^3$) of $\frac{\partial y}{\partial \rho}(\rho_i)$. Indeed by comparing (19) with (14), using (15)-(17), it is seen that

$$\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) = \frac{\partial y}{\partial \rho}(\rho_i) + \frac{S_0(\rho_i)}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right)v_i^3 + \frac{\partial C_y}{\partial \rho}(\rho_i)v_i^2\right]. \tag{20}$$

[1]The signals $\{r_i^j\}$ mentioned here are all considered to be small signal deviations with respect to constant reference values.

Two things are worth observing. First, the disturbance generated in the first experiment is not a nuisance. The output of the first experiment is used in (18) to create an exact version of the signal $\tilde{y}(\rho_i)$ which is used in the criterion $J$: see (4). Secondly, the output of the first experiment (with the disturbance) is exactly what is needed as reference signal in the second experiment to generate an estimate of $\frac{\partial \tilde{y}}{\partial \rho}$: compare (16) with the second term of (13). The only nuisances that are introduced are the disturbance contributions from the second and third experiments.

### Input related signals

It is possible to use the measurements of the process input generated from the three experiments using the reference signals (15) –(17) to generate an estimate of the sensitivity function $\frac{\partial u}{\partial \rho}(\rho_i)$. From

$$u(\rho) = \frac{C_r(\rho)}{1 + C_y(\rho)G_0} r - \frac{C_y(\rho)}{1 + C_y(\rho)G_0} v = S_0(\rho)\left[C_r(\rho)r - C_y(\rho)v\right]$$

and

$$\frac{\partial S_0}{\partial \rho}(\rho) = -\frac{1}{C_r} T_0(\rho) S_0(\rho) \frac{\partial C_y}{\partial \rho}(\rho)$$

it follows that

$$\begin{aligned}
\frac{\partial u}{\partial \rho}(\rho) &= S_0(\rho)\left[\frac{\partial C_r}{\partial \rho} r - \frac{\partial C_y}{\partial \rho}(\rho)\left(T_0(\rho)r + S_0(\rho)v\right)\right] \\
&= S_0(\rho)\left[\frac{\partial C_r}{\partial \rho} r - \frac{\partial C_y}{\partial \rho}(\rho) y\right] \\
&= S_0(\rho)\left[\left(\frac{\partial C_r}{\partial \rho}(\rho) - \frac{\partial C_y}{\partial \rho}(\rho)\right) r + \frac{\partial C_y}{\partial \rho}(\rho)(r - y)\right]. \qquad (21)
\end{aligned}$$

The experiments with reference signals defined as in (15)-(17) give the following input signals

$$\begin{aligned}
u^1(\rho_i) &= S_0(\rho_i)\left[C_r(\rho_i)r - C_y(\rho_i)v_i^1\right], \qquad (22)\\
u^2(\rho_i) &= S_0(\rho_i)\left[C_r(\rho_i)(r - y^1(\rho_i)) - C_y(\rho_i)v_i^2\right], \qquad (23)\\
u^3(\rho_i) &= S_0(\rho_i)\left[C_r(\rho_i)r - C_y(\rho_i)v_i^3\right]. \qquad (24)
\end{aligned}$$

Thus $u^1(\rho_i)$ is a perfect realization of $u(\rho_i)$,

$$u(\rho_i) = u^1(\rho_i), \qquad (25)$$

while

$$\widehat{\frac{\partial u}{\partial \rho}}(\rho_i) = \frac{1}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right) u^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i) u^2(\rho_i)\right] \qquad (26)$$

is a perturbed version of $\frac{\partial u}{\partial \rho}(\rho_i)$. Indeed a comparison of (26) with (21) shows that

$$\widehat{\frac{\partial u}{\partial \rho}}(\rho_i) = \frac{\partial u}{\partial \rho}(\rho_i) - \frac{C_y(\rho_i)S_0(\rho_i)}{C_r(\rho_i)}\left[\frac{\partial C_y}{\partial \rho}(\rho_i) v_i^2 + \left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right) v_i^3\right]. \qquad (27)$$

### An estimate of the gradient

With the signals defined in the preceding subsections, an experimentally based estimate of the gradient of $J$ can be formed by taking

$$\widehat{\frac{\partial J}{\partial \rho}}(\rho_i) = \frac{1}{N}\sum_{t=1}^{N}\left(\tilde{y}_t(\rho_i)\widehat{\frac{\partial y_t}{\partial \rho}}(\rho_i) + \lambda u_t(\rho_i)\widehat{\frac{\partial u_t}{\partial \rho}}(\rho_i)\right). \tag{28}$$

For a stochastic approximation algorithm to work, it is required that this estimate be unbiased, that is we need:

$$E\left[\widehat{\frac{\partial J}{\partial \rho}}(\rho_i)\right] = \frac{\partial J}{\partial \rho}(\rho_i), \tag{29}$$

The key feature of our construction of $\widehat{\frac{\partial J}{\partial \rho}}(\rho_i)$, and also the motivation for the third experiment, is that this unbiasedness property holds. It would indeed be tempting to use the data from the first experiment instead of the third one in (19) and (26), but then (29) would not hold because the error between $\widehat{\frac{\partial y}{\partial \rho}}(\rho_i)$ and $\frac{\partial y}{\partial \rho}(\rho_i)$ would be correlated with $\tilde{y}(\rho_i)$, and the error between $\widehat{\frac{\partial u}{\partial \rho}}(\rho_i)$ and $\frac{\partial u}{\partial \rho}(\rho_i)$ would be correlated with $u_i$.

### The algorithm

We now summarize the algorithm.

**Algorithm 3.1** *With a controller $C(\rho_i) = \{C_r(\rho_i), C_y(\rho_i)\}$ operating on the plant, generate the signals $y^1(\rho_i), y^2(\rho_i), y^3(\rho_i)$ of (15)-(17), the signals $u^1(\rho_i), u^2(\rho_i), u^3(\rho_i)$ of (22)-(24) and compute $\tilde{y}(\rho_i)$, $\widehat{\frac{\partial y}{\partial \rho}}(\rho_i)$, $u(\rho_i)$ and $\widehat{\frac{\partial u}{\partial \rho}}(\rho_i)$ using (18), (19), (25) and (26). Let the next controller parameters be computed by:*

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1}\widehat{\frac{\partial J}{\partial \rho}}(\rho_i) \tag{30}$$

*where $\widehat{\frac{\partial J}{\partial \rho}}(\rho_i)$ is given by (28), where $\{\gamma_i\}$ is a sequence of positive real numbers that determines the step size and where $\{R_i\}$ is a sequence of positive definite matrices that are, for example, given by (32). Repeat this step, replacing $i$ by $i+1$.*

## 4 Implementation issues

In this section we briefly comment on some aspects of the implementation of the scheme.

### Non-minimum phase or unstable controllers

Notice that the computation of $\widehat{\frac{\partial y}{\partial \rho}}(\rho_i)$ in (19) and $\widehat{\frac{\partial u}{\partial \rho}}(\rho_i)$ in (26) requires the filtering with the inverse of $C_r$. If $C_r$ is non-minimum phase, as may happen, this is not feasible. A similar problem occurs if the gradients of $C_y$ and/or $C_r$ are unstable. These problems can be overcome by extending $L_y$ and $L_u$ with an all-pass frequency weighting filter $L_a$, which leaves the objective function $J(\rho)$ of (6) unchanged. We illustrate the procedure for the case of a non-minimum phase $C_r$.

Let $C_r(\rho_i)$ be factorized as

$$C_r(\rho_i) = \frac{n_u n_s}{d},$$

where the factor $n_u$

$$n_u = \Pi_{k=1}^{m}(1 - z_k q^{-1})$$

contains all the unstable zeros and nothing else. At iteration $i$ let $L_a$ be the following all-pass filter

$$L_a \stackrel{def}{=} \frac{n_u}{n_u^*},$$

where

$$n_u^* = \Pi_{k=1}^{m}(1 - z_k^{-1} q^{-1}).$$

Then

$$L_a \widehat{\frac{\partial y}{\partial \rho}}(\rho_i) = \frac{d}{n_u^* n_s}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right) y^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i) y^2(\rho_i)\right] \tag{31}$$

which is stable. Thus, if $L_y$ and $L_u$ both contain $L_a$ it is possible to compute the gradients. If necessary, this filtering operation should be performed at each iteration.

### Modification of the search direction

There are many possible choices for the matrix $R_i$ in the iteration (12). The identity matrix gives the negative gradient direction. Another interesting choice is

$$R_i = \frac{1}{N}\sum_{t=1}^{N}\left(\widehat{\frac{\partial y_t}{\partial \rho}}(\rho_i)\left[\widehat{\frac{\partial y_t}{\partial \rho}}(\rho_i)\right]^T + \lambda \widehat{\frac{\partial u_t}{\partial \rho}}(\rho_i)\left[\widehat{\frac{\partial u_t}{\partial \rho}}(\rho_i)\right]^T\right), \tag{32}$$

for which the signals are available from the experiments described above. This will give a biased (due to the disturbance in the second experiment) approximation of the Gauss-Newton direction. It is the authors' experience that this choice is superior to the pure gradient direction.

### One degree of freedom controllers

In the case where the simplified controller structure $C_r = C_y \triangleq C$ is used, i.e.

$$u = C(\rho)(r - y),$$

the algorithm simplifies because the third experiment becomes unnecessary. Indeed, it follows immediately from expressions (14), (19), (21) and (26) that the first term in all these expressions is zero. Therefore, in the case of a one degree of freedom controller, the first two experiments are run with the same reference signals as indicated in (15) and (16), and the gradient estimates are obtained as special cases of (19) and (26):

$$\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) = \frac{1}{C(\rho_i)}\frac{\partial C}{\partial \rho}(\rho_i) y^2(\rho_i) \tag{33}$$

$$\widehat{\frac{\partial u}{\partial \rho}}(\rho_i) = \frac{1}{C(\rho_i)}\frac{\partial C}{\partial \rho}(\rho_i) u^2(\rho_i). \tag{34}$$

These are perturbed estimates of the actual gradients:

$$\begin{aligned}\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) &= \frac{\partial y}{\partial \rho}(\rho_i) + \frac{1}{C(\rho_i)}\frac{\partial C}{\partial \rho}(\rho_i)S_0(\rho_i)v_i^2 \\ \widehat{\frac{\partial u}{\partial \rho}}(\rho_i) &= \frac{\partial u}{\partial \rho}(\rho_i) - \frac{\partial C}{\partial \rho}(\rho_i)S_0(\rho_i)v_i^2\end{aligned} \tag{35}$$

### Disturbance rejection problem

The disturbance rejection problem is a special case of the one degree of freedom controller. The controller can be optimally tuned using iterations consisting of the same two experiments as just described in which the reference signal is put to zero. Thus, do two experiments with reference signals

$$r_i^1 = 0, \tag{36}$$

$$r_i^2 = -y_1(\rho_i) \tag{37}$$

Then take (33)-(34) as gradient estimates. Observe that, in the disturbance rejection case, the tuning of the controller parameter vector is entirely driven by the disturbance signal. This is in contrast with all identification-based iterative controller tuning schemes, where identifiability requires the injection of a sufficiently rich reference signal even in a disturbance rejection framework.

### Disturbance attenuation

As noted earlier, (20) contains an undesirable perturbation from the disturbances in the second and third experiments. Even though the influence of these disturbances is partly averaged out when $\frac{\widehat{\partial J}}{\partial \rho}(\rho_i)$ is formed, it is of course of interest to make this perturbation as small as possible. One way to decrease the influence is to increase the signal-to-noise ratio in these experiments. Let $W_i^j$, $j = 2, 3$ be two stable and inversely stable filters and replace (16) and (17) by

$$r_i^2 = W_i^2(r - y^1(\rho_i)), \qquad r_i^3 = W_i^3 r \tag{38}$$

respectively, and replace $y^j(\rho_i)$ in (19) by $[W_i^j]^{-1}y^j(\rho_i)$. Then

$$\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) = \frac{\partial y}{\partial \rho}(\rho_i) + \frac{S_0(\rho_i)}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right)[W_i^3]^{-1}v_i^3 + \frac{\partial C_y}{\partial \rho}(\rho_i)[W_i^2]^{-1}v_i^2\right]. \tag{39}$$

Thus, for frequencies where $W_i^j$ has a gain larger than one, the influence of the nuisance disturbances is decreased.

## 5 Design choices

The scheme is so simple that it might appear not to leave much room for freedom in the achievement of specific performance specifications. Indeed, once the criterion is posed, the procedure is fully automatic, except for the choice of the step size. In this section, we discuss the various design choices which allow the user to inject prior information or to translate time or frequency domain performance specifications in the framework of the minimization of a LQG-like performance index.

### Adjusting the reference model or reference trajectory

The choice of reference model or of reference trajectory is perhaps the key design choice. This is where the user can increase the speed of convergence of the algorithm significantly by injecting prior information (if any) about quantities like the delay of the system or the achievable closed loop bandwidth. If the initial controller gives bad performance, it can be quite tricky to find the optimal controller, *i.e.* the surface of the criterion can be very rough, thus allowing only small steps in each iteration. However, it is the authors' experience that the problem is simplified by starting with an objective that is easier to achieve (lower bandwidth) and then successively increasing the bandwidth as the achieved performance is increased. The easiest method of implementation of this principle is not to use a reference model, but rather to draw the new desired reference trajectory $y^d$ as a small modification (i.e. a small improvement) over the last achieved output response $y$. This has close ties with the so called windsurfing approach [15] to iterative control design.

### Frequency weighting

In Section 3 the algorithm has been derived under the assumption $L_y = L_u = 1$, for simplicity. In the general case we obtain the following.

$$\tilde{y}(\rho_i) = L_y(y^1(\rho_i) - y^d) \tag{40}$$

is a realization of $\tilde{y}(\rho_i)$, and the gradient signal is obtained by the filtering operation

$$\widehat{\frac{\partial y}{\partial \rho}}(\rho_i) \triangleq \frac{L_y}{C_r(\rho_i)} \left[ \left( \frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i \right) y^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i) y^2(\rho_i) \right] \tag{41}$$

Thus, a frequency weighting of the output is obtained by simply filtering all output signals through $L_y$. By the same arguments, a frequency weighting on $u$ is obtained by filtering the input signals from the three experiments (22)-(24) through $L_u$.

The frequency weighting filters can be used to focus the attention of the controller on specific frequency bands in the input and/or output response of the closed loop system, e.g. to suppress undesirable oscillations in these signals. Conversely, they can be used as notch filters in the frequency bands where the measurement noise dominates. They can also be used to meet specific frequency domain performance specifications, such as constraints on the sensitivitiess. The use of these filters has been illustrated in the benchmark application described in [13].

### Controller complexity modification

The method has been described as one in which successive adjustments are being made to the controller parameter vector of a controller of fixed complexity. However, it is straightforward to extend the complexity of the controller at any given iteration if the parametrization of the new one is an extension of the old one. This is useful if one realizes that the current controller is incapable of achieving the desired objective even after convergence to its optimal value. This idea has also been illustrated in [13].

### Interactive controller update

The step size can be used to control how much a controller changes from one iteration to another. Before actually implementing a controller it is possible to compare the Bode plots of the new controller with the previous ones to see whether they are reasonably consistent. If one doubts whether it will work or not one has the possibility of decreasing the step size and/or of extending the experiment so as to reduce the effects of the disturbances in the gradient calculation. The situation is quite comforting: one is backed up by the knowledge that for a small enough step size and large enough data set one will always go in a descent direction of the criterion. The step size can also be optimized along the gradient direction by line search optimization.

### Prediction of the new control performance

In addition to plotting the Bode plot of a new controller, one can also predict its effect on the closed loop response and on the achieved cost using a Taylor series expansion. To see this, we denote

$$\Delta\rho_i = \rho_{i+1} - \rho_i. \tag{42}$$

Using Taylor series expansions, we have the following predictions:

$$\tilde{y}_t(\rho_{i+1}) \approx \tilde{y}_t(\rho_i) + \left[\widehat{\frac{\partial \tilde{y}_t}{\partial \rho}}(\rho_i)\right]^T \Delta\rho_i \tag{43}$$

$$u_t(\rho_{i+1}) \approx u_t(\rho_i) + \left[\widehat{\frac{\partial u_t}{\partial \rho}}(\rho_i)\right]^T \Delta\rho_i \tag{44}$$

$$J_N(\rho_{i+1}) \approx J_N(\rho_i) - \gamma_i \left[\widehat{\frac{\partial J}{\partial \rho}}(\rho_i)\right]^T R_i^{-1} \left[\widehat{\frac{\partial J}{\partial \rho}}(\rho_i)\right], \tag{45}$$

where the last expression follows from (30). A comparison of $\tilde{y}_t(\rho_{i+1})$ with $\tilde{y}_t(\rho_i)$, of $u_t(\rho_{i+1})$ with $u_t(\rho_i)$, and of $J_N(\rho_{i+1})$ with $J_N(\rho_i)$ can help the user decide whether the step size that has led to the new controller was appropriate or not. In section 7 we shall illustrate on an industrial application how the predicted performance compares with the performance that was actually achieved with the new controller.

## 6 Engineering aspects

The IFT method is simple and applies to tuning of simple PID controllers as well as more complex controllers. For a given controller structure (example, a PID controller) it will converge to the controller that minimizes the mean square criterion for the signals (reference signals and noise signals) that are applied to the system during the controller parameter tuning. In section 7 we shall, for example, illustrate the use of this scheme for the optimal tuning of a PID controller that is aimed at obtaining ramp-like changes from a desired setpoint to another one.

### On-line considerations

The second experiment is the only *special purpose experiment*, in that it uses a different reference signal than the desired one, namely $r - y^1$. This experiment reinjects into the closed loop system a signal, $y^1$, that contains noise, thereby producing an output, denoted $y^2$, that contains the sum of two noise contributions. However, note that the contribution from the disturbance $v_i^2$ is exactly as under normal operating conditions. As for the contribution from the disturbance in the first experiment, $T_0(\rho_i)S_0(\rho_i)v_i^1$, it is essentially a bandpass filtered version of the normal disturbance contribution $S_0(\rho_i)v_i^1$ and should normally be small since (at least for a one degree of freedom controller) $S_0 + T_0 = 1$.

There are cases, however, where the additional noise injected in the reference input during the second experiment causes unacceptable behaviour in some of the states or even in the output of the system during that experiment. This has been observed, for example, in mechanical applications with flexible structures, where the noise present in the reference input during the second experiment caused excessive vibrations. This problem essentially arises during the initial iterations of the controller tuning, i.e. before the improvements in achieved controller performance outweigh the deterioration due to the noisy reference signal. One way to address this problem is to replace, in the initial iterations, the data-driven computations of the gradient of the cost criterion by an estimate of this gradient based on an identified model of the closed loop system. As soon as the improvement in closed loop performance achieved by the successive controllers outweighs the degradation due to the second experiment, one can then switch to the data-driven (i.e. IFT-based) computation of the gradient. This idea of using identified models during the initial iterations has been proposed and studied in [7].

## 7 Applications in the chemical industry

The IFT scheme has been applied by the chemical multinational Solvay S.A. for the optimal tuning of PID controllers operating on a range of different control loops. In each of these loops, PID controllers were already operating. Important performance improvements were achieved using the IFT method, both in tracking and in regulation applications. The reductions in variance achieved after a few (typically 2 to 6) iterations of the algorithm range from 25 % in a flow regulation problem in an evaporator, to 87 % in a temperature control problem for the tray of a distillation column, with other applications involving temperature control in furnaces. Here we present the results obtained on two such control loops. The first one is a temperature regulation problem for a tray of a distillation column, while the second illustrates the application of the algorithm to a setpoint modification problem in the flow of an evaporator.

### The PID controller

The same controller has been used in both loops. It differs slightly from standard PID in the following aspects :

- The derivative action is calculated on $y$ and not on the control error.
- In order to limit the gain of the controller at high frequencies when the derivative action is used, a first order filter is applied to $y$ before any calculation. The time constant of this filter is expressed as $Td/N$, $Td$ being the derivative time constant and $N$ the high frequency gain (fixed to 8 in our case).

The PID must therefore be considered as a 2-degree-of-freedom controller with common parameters.

### Temperature regulation in a distillation column

This first industrial application is a temperature regulation problem in a tray of a distillation column. Figure 1 presents temperature deviations with respect to setpoint in a tray of a distillation column, over a 24-hour period, first with the original tuning, then with the PID controller obtained after 6 iterations of the new scheme. Figure 2 shows the corresponding histograms of these deviations over 2-week periods. The control error has been reduced by 70 %.

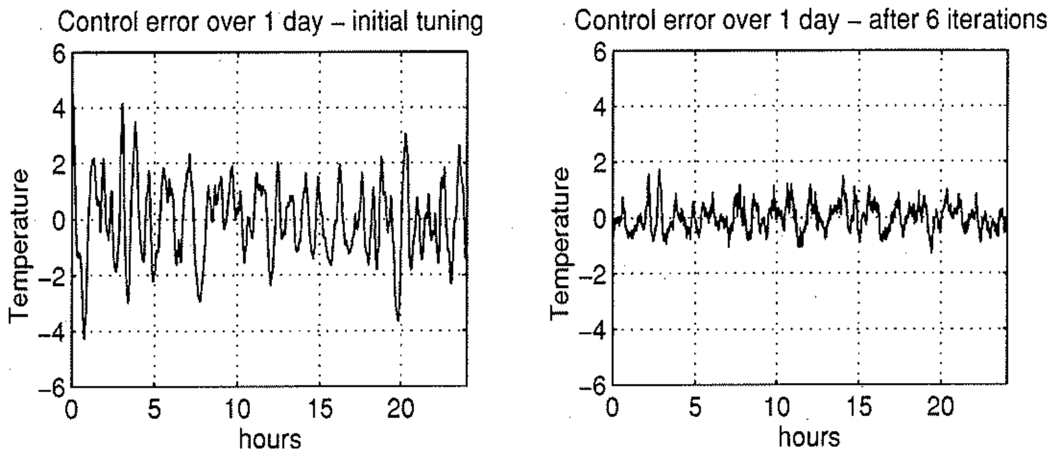


Figure 1: Control error over a 24-hour period before optimal tuning and after 6 iterations of the IFT algorithm

In Figure 3 we show the Bode plots of the two-degree of freedom controller $(C_r, C_y)$ before optimal tuning (full line), after 3 iterations of the IFT algorithm (dashed line) and after 6 iterations (dotted line). The gain was too low and the derivative action underused.

As mentioned in Section 5, an estimation of the new cost $J$ can be made at the end of each iteration using a Taylor series expansion. Table 1 shows, for the 6 iterations, the cost $J$ calculated with the first experiment as well as the predicted value with the new controller

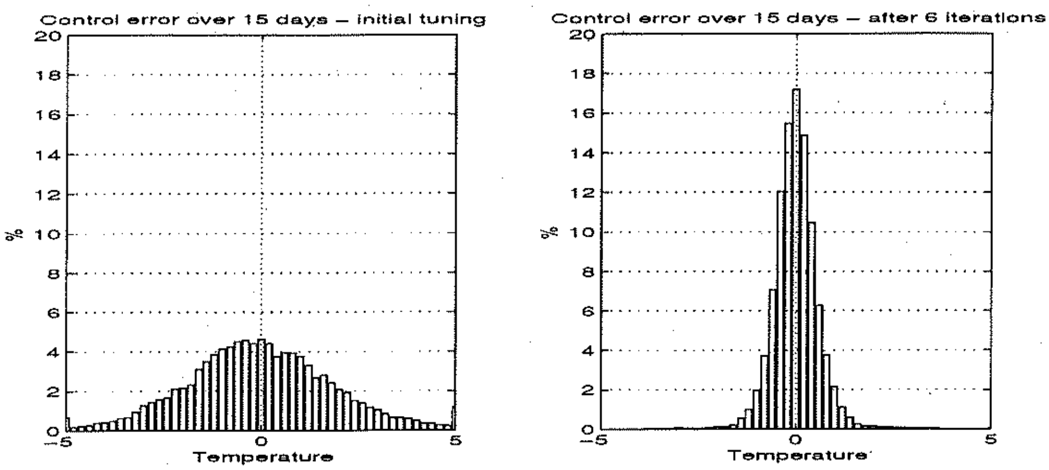


Figure 2: Histogram of control error over 2-week period before optimal tuning and after 6 iterations of the IFT algorithm

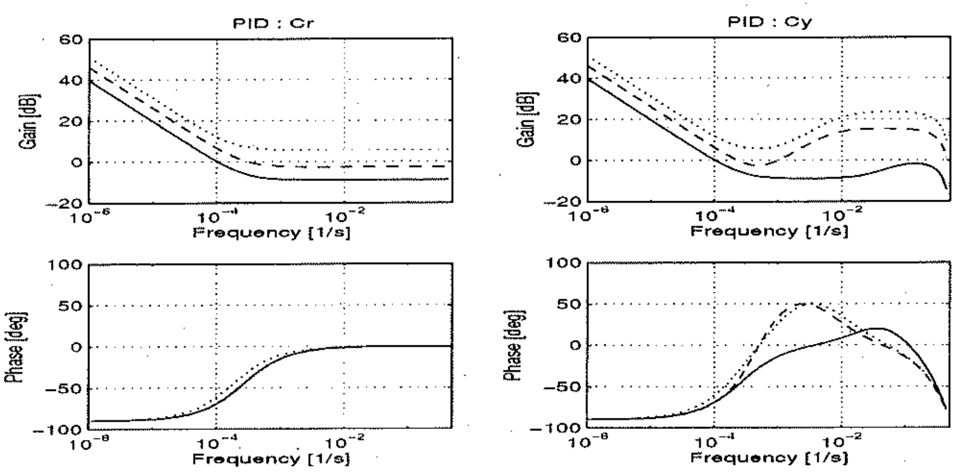


Figure 3: Bode diagram of the two-degree-of-freedom controller before tuning (full), after 3 iterations (dashed) and after 6 iterations of the algorithm (dotted).

parameters. The prediction is good except for the 2nd iteration which was perturbed by an abnormal disturbance.

| Iteration | Cost (measured) | Next cost (predicted) |
|---|---|---|
| 1 | 0.80 | 0.36 |
| 2 | 1.00 | 0.59 |
| 3 | 0.57 | 0.35 |
| 4 | 0.37 | 0.18 |
| 5 | 0.22 | 0.15 |
| 6 | 0.14 | 0.11 |

Table 1 : Calculated and predicted cost

### Flow control of an evaporator

In this case, the objective was to increase the tracking performance of the control loop during changes of production rate. We chose a 2-phase reference signal $r_d$: a ramp of 3 minutes followed by a constant value of 12 minutes. The top part of Figure 4 shows the closed loop response during the transient (first 5 minutes of the experiments) with the initial tuning and after 3 iterations. The bottom part represents a histogram of the corresponding tracking error $y_d - y$.

Figure 5 represents the control error over a 5-day period. The dispersion has been reduced by more than 25 %.

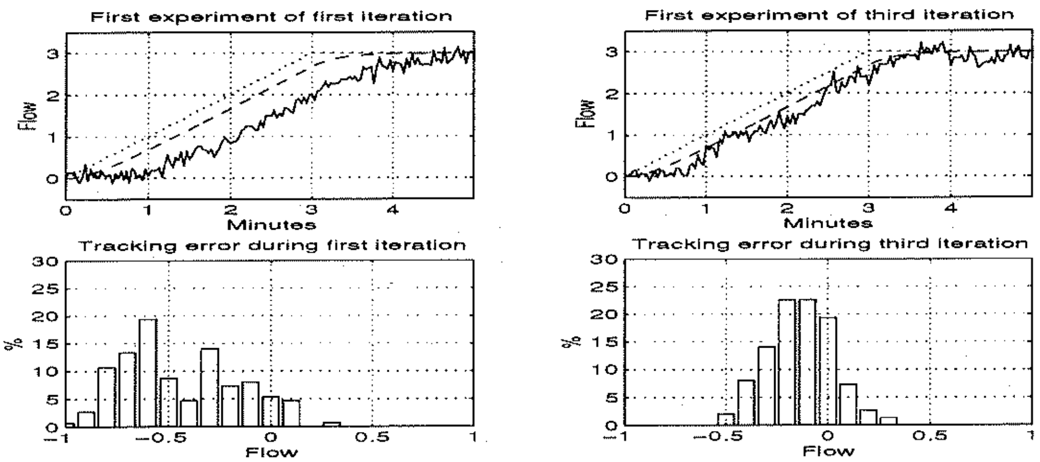


Figure 4: Evaporator: reference signal $r$ (dotted), desired response $y^d$ (dashed) and closed loop response (full) during first experiment of first and third iteration, with corresponding histograms

## 8 Final Discussion

In this paper we have examined an optimization approach to iterative control design. The important ingredient is that the gradient of the design criterion is computed from measured

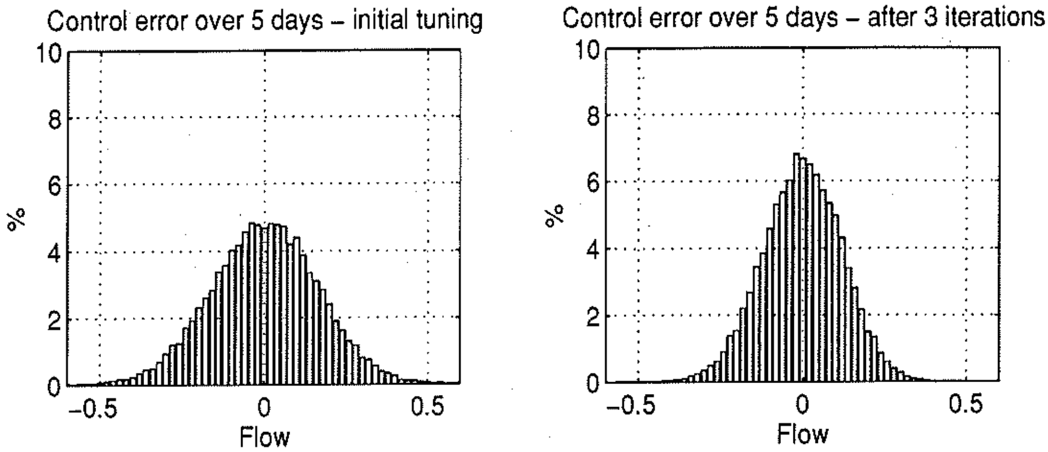


Figure 5: Evaporator: histogram of the control error over a 5-day period, with initial tuning and after 3 iterations

closed loop data. The approach is thus not model-based. The scheme converges to a local minimum of the design criterion under the assumption of boundedness of the signals in the loop.

From a practical viewpoint, the scheme offers several advantages. It is straightforward to apply. It is possible to control the rate of change of the controller in each iteration. The objective can be manipulated between iterations in order to tighten or loosen performance requirements. Certain frequency regions can be emphasized if desired.

This direct optimal tuning algorithm is particularly well suited for the tuning of the basic control loops in the process industry. These primary loops are often very badly tuned, making the application of more advanced (e.g. multivariable) techniques rather useless. A first requirement in the successful application of advanced control techniques is that the primary loops be tuned properly. This new technique appears to be a very practical way of doing this, with an almost automatic procedure. The application of the method at Solvay, of which we have presented a few typical results here, certainly appears promising.

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