Virtual Reference Feedback Tuning for Non Minimum Phase Plants

Lucíola Campestrini, Michel Gevers and Alexandre Sanfelice Bazanella

***Abstract*— Model Reference control design methods fail when the plant has one or more non minimum phase zeros that are not included in the reference model, leading possibly to an unstable closed loop. This is a very serious problem for data-based control design methods where the plant is typically unknown. For Iterative Feedback Tuning a procedure was proposed in [1] to overcome this difficulty. In this paper we extend this idea for Virtual Reference Feedback Tuning, another data-based control design method. We present a very simple two-step procedure that can cope with the situation where the unknown plant may or may not have non minimum phase zeros.**

I. INTRODUCTION

When Model Reference control design is used, it is important that the possible Non Minimum Phase (NMP) zeros of the plant to be controlled be included in the reference model. Failure to do so may even result in an unstable closed loop system. Thus, a good knowledge of the NMP zeros of the plant is essential.

In the last 15 years, a number of data-based control design methods have been proposed [2], [3], [4], [5], where a parametrized controller structure is chosen a priori, and the controller tuning is based directly on input and output data collected on the plant without the use of a model of this plant. These data-based controller tuning methods will fail if the plant contains one or more NMP zeros that have not been included in the Reference Model. To overcome this difficulty in the case of the Iterative Feedback Tuning (IFT) method [3], a procedure was proposed in [1]. It involves adding to the classical H_2 criterion of IFT an additional term that penalizes the mean square error between the achieved output of the closed loop system and a *flexible reference model* whose poles are the same as those of the desired reference model, but whose zeros are entirely free. Actually, the numerator polynomial of this flexible reference model has all its parameters free. The global criterion is a weighted version of the standard criterion and of this *flexible criterion*; it contains the controller parameters and the coefficients of the flexible reference model. This global reference model is then minimized jointly with respect to these two sets of parameters. A convergence analysis for this modified IFT criterion is quite difficult; it was performed in [1] only for the case where the controller is tuned for step changes in the reference. However, simulations have shown that this modified scheme performs remarkably well: in the case where the plant has NMP zeros, the simulations show that the parameters of the flexible reference model actually converge to values that reproduce the NMP zeros of the plant.

The objective of the work reported in this paper was to examine whether a similar idea could be developed for the VRFT method [4]. The application of the flexible reference model idea to VRFT is more difficult, because in the VRFT scheme the criterion that is minimized is different from the desired criterion; it can be made to approximate the desired criterion only by a proper prefiltering of the data. However, we will show in this paper that the idea of a flexible reference model can in fact be adapted to the VRFT method of controller tuning. Just like in the case of IFT with a flexible criterion, we will introduce a flexible VRFT criterion that contains a reference model whose numerator is a polynomial parametrized with a set of free parameters.

We will first show that the expression appearing in this flexible H_2 criterion is a bilinear function of the parameters of the numerator of the flexible reference model and of the controller parameters. This means that the minimum of this flexible part of the criterion can be obtained using an appropriate iterative least squares procedure. We will then show that the global criterion can similarly be minimized by an iterative least squares procedure.

We have applied this flexible VRFT scheme to a number of simulation examples reflecting the two main situations: one where the unknown plant contains NMP zeros, one where it does not. This leads us to propose a two-step procedure that applies to these two situations. In the first step, only the flexible part of the criterion is minimized with respect to the numerator coefficients of the reference model and the controller parameters. All our simulations have shown that when the plant contains NMP zeros, the numerator coefficients of the flexible reference model converge to a polynomial that contains these NMP zeros. Thus, the user is immediately alerted to the existence of these zeros and, more importantly, their precise locations. The second step then proceeds as follows: (i) if the first step shows that the system contains NMP zeros, the desired reference model is modified so as to contain these NMP zeros, while the poles are kept at their desired values; (ii) if the flexible reference model has converged to a value that does not exhibit NMP zeros, the standard VRFT can be used with desired fixed reference model.

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L. Campestrini and A. S. Bazanella are with the Department of Electrical Engineering, Universidade Federal do Rio Grande do Sul, Brazil {luciola,bazanela}@ece.ufrgs.br

M. Gevers is with the Center for Systems Engineering and Applied Mechanics (CESAME), Universite catholique de Louvain, B1348 Louvain la Neuve, Belgium. michel.gevers@uclouvain.be

Our analysis is so far limited to the situation where the controller set is able to produce an exact matching between the closed loop system and the flexible reference model for some values of the controller parameters and of the numerator coefficients of this reference model, i.e. the matching controller is in the controller set. Our simulations will show, however, that the method works well also when that is not the case.

The paper is organized as follows. Definitions and the problem formulation are presented in Section II. Section III reviews the standard VRFT method and the proposed flexible criterion for VRFT is then presented in Section IV. The iterative procedure used for the minimization of the flexible criterion is presented in Section V, while Section VI shows some examples of the application of the proposed method. In the end, we present some conclusions.

II. PRELIMINARIES

A. Definitions

Consider a linear time-invariant discrete-time single-input-single-output process

$$y(t) = G_0(z)u(t) + v(t), \tag{1}$$

where z is the forward-shift operator, $G_0(z)$ is the process transfer function, $u(t)$ is the control input and $v(t)$ is the process noise. The noise is a quasi-stationary process which can be written as $v(t) = H_0(z)e(t)$ where $e(t)$ is white noise with variance σ_e^2. Both transfer functions, $G_0(z)$ and $H_0(z)$, are rational and causal.

This process is controlled by a linear time-invariant controller which belongs to a given - user specified - class $\mathcal{C}$ of linear transfer functions. This class is such that $C(z)G_0(z)$ has positive relative degree for all $C(z) \in \mathcal{C}$; equivalently, the closed loop is not delay-free. The controller is parameterized by a parameter vector $\rho \in \mathbb{R}^n$, so that the control action $u(t)$ can be written as

$$u(t) = C(z,\rho)(r(t) - y(t)), \tag{2}$$

where $r(t)$ is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is

$$\bar{E}\left[r(t)e(s)\right] = 0 \;\forall t, s$$

where $\bar{E}[\cdot]$ is defined as

$$\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{\infty} E[f(t)]$$

with $E[\cdot]$ denoting expectation [6]. The system (1)-(2) in closed loop becomes

$$\begin{aligned} y(t,\rho) &= T(z,\rho)r(t) + S(z,\rho)v(t) \\ T(z,\rho) &= \frac{C(z,\rho)G_0(z)}{1 + C(z,\rho)G_0(z)} = C(z,\rho)G_0(z)S(z,\rho) \end{aligned}$$

where we have now made the dependence on the controller parameter ρ explicit in the output signal $y(t,\rho)$. It is also assumed that the controller has a linear parametrization, i.e. it belongs to a controller class $\mathcal{C}$ as specified below

$$\mathcal{C} = \{C(z,\rho) = \rho^T \beta(z), \rho \in \Re^n\}, \tag{3}$$

where $\beta(z)$ is a n-column vector of fixed causal rational functions.

Some of the most common controller structures are indeed linearly parametrized, PID with fixed derivative pole being the most popular,

$$C(z,\rho) = [k_p \; k_i \; k_d] \begin{bmatrix} 1 & \frac{z}{z-1} & \frac{z-1}{z} \end{bmatrix}^T.$$

B. Problem Statement

A good control system is one that can lead the control variable to its desired value as fast as possible with low input power. To reach this goal, we design a controller for which the closed loop system presents a desired performance, that is specified through a "desired" closed loop transfer function $\bar{M}(z)$, also known as the *reference model*. One way of finding this controller is through the solution of an optimization problem

$$\min_{\rho} J^{MR}(\rho) \tag{4}$$

$$J^{MR}(\rho) \triangleq \bar{E}\left[\left((T(z,\rho) - \bar{M}(z))r(t)\right)^2\right]. \tag{5}$$

The model matching controller $C_d^{MR}(z)$ is the one that allows the closed loop system to match exactly $\bar{M}(z)$ and is given by

$$C_d^{MR}(z) = \frac{\bar{M}(z)}{G_0(z)(1 - \bar{M}(z))}. \tag{6}$$

However, without some constraints on the choice of $\bar{M}(z)$ as a function of $G_0(z)$, this model matching controller may not be causal, or may produce an unstable closed loop system. The latter will happen if the plant $G_0(z)$ contains NMP zeros that are not included as zeros of $\bar{M}(z)$.

The design formulation (4)-(5) is used by some model-based control methods. It can be solved using a Linear Quadratic Regulator (LQR) and is then called Model Matching by LQR [7]. Such model-based designs require the knowledge of the process model $G_0(z)$. On the other hand, data-based control methods address the minimization of the criterion (5) directly from data collected from the system, without deriving a process model from these data [3], [4], [8].

This paper considers a solution to the minimization of the H_2 criterion (5) using the VRFT control design method [4]). Rather, we consider a modified version of the VRFT criterion, inspired by [1], to cope with the possible occurrence of NMP zeros in $G_0(z)$. First we present the standard VRFT method.

III. THE STANDARD VRFT METHOD

Through either an open loop or a closed loop experiment, input data $u(t)$ and output data $y(t)$ are collected on the process. Given the measured $y(t)$, we define a reference signal $\bar{r}(t)$ such that

$$\bar{M}(z)\bar{r}(t) = y(t).$$

This signal is called a *virtual reference* since it is not used to generate $y(t)$. If we now apply $\bar{r}(t)$ to the closed loop system with the controller $C(z,\rho)$, we want it to present $y(t)$ (the measured output signal) as its output. In this case, the reference tracking error is given by

$$e(t) = \bar{r}(t) - y(t).$$

Even though the plant $G_0(z)$ is unknown, when it is fed by $u(t)$ (the measured input signal), it generates $y(t)$ as output. So, a "good" controller is one that generates $u(t)$ when fed by $e(t)$. Since both signals $u(t)$ and $e(t)$ are known, the controller design can be seen as the identification of the dynamical relation between $e(t)$ and $u(t)$. As a result of this reasoning, the VRFT method minimizes the following criterion

$$\begin{aligned} J^{VR}(\rho) &= \bar{E}\left[u(t) - C(z,\rho)e(t)\right]^2 \\ &= \bar{E}\left[u(t) - \left(\frac{1-\bar{M}(z)}{\bar{M}(z)}C(z,\rho)\right)y(t)\right]^2 \end{aligned} \quad (7)$$

The criterion in (7) is a quadratic function of the parameter vector ρ and the solution of the optimization problem can be obtained through the application of the least squares method [6], which is an advantage over methods like IFT or CbT.

Consider now the following simplifying assumption.

Assumption 1: $C_d^{MR} \in \mathcal{C}$ or, equivalently,

$$\exists \rho_d : C(\rho_d) = C_d^{MR} = \rho_d^T \beta(z).$$

Under Assumption 1, the criteria (5) and (7) have the same minimum. When Assumption 1 does not hold, the minima of the two criteria can be made close provided the signals $u(t)$, $e(t)$ and $y(t)$ in (7) are filtered by a filter $L(z)$ defined by [4]:

$$|L(z)|^2 = |1-\bar{M}(z)|^2 |\bar{M}(z)|^2 \frac{\Phi_r}{\Phi_u}, \quad (8)$$

where Φ_u is the power spectrum of the signal $u(t)$ and Φ_r is the power spectrum of $r(t)$. The key advantage of the VRFT criterion (7) over the MR criterion (5) is that $J^{VR}(\rho)$ is quadratic in ρ. The optimal ρ is thus computed as a least squares solution

$$\hat{\rho} = \bar{E}\left[\varphi_L(t)\varphi_L(t)^T\right]^{-1} \bar{E}\left[\varphi_L(t)u_L(t)\right] \quad (9)$$

where $\varphi_L(t) = \beta(z)L(z)e(t)$ and $u_L(t) = L(z)u(t)$. The formulation of the VRFT method is based on signals obtained from a plant which is not affected by noise. In the presence of noise, an instrumental variable can be used instead of the least squares solution: see [4] for details.

IV. FLEXIBLE CRITERION FOR VRFT

By analogy to the flexible IFT criterion proposed in [1], the flexible VRFT criterion can be defined as follows

$$J_\lambda^{VR}(\eta,\rho) = (1-\lambda)J_0^{VR}(\eta,\rho) + \lambda J^{VR}(\rho) \quad (10)$$

where

$$\begin{aligned} J_0^{VR}(\eta,\rho) &= \bar{E}\left\{L(z)\left[u(t) - \left(\frac{1-M(z,\eta)}{M(z,\eta)}C(z,\rho)\right)y(t)\right]\right\}^2 \\ &= \bar{E}\left[u_L(t) - C(z,\rho)e_L(\eta,t)\right]^2 \end{aligned} \quad (11)$$

Here $M(z,\eta)$ is parametrized as

$$M(z,\eta) = \eta^T F(z), \quad (12)$$

where $F(z)$ is a vector of basis functions, $u_L(t) = L(z)u(t)$, and

$$\begin{aligned} e_L(\eta,t) &= (1-M(z,\eta))\bar{r}_L(t) \\ &= \frac{(1-M(z,\eta))}{M(z,\eta)}L(z)y(t) \end{aligned} \quad (13)$$

Note that when minimizing $J_0^{VR}(\eta,\rho)$ with complete freedom in the position of the zeros, we solve a pole placement problem while minimizing $J^{VR}(\rho)$ yields the solution of a model reference problem.

From (10), we can see that only the first term depends on η. Therefore, the flexible criterion for the VRFT problem can be reformulated as follows:

$$\min_{\eta,\rho} J_\lambda^{VR}(\eta,\rho) = \min_\rho \{(1-\lambda)\min_\eta J_0^{VR}(\eta,\rho) + \lambda J^{VR}(\rho)\} \quad (14)$$

V. MINIMIZING THE FLEXIBLE CRITERION

We can first minimize $J_0^{VR}(\eta,\rho)$ with respect to η, and then the optimization problem (14) is reduced to a minimization problem in ρ only. In order to proceed, we make the following model matching assumption.

Assumption 2: There exists a pair (η^*,ρ^*) such that $J_0^{VR}(\eta^*,\rho^*) = 0$, i.e. for some (η^*,ρ^*) we have

$$C(z,\rho^*) = \frac{M(z,\eta^*)}{[1-M(z,\eta^*)]G_0(z)}. \quad (15)$$

Under Assumption 2, $\min_{\eta,\rho} J_0^{VR}(\eta,\rho) = 0$, and therefore

$$\arg\min_{\eta,\rho} J_0^{VR}(\eta,\rho) = \arg \min_{\substack{\eta,\rho \\ (\eta,\rho)\neq\{0,\,0\}}} \tilde{J}_0^{VR}(\eta,\rho) \quad (16)$$

where

$$\tilde{J}_0^{VR}(\eta,\rho) = E\left[LM(\eta)u(t) - LC(\rho)(1-M(\eta))y(t)\right]^2. \quad (17)$$

For readability we have omitted the dependence on z in (17). Note that $\tilde{J}_0^{VR}(\eta,\rho)$ is obtained by multiplying $J_0^{VR}(\eta,\rho)$ by $M(z,\eta)$, which acts like a frequency weighting variable function of the unknown η and introduces an undesired minimum in zero. In order to avoid this minimum $(\eta,\rho) = \{0,0\}$, some restrictions should be imposed, for example by forcing the reference model to have steady-state gain $M(\eta,1) = 1$.

Inserting (12), one can rewrite (17) as

$$\tilde{J}_0^{VR}(\eta,\rho) = E\left[\eta^T F(z)w(\rho,t) - C(z,\rho)L(z)y(t)\right]^2 \quad (18)$$

where $w(\rho,t) \triangleq L(z)[u(t) + C(z,\rho)y(t)]$. We note that $w(\rho,t)$ can be generated from the data, since $u(t), y(t), L(z)$

and $C(z,\rho)$ are all known. Minimizing (18) yields η^* as a function of ρ, since (18) is linear in the parameter η:

$$\begin{aligned}\eta^*(\rho) \triangleq& \arg\min_{\eta} \tilde{J}_0^{VR}(\eta,\rho)\\ =&\bar{E}\left\{[F(z)w(\rho,t)][F(z)w(\rho,t)]^T\right\}^{-1}\times\\ &\bar{E}\left\{[F(z)w(\rho,t)][C(z,\rho)L(z)y(t)]\right\}.\end{aligned}$$

A. Minimizing the flexible criterion only

The expression in (18) is bilinear in the parameters η and ρ. Therefore, the minimization of $\tilde{J}_0^{VR}(\eta,\rho)$ can be treated as a sequence of least squares problems:

$$\hat{\eta}^{(i)} = \arg\min_{\eta} \tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)}) \quad (19)$$

$$\hat{\rho}^{(i)} = \arg\min_{\rho} \tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho) \quad (20)$$

This iterative minimization method, starting from some initial value $\hat{\rho}^{(0)}$, leads to a local minimum [6], [9]. Since the data are collected in closed loop, it is natural to use the parameters of the controller that is in the control loop during the experiment as the initial value, but this is not the only possible choice. It is worth stressing that even though the minimization algorithm is iterative, the data from the system is collected just once, thereby keeping the “one-shot” property of the VRFT method.

B. Minimizing the global criterion

Note that only the term $J_0^{VR}(\eta,\rho)$ is a function of η in the criterion $J_\lambda^{VR}(\eta,\rho)$ (see (14)) and that, for given η, this criterion is quadratic in the parameter ρ. Thus, an iterative algorithm can again be used to minimize $J_\lambda^{VR}(\eta,\rho)$ by proceeding as follows

$$\hat{\eta}^{(i)} = \arg\min_{\eta} \tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)}) \quad (21)$$

$$\hat{\rho}^{(i)} = \arg\min_{\rho} J_\lambda^{VR}(\hat{\eta}^{(i)},\rho) \quad (22)$$

where

$$J_\lambda^{VR}(\hat{\eta}^{(i)},\rho) = (1-\lambda)J_0^{VR}(\hat{\eta}^{(i)},\rho) + \lambda J^{VR}(\rho).$$

C. Two-step procedure

We propose the following two-step procedure for the application of VRFT to a system that may or may not have NMP zeros.

Step 1. Use $\lambda = 0$. Extensive simulations have shown that, when the system $G_0(z)$ does have NMP zeros or a delay, the iterative algorithm (19)-(20) converges with high precision to a model $M(z,\bar{\eta})$ that reflects this. If $G_0(z)$ does not have NMP zeros, then $M(z,\bar{\eta})$ will also reflect this, i.e. its zeros will be minimum phase. If the step response of $M(z,\bar{\eta})$ is not good enough, go to Step 2.
Step 2. If $M(z,\bar{\eta})$ obtained in Step 1 has NMP zeros, then the desired reference model $\bar{M}(z)$ should be modified so that it contains these NMP zeros. If not, keep the chosen $\bar{M}(z)$. Now, apply the standard VRFT with $\bar{M}(z)$, i.e. use $\lambda = 1$.

VI. ILLUSTRATIVE EXAMPLES

In this section we present some simulations using the flexible VRFT scheme, proposed in this paper for handling unknown plants that may be non minimum phase. If the plant has NMP-zeros, the proposed method estimates these zeros and then they can be included in the fixed reference model.

A. Process with one non-minimum phase zero

Suppose that we design a controller for a process whose transfer function is given by

$$G_1(z) = \frac{(z-1.2)(z-0.4)}{z(z-0.3)(z-0.8)}. \quad (23)$$

We want to control it with a PID controller

$$C(z,\rho) = \rho^T\beta(z) = [\rho_1\ \rho_2\ \rho_3]\begin{bmatrix}\frac{z^2}{z^2-z}\\ \frac{z}{z^2-z}\\ \frac{1}{z^2-z}\end{bmatrix}. \quad (24)$$

The experiment from which we get data is a closed loop experiment, where a step is applied as the reference signal, and the controller in the loop is given by

$$C_{init}(z) = \frac{-0.7(z-0.4)(z-0.6)}{z^2-z}.$$

1) Assumption 2 is satisfied: For the first design, we choose the poles of $M(z,\eta)$ such that Assumption 2 is satisfied:

$$M(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.885)(z^2-0.706z+0.32)}. \quad (25)$$

For the fixed reference model we choose the same poles, but all zeros at the origin

$$\bar{M} = \frac{0.07061z^2}{(z-0.885)(z^2-0.706z+0.32)}. \quad (26)$$

The gain of $\bar{M}(z)$ is chosen such that $\bar{M}(1) = 1$. If the standard VRFT criterion is used, (26) is the reference model, and the controller obtained is

$$C(z,\rho) = \frac{-2.2693(z^2-1.655z+0.7007)}{z^2-z},$$

which causes the closed loop response to be unstable, due to the non-minimum phase zero present in the process but not in the reference model.

Let us now use the proposed procedure. We set $\lambda = 0$ and minimize $\tilde{J}_0^{VR}(\eta,\rho)$ w.r.t η and ρ using the iterative procedure (19)-(20). The step responses of $M(z,\hat{\eta}^{(i)})$ and the closed loop $T(z,\hat{\rho}^{(i)})$ obtained at iterations 1 and 20 are presented in Fig. 1. Table I shows the evolution of the corresponding parameters, by means of the numerators of the controller and the flexible reference model, obtained in different iterations. Note that the cost $\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho)$ is higher than $\tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)})$ due to the fact that we use the filter $L(z)$ when minimizing the cost w.r.t. ρ, but not w.r.t. η. The

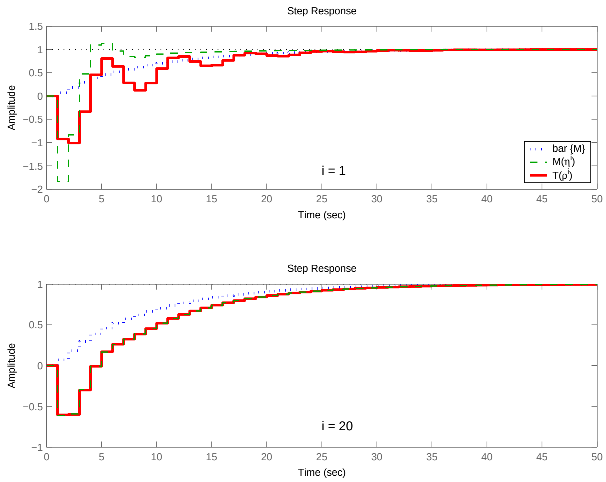

Fig. 1. Step responses of the fixed reference model (26) (bar$\{M\}$), the flexible model $M(z,\hat{\eta}^{(i)})$ ($M(\eta^i)$) and the closed loop system $T(z,\hat{\rho}^{(i)})$ ($T(\rho^i)$) for $G_1(z)$ with $\lambda = 0$ for iterations 1 and 20.

values of $M(z,\hat{\eta}^{(20)})$ and $C(z,\hat{\rho}^{(20)})$ at iteration 20 are as follows:

$$M(z,\hat{\eta}^{(20)}) = \frac{-0.6145(z-1.2)(z-0.4258)}{(z-0.885)(z^2-0.706z+0.32))},$$
$$C(z,\hat{\rho}^{(20)}) = \frac{-0.6055(z-0.8001)(z-0.3148)}{z^2-z}.$$

Observe in Table I that $M(z,\hat{\eta}^{(60)})$ reproduces both zeros of $G_1(z)$, and the controller $C(z,\hat{\rho}^{(60)})$ is such that its zeros cancel the poles of the process. However, the NMP zero is already perfectly estimated at iteration $i = 10$, while convergence to the minimum phase zero is slower. This observation is consistent with the findings of [10], [11] where it is shown that NMP zeros are easier to estimate than minimum phase zeros. Once we know that the process actually has a NMP-zero and where it is, we can add this zero to $\bar{M}(z)$ and use (14) for a different value of λ. If we choose $\lambda = 1$ we then minimize the standard VRFT criterion (7), but now with a "safer" reference model. The new reference model is defined by

$$\bar{M}_m(z) = \frac{-0.35303z(z-1.2)}{(z-0.885)(z^2-0.706z+0.32)}, \quad (27)$$

where the gain is chosen so that $\bar{M}_m(1) = 1$. Fig. 2 shows the step responses obtained at the end of Step 1 with $T(z,\hat{\rho}^{(20)}) = M(z,\hat{\eta}^{(20)})$ and Step 2 with $T(z,\hat{\rho})$ where $\hat{\rho}$ minimizes (9), and the step response of $\bar{M}_m(z)$. Both responses are very similar, but for $\lambda = 1$ we obtain a smaller negative response.

2) Assumption 2 is not satisfied: In the previous example, the reference model (25) was chosen in a way that the matching condition (15) is satisfied for some (η^*, ρ^*) pair. Since the process $G_0(z)$ is typically unknown, it is unlikely that any chosen poles of $M(z,\eta)$ will allow Assumption 2 to be satisfied.

Suppose then that we choose a fixed reference model

$$\bar{M}_f(z) = \frac{0.064z^2}{(z-0.6)^3}, \quad (28)$$

and a flexible one defined as

$$M_f(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.6)^3},$$

for which Assumption 2 is not satisfied; for $\lambda = 0$ we obtain, in 20 iterations,

$$M_f(z,\hat{\eta}^{(20)}) = \frac{-0.7650(z-1.201)(z-0.5846)}{(z-0.6)^3},$$
$$C(z,\hat{\rho}^{(20)}) = \frac{-0.6799(z-0.7917)(z-0.1817)}{z^2-z}.$$

The step responses for iterations 1 and 20 are presented in Fig. 3. Table II presents the numerators of the controller and the flexible reference model, obtained in different iterations. Note that, even though Assumption 2 is not satisfied, the NMP-zero is still identified with high precision by the minimization of $\tilde{J}_0(\eta,\rho)$, in 20 iterations. Besides, the closed loop $T(z,\hat{\rho}^{(20)})$ presents a response that is not exactly, but very similar to the reference model $M_f(z,\hat{\eta}^{(20)})$ response (see Fig. 3).

B. *Process with two minimum-phase zeros*

Finally, we apply the method to an example in which the plant zeros are both minimum phase:

$$G_2(z) = \frac{(z+0.2)(z-0.4)}{z(z-0.3)(z-0.8)}. \quad (29)$$

It is initially in closed loop with a PID controller

$$C_{init}(z) = \frac{0.7(z-0.4)(z-0.6)}{z^2-z},$$

which we want to retune so that the closed loop response is as close as possible to a given $\bar{M}(z)$, using a controller $C(z,\rho)$ of the form (24).

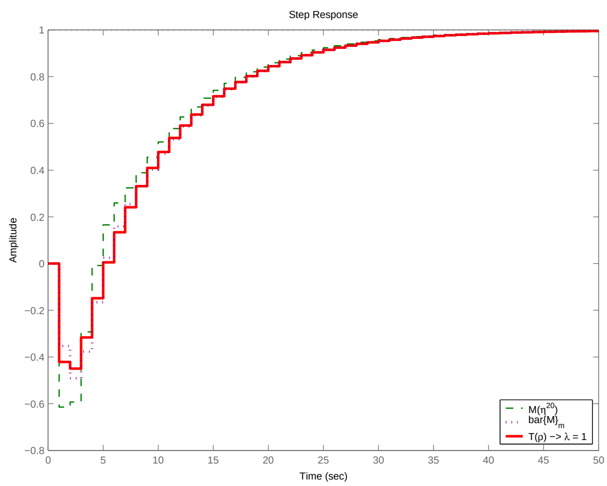

Fig. 2. Step responses obtained in Step 1: $T(z,\hat{\rho}^{(20)}) = M(z,\hat{\eta}^{(20)})$ ($M(\eta^{20})$); and in Step 2: $T(z,\hat{\rho})$ ($T(\rho)$ -> $\lambda = 1$) with the fixed reference model (27) (bar$\{M\}_m$).

TABLE I

EVOLUTION OF $M(z,\eta)$ AND $C(z,\rho)$ IN THE ITERATIVE PROCEDURE FOR $G_1(z)$.

i	num$(M(z,\hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)})$	num$(C(z,\hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho)$
1	$-1.8349(z-1.275)(z-0.8602)$	5.826165	$-0.9245(z-0.7831)(z-0.5482)$	23.102193
2	$-0.9143(z-1.208)(z-0.6291)$	0.083319	$-0.7627(z-0.8018)(z-0.4200)$	0.134643
5	$-0.7552(z-1.201)(z-0.5353)$	0.005507	$-0.6854(z-0.8009)(z-0.3760)$	0.008832
10	$-0.6677(z-1.2)(z-0.4723)$	0.000989	$-0.6376(z-0.8003)(z-0.3421)$	0.001324
20	$-0.6145(z-1.2)(z-0.4258)$	0.000088	$-0.6055(z-0.8001)(z-0.3148)$	0.000108
60	$-0.5902(z-1.2)(z-0.4018)$	6.4×10^{-8}	$-0.5900(z-0.8001)(z-0.3002)$	7.2×10^{-8}

TABLE II

EVOLUTION OF $M_f(z,\eta)$ AND $C(z,\rho)$ IN THE ITERATIVE PROCEDURE FOR $G_1(z)$.

i	num$(M_f(z,\hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)})$	num$(C(z,\hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho)$
1	$-1.3552(z-1.225)(z-0.7903)$	2.708518	$-0.9729(z-0.7678)(z-0.3676)$	9.972294
2	$-1.2833(z-1.206)(z-0.7580)$	0.159566	$-0.9217(z-0.7718)(z-0.3433)$	4.035966
20	$-0.7650(z-1.201)(z-0.5846)$	0.009044	$-0.6799(z-0.7917)(z-0.1817)$	0.077548

TABLE III

EVOLUTION OF $M(z,\eta)$ AND $C(z,\rho)$ IN THE ITERATIVE PROCEDURE FOR $G_2(z)$.

i	num$(M(z,\hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)})$	num$(C(z,\hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho)$
1	$0.8363(z-0.6470)(z+0.5586)$	0.039204	$0.8592(z-0.8093)(z-0.0364)$	0.002505
2	$0.8644(z-0.6381)(z+0.4707)$	0.000158	$0.8595(z-0.8198)(z-0.0664)$	0.016214
10	$0.6895(z-0.4117)(z+0.1343)$	0.000003	$0.6849(z-0.8006)(z-0.3223)$	0.000049
20	$0.6517(z-0.4040)(z+0.1846)$	3.08×10^{-7}	$0.6501(z-0.8002)(z-0.3051)$	6.85×10^{-6}

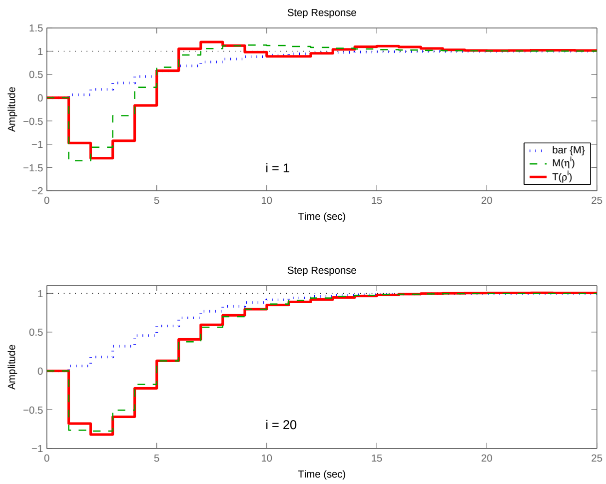

Fig. 3. Step responses of the fixed reference model (28) (bar$\{M\}$), the flexible model $M_f(z,\hat{\eta}^{(i)})$ ($M(\eta^i)$) and the closed loop system $T(z,\hat{\rho}^{(i)})$ ($T(\rho^i)$) with $G_1(z)$ at iterations 1 and 20.

1) Assumption 2 is satisfied: In this case, the fixed reference model is given by

$$\bar{M}(z) = \frac{0.46009z^2}{(z-0.6673)(z^2+0.3063z+0.07661)},$$

and the flexible reference model is chosen as

$$M(z,\eta) = \frac{\eta_1 z^2+\eta_2 z+\eta_3}{(z-0.6673)(z^2+0.3063z+0.07661)},$$

for which Assumption 2 is satisfied. In Step 1 the zeros of $M(z,\eta)$, estimated using (19)-(20), converge to the zeros of $G_0(z)$, but much more slowly than in the case of NMP-zeros: see Table III. Since $M(z,\hat{\eta}^{(20)})$ does not present a NMP-zero, we can safely go for Step 2 and use the standard VRFT method without modifying the reference model.

2) Assumption 2 is not satisfied: Suppose now we choose another fixed reference model:

$$\bar{M}_f(z) = \frac{0.216z^2}{(z-0.4)^3}, \tag{30}$$

and a corresponding flexible model having the same poles as $\bar{M}_f(z)$:

$$M_f(z,\eta) = \frac{\eta_1 z^2+\eta_2 z+\eta_3}{(z-0.4)^3}.$$

With $M_f(z,\eta)$ and the controller (24), Assumption 2 is not satisfied. Then Step 1 leads to

$$M_f(z,\hat{\eta}^{(10)}) = \frac{0.55939(z-0.697)(z+0.2742)}{(z-0.4)^3},$$
$$C_f(z,\hat{\rho}^{(10)}) = \frac{0.55643(z-0.6537)(z+0.1013)}{z^2-z}.$$

Note that $M_f(z,\hat{\eta}^{(10)})$ is far from $\bar{M}_f(z)$ (see Fig. 4 - Step 1), but it does not present a NMP zero. We can then safely go to Step 2 and apply the standard VRFT with the fixed reference model (30). The controller obtained is

$$C(z,\hat{\rho}) = \frac{0.19874(z+0.5094)(z-0.7791)}{z^2-z}.$$

Fig. 4 - Step 2 - shows the closed loop responses obtained with this controller compared to the fixed reference model $\bar{M}_f(z)$.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a flexible criterion $J_\lambda(\eta,\rho)$ for the tuning of controllers using the VRFT method

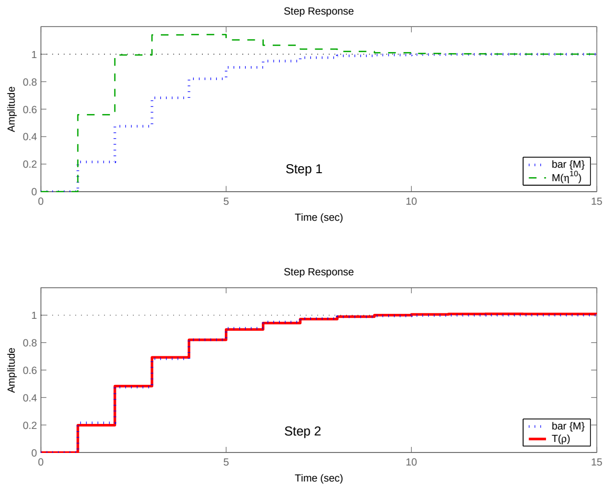

Fig. 4. Step responses of the fixed reference model (30) (bar$\{M\}$) and the flexible reference model $M_f(z, \hat{\eta}^{(10)})$ ($M(\eta^{10})$) in Step 1; step response of (30) (bar$\{M\}$) and of the closed loop system $T(z, \hat{\rho})$ ($T(\rho)$) in Step 2.

that allows the method to be used for the control of non-minimum phase plants. We have also proposed a two-step procedure for the tuning: in the first step we use $\lambda = 0$ and the flexible reference model will reproduce the NMP-zeros if there are some. If this is the case, then the fixed reference model should be modified so as to contain this NMP-zeros. In the second step, we use $\lambda = 1$ in order that the closed loop response can reach the desired reference model response. The efficiency of the flexible criterion is illustrated in some simulations.

Extending the analysis to the case where the matching controller is not in the controller set, for which we have already obtained good simulation results, as well as adapting the method to deal with signals corrupted by noise are some of the aims of future research.

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