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Model-free Tuning of a Robust Regulator for a Flexible Transmission System

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Recently, a data-driven model-free iterative control design method has been proposed [Hjalmarsson et al., Proc. 33rd IEEE CDC, Orlando, FL, 1994, 1735–1740]. This design method works directly with closed loop data from the plant and iteratively improves the performance. This contribution reports a simulation study of this method when applied to a flexible transmission system. The system is characterised by load dependent dynamics and certain performance specifications have to be satisfied for three different load cases. These specifications cannot be translated into a specific control criterion a priori. *However, by adaptively changing the design criterion it is shown that it is possible to tune the criterion so as to eventually obtain the desired closed loop performance for all three load cases with the same controller. The new concepts of synthetic noise and time delays are shown to be valuable tools when tuning the criterion.*

Keywords: Control design; Multiple systems; Optimisation

1. Introduction

In this paper we describe how a recently developed model-free, optimisation and experimental based control design scheme [3,4], originally designed for linear time-invariant systems, can be used to design a single controller for the control of multiple linear time-invariant plants.

The control objective in [3] and [4] is to minimise an *LQG* (like) control criterion for a reduced complexity controller. Generally, explicit solutions to such optimisation problems require full knowledge of the plant and disturbances and complete freedom in the complexity of the controller, *LQG* and H_∞ control being the standard examples. In the reduced controller complexity case such optimisation does in general not have explicit solutions and one has to resort to numerical minimisation procedures, see e.g. [1]. One approach is to employ some descent algorithm such as the Gauss–Newton method. In [3] and [4] it was observed that such an algorithm can be devised *without* explicit knowledge of the plant. It was shown that an *unbiased* estimate of the gradient of certain control criteria, such as the *LQG* criterion, can be computed from experimental data collected during (essentially) normal operating conditions. No models of the plant and the disturbance are required. This leads to an iterative procedure of experiments/controller updates which, under the assumption of boundedness of the signals in the loop, converges to a local minimum of the criterion. This procedure was originally developed for linear time-invariant systems but the theory can be applied to simultaneous control of multiple systems as well. This is the application considered in this paper.

The procedure is applied to a flexible transmission at the Laboratoire d'Automatique de Grenoble (LAG), where the system dynamics depend on the load. The amplitude and location of two resonant modes depend heavily on the load. This has to be

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taken into account when designing a controller for the system.

We have not had the opportunity to do experiments on the real plant. Instead the experiments were performed as computer simulations using (accurate) models of the true plant provided by I.D. Landau (LAG). This is thus a rather artificial application of the data-driven model-free control design scheme which is supposed to work directly with data collected on the real plant. However, these are the conditions under which the benchmark was organised, and we decided to participate in order to compare our new approach to model-based approaches. The characteristic features of our design scheme are that we work directly with low order controllers and that the plant models have not been used explicitly; only data generated from these models have been used.

The paper evolves as follows. In Section 2 we briefly present the design criterion for a single plant and in Section 3 we discuss how this criterion can be minimised using experimental data. The multiple plant case is discussed in Section 4. Section 5 contains the application example.

2. The Design Criterion for a Single Plant

Let the true system be given by

$$y(t) = G_0(q)u(t) + v(t) \tag{1}$$

where $\{v(t)\}$ is a (process) disturbance and where $G_0(q)$ is a linear time-invariant discrete-time system.[1] The output, $\{y(t)\}$, from the true system will be called the achieved response. We will use a two degrees of freedom linear time-invariant controller:

$$u(t) = C_r(q)r(t) - C_y(q)y(t) \tag{2}$$

where $\{r(t)\}$ is an external reference signal. The controller pair $C(q) = \{C_r(q), C_y(q)\}$ is parameterised by a parameter vector ρ. To ease the notation somewhat we will from now on omit the time argument of the signals and the operator argument q from the transfer functions. In addition, whenever signals are obtained from the closed loop system with the controller $\{C_r(\rho), C_y(\rho)\}$ operating, we will indicate this by using the ρ-argument; thus, e.g., $y(\rho)$ will denote the output of the system (1) in feedback with the controller (2).

Let T_d be a desired stable closed loop transfer operator from reference signal to output signal and let y_d represent the desired response

$$y_d = T_d\, r \tag{3}$$

The error between the achieved and desired response is

$$\tilde{y}(\rho) = y(\rho) - y_d = \frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} r - T_d\, r + \frac{1}{1 + C_y(\rho)G_0} v \tag{4}$$

It is natural to formulate the design objective as a minimisation of some norm of $\tilde{y}(\rho)$. Although not necessary from a procedural viewpoint we shall restrict the attention to the quadratic criterion, i.e. we will study the problem

$$\arg \min_{\rho} J(\rho) \tag{5}$$

where

$$J(\rho) = \frac{1}{2} E\left[(L_y \tilde{y}(\rho))^2\right] \tag{6}$$

Here E denotes expectation over v and r which we assume to be realisations of stationary stochastic processes. This criterion is, the by L_y, frequency weighted norm of the error between the desired response and the achieved response. For more general criteria see [3,4].

3. Criterion Minimisation

We now address the minimisation of $J(\rho)$, given by (6), with respect to the controller parameter vector ρ. To simplify matters we shall in this section assume that $L_y = 1$.

Notice here that we do not assume that we have a full order controller. Hence there may not be an explicit unique solution to the minimisation problem (5). Notice also that it is evident from (4) that $J(\rho)$ depends in a fairly complicated way on ρ and that the fundamental problem is that the true system G_0 and the spectrum of $\{v\}$ are unknown.

The problem we may hope to solve is to find a stationary point to (6), i.e. a solution for ρ to the equation

$$0 = J'(\rho) = E\left[\tilde{y}(\rho)\tilde{y}'(\rho)\right] \tag{7}$$

This can be done by taking successive steps in a descent direction

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} J'(\rho_i) \tag{8}$$

Here R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of J, such as a Gauss–Newton approximation of this Hessian. As stated this problem is intractable since it involves tak-

[1] q is the shift operator.

ing expectation. It is, however, exactly a problem that can be attacked with stochastic approximation procedures such as suggested by Robbins and Monro [8]. One replaces J' with an approximation based on the current samples. In order to do this, the signal $\tilde{y}(\rho_i)$ and its gradient $\tilde{y}'(\rho_i)$ are required. If a model of the plant is available, then this model can be used to compute these quantities. However, in [3] and [4] it is shown that $\tilde{y}(\rho_i)$ can be computed exactly and that $\tilde{y}'(\rho_i)$ can be computed approximately using experimental data from (essentially) normal operating conditions only. No explicit model is needed. The procedure is as follows.

In each iteration i we will use three experiments, each of duration N, say, with the fixed controller $C(\rho_i)$ operating on the actual plant; we denote the corresponding output signals and disturbances by $\{y_i^j\}$, $j = 1,2,3$ and $\{v_i^j\}$, $j = 1,2,3$, respectively. The reference signals that are to be used are

$$r_i^1 = r; \quad r_i^2 = y_i^1; \quad r_i^3 = r \tag{9}$$

This means that during the first experiment of iteration i, the reference signal r_i^1 applied to the closed loop system should be the reference signal of normal operation. During the second experiment, the reference signal should be the output signal of the first experiment, while a normal operating reference is used in the third experiment again.

With these experiments

$$\tilde{y}_i = y_i^1 - y_d \tag{10}$$

is a perfect realisation of $\tilde{y}(\rho_i)$ and

$$\hat{\tilde{y}}_i' = \frac{1}{C_r(\rho_i)}\left(C_r'(\rho_i)y_i^3 - C_y'(\rho_i)y_i^2\right) \tag{11}$$

is a perturbed version (by the disturbances v_i^2 and v_i^3) of $\tilde{y}'(\rho_i)$. For details we refer to [3] or [4].

3.1. An Estimate of the Gradient

With the signals defined in the preceding subsections, an estimate of the gradient of J can be formed by taking

$$\hat{J}_i' = \frac{1}{N}\sum_{t=1}^{N} \tilde{y}_i(t)\hat{\tilde{y}}_i'(t) \tag{12}$$

It can be shown that this is an *unbiased* estimate of the gradient $J'(\rho_i)$. From this it can be shown that, with J' replaced by $\hat{J}'$ in (8), the iterations (8) converge to a stationary point of the criterion J if the step size is chosen as $\mu_i = 1/i$ and if the closed loop system is stable during the iterations.

4. Simultaneous Control of Multiple Plants

So far the model-free scheme has been developed for the case of a single system. Assume now that M plants $\{G_k, H_k\}$, $k = 1, \ldots, M$, are to be controlled with the same controller $\{C_r(\rho), C_y(\rho)\}$. Let $J^k(\rho)$ be the control criterion (6) with the system $\{G_k, H_k\}$ in the loop. The objective is now to minimise the multi-objective function

$$J(\rho) = [J^1(\rho), \ldots, J^M(\rho)]^T$$

It is of course almost never possible to minimise all these criteria simultaneously. An alternative is to solve the worst case problem

$$\min_{\rho} \max_{k} \frac{J^k(\rho)}{c_k} \tag{13}$$

The weights $\{c_k\}$ are user specified in an iterative way. After each minimisation the user decides whether some criterion should be given more weight in order to satisfy the requirements. Now, (13) is a non-smooth problem and there are several alternatives to get around that. The simplest and most naive approach is to minimise the criterion

$$J(\rho) = \sum_{k=1}^{M} \frac{J^k(\rho)}{c_k} \tag{14}$$

For this criterion, the gradient is a weighted sum of the gradients of the criteria for the different plants, i.e.

$$J'(\rho) = \sum_{k=1}^{M} \frac{1}{c_k}\frac{dJ^k(\rho)}{d\rho} \tag{15}$$

Hence, if gradient approximations $\hat{J}_i^k(\rho_i)$ for the different criteria $J^k(\rho_i)$ are available, it is straightforward to compute a gradient approximation for the joint criterion (14) according to (15) and hence to update the controller parameters using (8).

Now, gradient approximations $\hat{J}_i^k(\rho_i)$ for the different plants can be computed according to (12) if the three experiments (9) are carried out for each plant in each iteration. It is obvious that as long as $J(\rho)$ is a smooth function of the individual criteria $J^k(\rho)$, the gradient of J will be a function of the gradients of the J^k. Hence, the data from the experiments (9) on each system will suffice to compute an estimate of the gradient of J also in less naive problem formulations than (14). An interesting possibility is the criterion

$$J(\rho) = \frac{1}{\alpha}\ln\left(\sum_{k=1}^{M} e^{\alpha\frac{J^k(\rho)}{c_k}}\right)$$

which can be shown to have a minimum close to the minimum of the minimax problem (13). See [5] and [7] for details.

5. Application to a Flexible Transmission

In this section we discuss how the scheme has been applied to the flexible transmission of the LAG benchmark. The dynamics of the transmission depend on the load and available were discrete-time ARX models for three different load cases, 0%, 50% and 100% load

$$A_k y(t) = B_k u(t) + e(t), \quad k = 1, 2, 3 \tag{16}$$

The system is sampled with a sampling frequency of $f_e = 20\,\text{Hz}$. For details of the system we refer to [6].

5.1. Performance Specifications

The performance specifications to be satisfied for all these cases were:

1. The maximum amplitude of the output sensitivity (S_{yp}) less than 6 dB.
2. The attenuation band (f_{att}), i.e. smallest frequency for which $S_{yp} \geqslant 1$, larger than 1% of the sampling frequency.
3. The maximum amplitude of the input sensitivity (S_{up}) less than 10 dB in the frequency range 8–10 Hz.
4. Delay margin (t_{del}) > 40 ms.
5. No steady state error.
6. Overshoot $y_{\max} < 10\%$ for a step reference change.
7. Rise time (t_{rise}) $\leqslant 1$ s.
8. Rejection time (t_{rej}) of step disturbances in the noise source e: < 1.2 s for 90% rejection of measured peak value.

5.2. Assumed Knowledge

We have tried to emulate a situation which is as close as possible to the real situation when the only source of information is input/output data from the true plant. Thus, only data from closed loop simulations of the models provided by I.D. Landau have been used in the design. *The models themselves have not been used explicitly*. The disturbances on the true system were not specified and we have taken the liberty to assume that they can be neglected compared to the synthetic noise (see below). Let us point out that if the disturbances acting on the true system cannot be neglected, it is advantageous for the algorithm to use real experimental data. These disturbances will then automatically be taken into account in the control design.

The first two conditions in the specifications are given in the frequency domain which thus cannot be checked with time domain data. This means that accurate models of the system must be used to check whether these specifications are satisfied or not. Just looking at the signals in the loop is not enough. However, it turned out that it was only condition 1 that had to be checked in the frequency domain. The second condition turned out to be automatically fulfilled when condition 7 was fulfilled. Condition 1 could, however, have been replaced by a time domain condition saying that no disturbance response should be too oscillatory. However, in the design we have allowed ourselves to use the sensitivity functions and the transfer function from the disturbance to the output as pieces of information in order to know how the design quantities should be adapted to fulfill condition 1.

5.3. The Closed Loop

The experiments were thus run as simulations on the computer. A two degree of freedom controller was employed. For load case k, the input signal u^k was generated according to

$$u^k = C_u(C_r r - C_y y^k) + w^k \tag{17}$$

The term w^k, which is user specified, was taken to be zero mean white noise of variance σ^2. This term was used as a *synthetic* disturbance[2] which influenced the shaping of the sensitivity function. Recall that the optimum is a trade-off between reference tracking and disturbance rejection. With this term it is possible to decide where this trade-off should be. By choosing w small, the tracking ability is emphasised whereas a large w will emphasise the disturbance rejection.

With the input (17), the closed loop output can be written

$$y^k = \frac{C_r C_u G_k}{1 + C_y C_u G_k}(r + w^k) + \frac{H_k}{1 + C_y C_u G_k} e$$

with $k = 1, 2, 3$, where, according to (16), $G_k = B_k / A_k$ and $H_k = 1/A_k$.

Two types of simulations were performed. During the optimisation procedure, only the synthetic disturbance w^k was present; the real disturbance e was assumed to be negligible. In order to check whether

[2]Notice that it is possible to perturb the input signal in this way also in practice.

the performance specification regarding output disturbance rejection was satisfied, w^k was set to zero and e was taken to be a step.

5.4. The Design Quantities

We now discuss the design quantities that are involved in the design.

Criterion Structure

Since the performance specifications are to be satisfied for all loads, some kind of simultaneous optimisation over the three closed loops must be done. The naive approach (14) was adopted. Each load case was given a criterion of the type (6). In order to ensure the required delay margin of at least 40 ms, the case of 0% load with one additional (synthetic) time delay was also simulated.[3] The reason why only the 0% load case was simulated with an additional time delay is that it was quite clear from initial experiments that this was the loop most likely to become unstable. Data from this additional simulation were used in a fourth criterion. These four criteria were then added together with weights of 30% for the first three cases and 10% for the case with one additional time-delay.

Number of Data

Each simulation consisted of 1200 samples from each load case, i.e. a total of 4800 samples. This gives a total of 14400 samples/iteration or 12 min of data collection/iteration.

Reference Signal

Since the closed loop was specified in terms of a certain rise time only, the reference signal was taken to be a unit amplitude square-wave with a period equal to half the simulation length, i.e. 600 samples.

Reference Model

The reference model was chosen to be a third order low-pass filter with a triple pole and the same time delay as the models. The bandwidth of the reference model was chosen so that it satisfied the required 1 s step response. More specifically, the triple pole location was chosen as $e^{-6\times 0.05}$.

Synthetic Disturbance

w was taken to be one realisation of Gaussian zero mean white noise with variance 0.002. The variance level was chosen such that the contribution of the disturbance to the output was relatively small compared to the contribution of the reference signal. The purpose was mainly to alert the scheme that the disturbance rejection properties could not be disregarded completely. The same realisation was used in all iterations.

The reason why this synthetic disturbance was added to the input side and not to the output side of the system was that the design should take into account disturbances in e (see (16)) which is a disturbance passing through the denominator $1/A$ of the system.

Since this disturbance enters on the input side of the system, it means that the energy content of the disturbance on the output side was high around the resonance peaks of the system. Thus the criterion was initially tuned to make the sensitivity low around the resonance frequencies.

Frequency Weighting Filter

The frequency weighting filter was initially set to one.

Controller Structure

Condition 5 in the specifications requires an integrator in the controller. Therefore C_u was taken to be a pure integrator[4] $C_u = 1/(1-q^{-1})$. The other two compensators C_r and C_y were chosen to be of finite impulse response type[5]

$$C_r = r_0 + r_1 q^{-1} + \ldots + r_{n_r} q^{-n_r}$$
$$C_y = s_0 + s_1 q^{-1} + \ldots + s_{n_y} q^{-n_y}$$

where the orders n_r and n_y were chosen adaptively (see below)

Update Direction

The update direction in (8) was taken to be of Gauss–Newton type, see [3] for details.

[3] Notice that it is possible to add synthetic time-delays to real systems also.

[4] q^{-1} denotes the shift operator.

[5] In the R-S-T format, C_u, C_r, C_y correspond to $1/S$, T, R, respectively.

5.5. The Iterations

Since no controller at all was known, the initial controller was taken to be a low gain PI controller $C_r = C_y = 0.1$ and $C_u = 1/(1 - q^{-1})$. This controller yields very poor performance. One iteration improved the performance but it was quite clear that this simple controller would not be able to meet all requirements. Consequently the complexity of the controller was increased to $n_r = n_y = 2$.

A local optimum was reached after three more iterations. As evidenced by Fig. 1, the performance has improved significantly. But it seemed difficult to reach the performance specifications other than by increasing the controller complexity.

With n_r and n_y both increased to 4 it took eight more iterations to reach a new (local) minimum of the criterion. The corresponding controller satisfied nearly all specifications. The main violation was on the maximum bound on the amplitude gain of the output sensitivity function. The output sensitivity exceeds the 6 dB bound around $f/f_e = 0.2$ with 3 dB. To decrease the sensitivity functions at high frequencies, the frequency weighting filter L_y was taken to be a fourth order high-pass filter with a 10 dB difference in the gain between low and high frequencies and a cut-off frequency of $f/f_e = 0.15$. Three more iterations were performed with this filter L_y acting. The final controller was

$$\begin{aligned} C_r = {} & 0.1040212 - 0.08570247q^{-1} \\ & -0.04273886q^{-2} + 0.03793123q^{-3} \\ & +0.03612251q^{-4} \end{aligned}$$

$$\begin{aligned} C_y = {} & 0.5517005 - 1.764544q^{-1} + 2.112755q^{-2} \\ & -1.296223q^{-3} + 0.4457450q^{-4} \end{aligned}$$

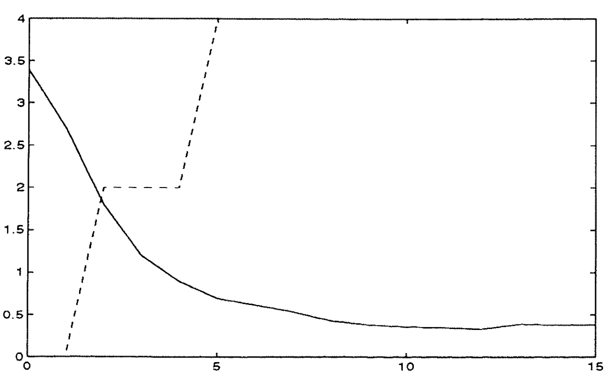

Fig. 1. Solid line: criterion function $10J(\rho_i)$; dashed line: $n_r(= n_y)$, number of coefficients in the controllers.

As can be seen from Table 1 and the sensitivity plots in Figs 2 and 3, all specifications have been met, except that the rejection time in the full load case (1.35 s) is slightly too slow and f_{att} is slightly below 0.2 Hz. Figure 4 shows the Bode plots of the final controller and the location of the closed loop poles can be found in Fig. 5. Figures 6–8 show the output/input response to a step change in the reference signal (occurring at time 10) and a step change in the disturbance e (occurring at time 120) for the three different load cases.

Table 1. Performance of closed loop for different load conditions. Δ_M denotes modulus margin. For a definition of the other quantities see Subsection 5.1.

Load (%)	t_{rise} (s)	y_{max} (%)	t_{rej} (s)	t_{del} (s)	Δ_M (dB)	f_{att} (Hz)	S_{up} (dB)
0	0.95	0.6	1.1	0.05	−5.76	0.18	9.9
50	0.85	0.4	0.75	0.12	−5.14	0.18	9.5
100	0.70	1.3	1.35	0.39	−6.11	0.18	9.7

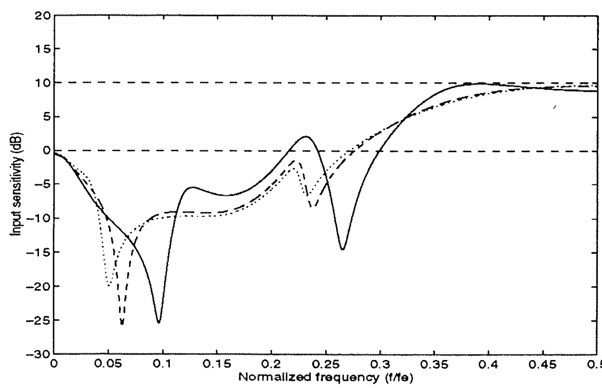

Fig. 2. Input sensitivity function. Solid line: 0% load; dashed line: 50% load; dotted line: 100% load.

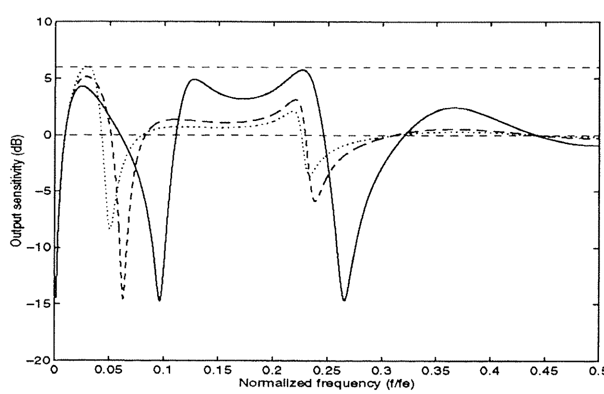

Fig. 3. Output sensitivity function. Solid line: 0% load; dashed line: 50% load; dotted line: 100% load.

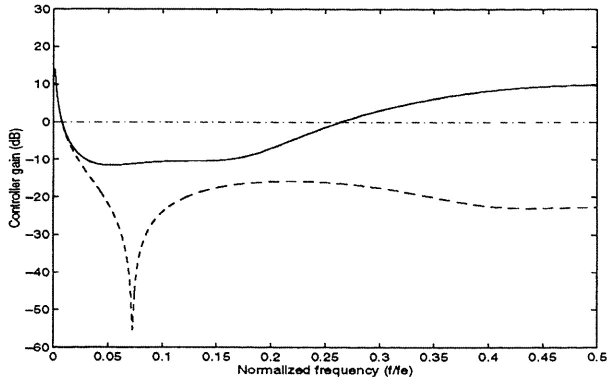

Fig. 4. Frequency responses of controller. Solid line: $C_y C_u$; dashed line: $C_r C_u$.

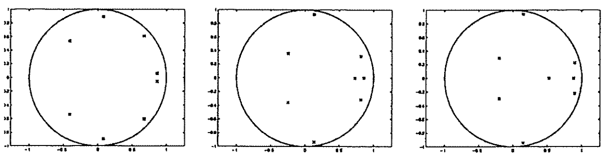

Fig. 5. Closed loop poles. Left: no load; center: half load; right: full load.

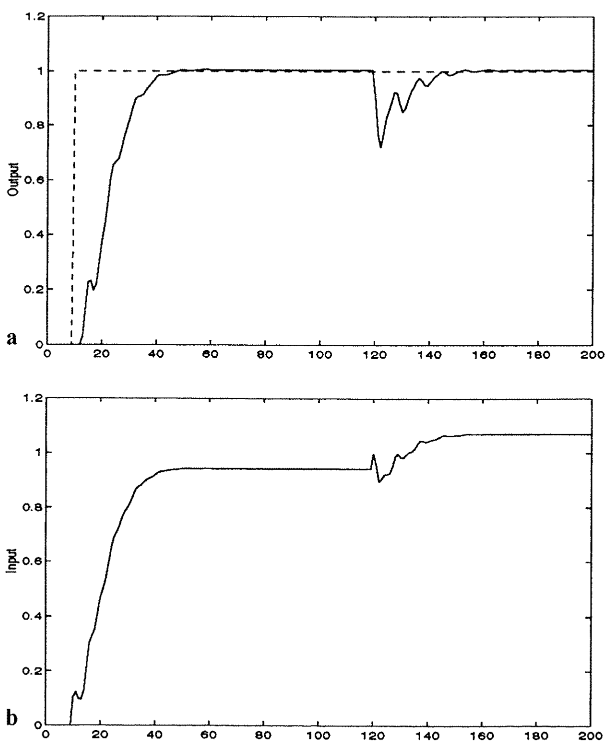

Fig. 6. Step/disturbance response. No load. Dashed line: reference signal. (a) Output; (b) input.

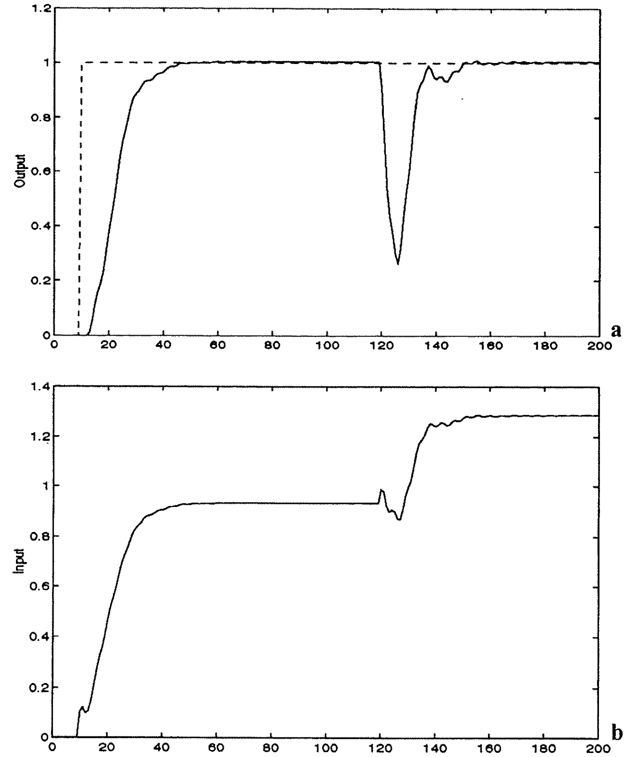

Fig. 7. Step/disturbance response. Half load. Dashed line: reference signal. (a) Output; (b) input.

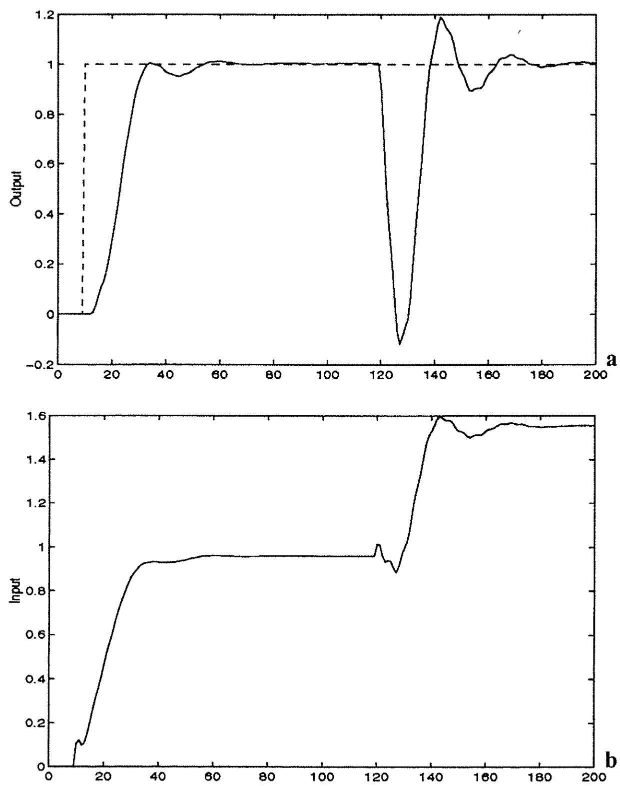

Fig. 8. Step/disturbance response. Full load. Dashed line: reference signal. (a) Output; (b) input.

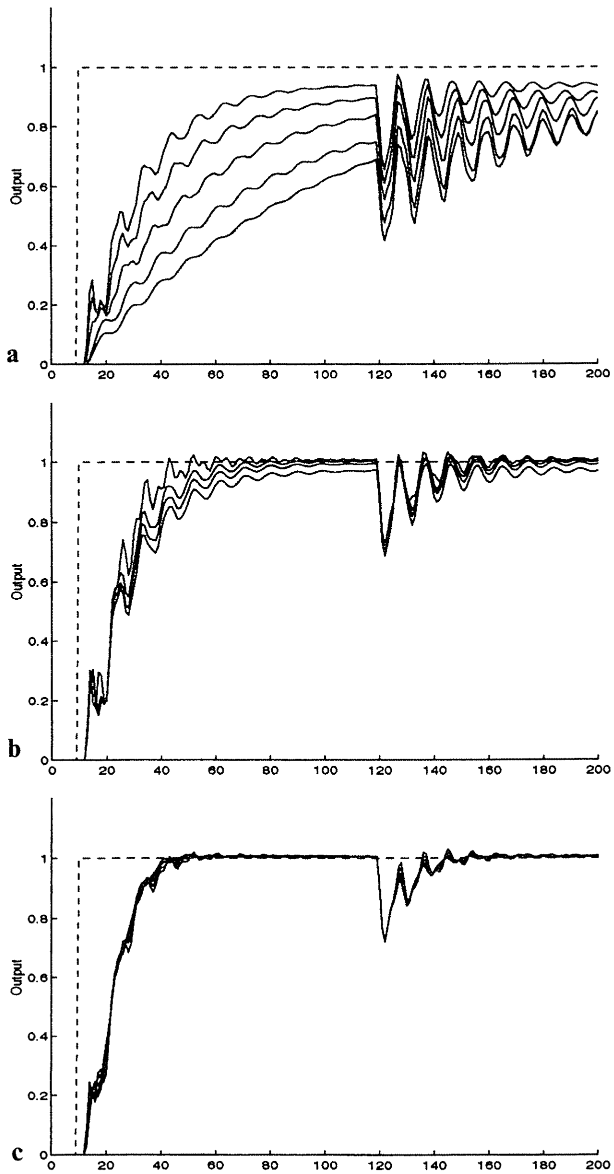

Fig. 9. Evolution of step/disturbance response. (a) Initial response and response after 1–4 iterations; (b) response after 5–8 iterations; (c) response after 9–15 iterations. In (a) and (b), the rise time increases monotonically with the iteration number.

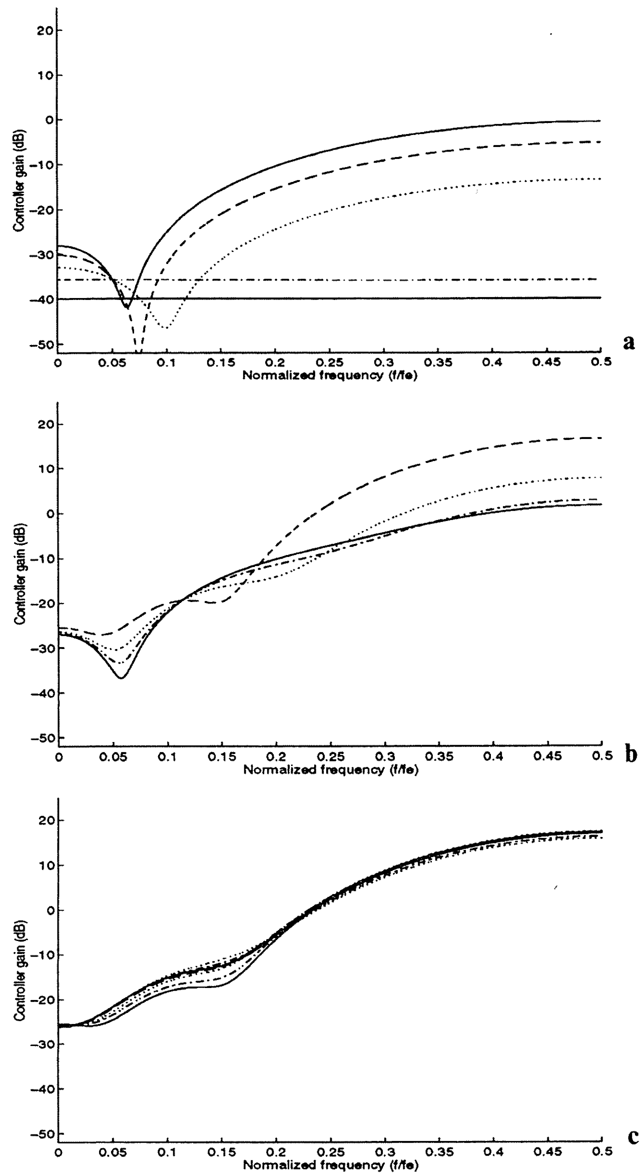

Fig. 10. Evolution of controller C_y with iterations. (a) Initial controller and controller after iterations 1–4; (b) controller after iterations 5–8; (c) Controller after iterations 9–15. In (a) and (b), the high frequency gain of the controller increases monotonically with the iteration number.

Figure 9 shows how the step response and the response to a step disturbance change with the iterations for the no-load configuration. It can be seen that every time the order of the controller is increased, the new freedom is immediately taken advantage of. After 10 or so iterations, there are only marginal changes in the step and disturbance responses and the oscillation in the step response is very small. In Fig. 10, the corresponding evolution of the feedback compensator C_y is shown. Here it is seen that when $n_r = n_y = 2$, the control strategy is a notch filter around the first resonance of the system but that when the freedom in the controller is increased to $n_r = n_y = 4$, the additional freedom allows this strategy to be abandoned for a more sophisticated one.

6. Discussion

For each iteration, the scheme requires experiments for all load cases which practically may be very inconvenient. Thus this application is most certainly not very realistic. However, we found it interesting that, disregarding the aforementioned practical inconvenience, it was quite easy to obtain satisfactory performance with this tuning scheme. With simple, intuitively appealing choices of the design quantities based on the performance specifications we almost directly ended up with a good controller. Only minor modifications of these quantities had to be made during the iteration process.

It was also demonstrated that the design can be modified by inserting dynamic simulation blocks on the input side, i.e. before the plant input signal u, or the output side, i.e. after the plant output signal y. These synthetic modifications of the plant are implementable also in the case of a real plant. In the design, a synthetic (user generated) disturbance on the input side was used to influence the sensitivity function. Furthermore a synthetic time-delay was introduced to ensure the required delay margin. These synthetic modifications can thus be used to make the design robust against certain future changes in the dynamics of the plant.

How to use the model-free design scheme for truly time-varying systems, and not only for certain plant configurations as in the benchmark application, is described in [2].

Finally, it should be stressed that we do not claim that this design procedure guarantees robust performance or stability for plants other than those included in the multicriterion.

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