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# MODELING, IDENTIFICATION AND CONTROL

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## ABSTRACT

We study the interactions between modeling, identification and control, in the situation where the only purpose of the modeling or identification is the design of a high performance controller. This leads us to suggest that the model building criterion should be determined by the control objective, leading to identification on the basis of closed loop data. We present three different approaches to this 'identification for control' paradigm: a dual control approach, an optimal experiment design approach and a robust control approach. The connections and distinctions between these three viewpoints are discussed, and recent results for each approach are briefly presented.

## 1 INTRODUCTION

Until about 1990, identification and control were developed as two separate fields of control science. With a few exceptions, there was very little research activity on the interconnection between the two areas. The exceptions (dual control, indirect adaptive control) considered essentially the rather unrealistic situation where the true system is in the model set ($\mathcal{S} \in \mathcal{M}$). The dominant idea was: identify the "best" model, then design the controller on the basis of that best model. "Best" was not "best for control design".

Until the mid-1980's, the concept of identification "design" was essentially non-existent, because the mainstream thinking was that one should identify the 'true system'. Thus, identification was not viewed as an *approximation* theory but as an alternative to modeling from physical principles, with the idea that any decent identification method should have the property that the model converges to this elusive 'true system'. L. Ljung's book [13] presented the first formal identification design concepts. The few available results on identification design for control were again limited to the case $\mathcal{S} \in \mathcal{M}$, i.e. only noise-induced errors were considered. The new theories of robust control were not taken into account in identification design. The interplay between identification design and robust control design was probably first discussed in [3], in the context of adaptive control with unmodelled dynamics ($\mathcal{S} \notin \mathcal{M}$).

The study of the interactions between modeling, identification and control design can be performed at various levels of generality and idealisation. First one should examine whether a model is really necessary for control design, or whether one cannot obtain better performance by the direct tuning of controller parameters towards the minimization of a closed loop performance criterion. We shall return to the idea of direct controller parameter tuning at the end of this paper. Assuming now that one takes a model-based approach to controller design, then the following observations are worthwhile making.

1. Experience shows that extremely simple models often lead to high performance controllers on complex processes. Thus, to compute a high performance controller it is not always necessary to have a very accurate model. The key feature is that the model should capture with high precision the dynamic characteristics that are essential for control.

2. Once a first controller operates on a process, it is typical that large quantities of data continue to be fed to the computer. Why not use these data to compute controllers with higher performance ?

3. The best open loop model is not necessarily good for control design. A famous example, produced in his PhD thesis by R. Schrama [14], has illustrated how a 5th order model of an 8th order system, identified using open loop data and validated by commonly used open loop validation criteria, led to a controller that destabilized the 'true' 8th order system, despite the fact that the two (true and model) Nyquist plots were apparently indistuinguishable.

4. Since the model is only a vehicle for control design, it is only natural that the modeling or identification criterion should be a function of the control design objective. One should view the identification and control problem as a *combined optimization problem.* However, except in the simplest of cases, the solution of such problem is presently beyond our computational reach.

To illustrate the first point, let us mention the modeling, identification and control of the Philips Compact Disc (CD) Player. Following the track on a CD involves two control loops: see e.g. [5]. A first permanent magnet/coil system mounted on the radial arm positions the laser spot in the direction orthogonal to the track. A second permanent magnet/coil system controls an objective lens which focuses the laser spot on the disc. The control system therefore consists of a 2-input/2-output system, with the spot position errors (in both radial and focus directions) as the variables to be controlled, and the currents applied to the magnet/coil actuators as the control variables. The modeling of this system using finite element methods or its estimation using spectral analysis techniques would lead to a 2-input/2-output model whose McMillan degree would be of the order of 150. However, by using an identification for control design criterion, a 16-th order model has been identified that leads to excellent control performance: see [5] for details. A comparison between the spectral estimates and the identified models for the 4 input-output channels is presented in Figure 1. The spectral estimates have been obtained by taking 100 averages over 409,600 time samples. The parametric models have been identified using 2,000 closed loop data samples.

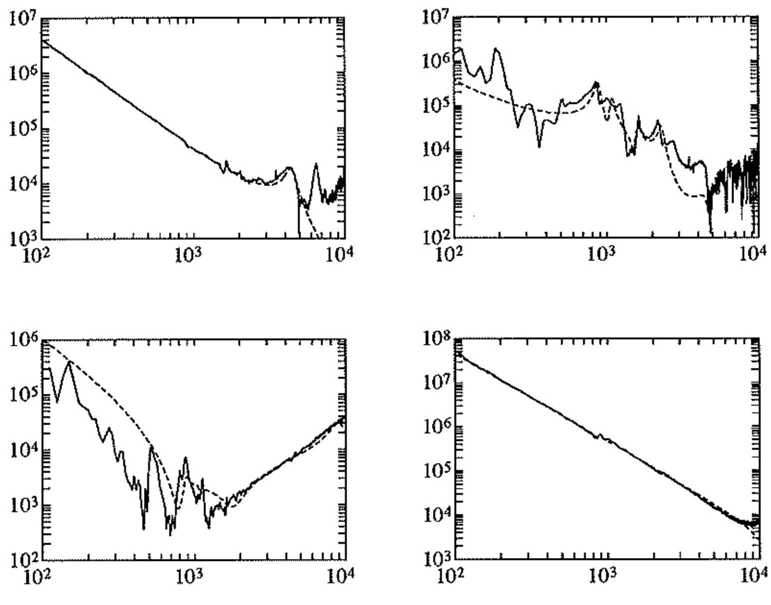


**Figure 1** Amplitude of spectral estimate (—) and of parametric model (- -)

It follows from our observations that the *modeling* techniques, which typically aim at reproducing the dynamical behavior of the system as accurately as possible on the basis of physical laws, should give way to *identification* techniques, which are data-based, and in which the identification criterion should be tuned towards the control performance objective. This will lead to an approximate (and simplified) model, and hence to a reduced order controller. To return to the example of the CD player, a controller of degree 150, say, based on a full-order model obtained from physical considerations, would be practically useless.[1]

In the remainder of this paper we therefore discuss the concept of control-oriented identification design. The concept of *identification design* was essentially introduced by L. Ljung [13] who laid down some formal design concepts. It concerns the choice of experimental conditions, model set, data filters, criterion, etc so that the approximate model that results from the identification experiment is tuned towards the objective for which it is to be used. However, the contributions to control-oriented identification design were very modest in [13].

Since 1990, there has been a tremendous research activity in control-oriented identification, and it is impossible to cover the whole spectrum of this activity in this single contribution. Rather we shall attempt to present three different conceptual approaches to the problem of *identification for control.*

At the most general (or ideal) level, one can pose the problem as a *dual control* problem, in which identification (or, more precisely, parameter estimation) and control design are posed as a *combined problem.* The solution to this problem, although reasonably easy to formulate, typically leads to intractable computational difficulties. We will explain the basic concepts of dual control and present some recent results in Section 3. We should add that all available results are limited to the case where full-order models are used, i.e. where 'the system is in the model set'.

At the next level of idealisation, one can consider to design the identification such that the ensuing model-based controller performs as close as possible to the 'ideal controller' on the actual closed loop system. By 'ideal controller' is meant the controller that would result from the model-based control design criterion if the true system were known. This problem formulation is often referred to as *optimal experiment design.* We will present some recent results

[1] Of course, an alternative to this reduced order control-oriented identification is to compute a high-order controller on the basis of an accurate high-order model of the system, and then to perform controller reduction using a closed loop performance criterion.

in this direction in Section 4; they are again limited to the case where the 'system is in the model set'.

The next and more realistic approach is to formulate the identification design on the basis of the closeness between the *achieved closed loop system* and the *nominal closed loop sytem* (= identified model with its model-based controller), rather than the unavailable *optimal closed loop system* of the previous approach. This third approach is in line with robust control design thinking. Of course, there is no guarantee that the performance achieved by the model-based designed controller will be anywhere near the optimal performance, only that it will be close to the designed performance. In this approach, it is not necessary to restrict oneself to the case where 'the system is in the model set'.

It is now time to introduce some technical information. In Section 2 we set some notations and recall the basics about Least Squares identification. In Section 3 we present the concepts of dual control, as well as some recent results. Section 4 presents some key ideas of optimal experiment design for control. In Section 5 we present the 'robust control' framework, and examine how the identification criterion should be determined by the control design criterion.

## 2 LEAST SQUARES IDENTIFICATION

We consider that the task is to design a controller for some "true" linear time-invariant scalar system described by

$$\mathcal{S} : y_t = P(q)u_t + H(q)e_t \tag{1.1}$$

where $P(q)$ and $H(q)$ are scalar rational transfer function operators, with $H(q)$ normalized such that $H(\infty) = 1$. Here $q^{-1}$ is the delay operator ($q^{-1}u_t = u_{t-1}$), $u_t$ is the control input signal, $y_t$ is the observed output signal, $e_t$ is white noise of zero mean and variance $\sigma^2$, and $v_t \triangleq H(q)e_t$ is the noise acting on $y_t$.

A controller is to be designed on the basis of a model of the plant identified using a finite set of $N$ input and output data $\{y_t, u_t, t = 1, 2, \ldots, N\}$ collected on the plant. A parametrized model set $\mathcal{M} = \{M(\theta) : \theta \in D_\theta \subset \mathbb{R}^d\}$ is used, where $D_\theta$ is a set of admissible values and $M(\theta)$ is described by:

$$M(\theta) : y_t = P(q, \theta)u_t + H(q, \theta)e_t. \tag{1.2}$$

If there exists a $\theta_0 \in D_\theta$ such that $P(q) = P(q, \theta_0)$, $H(q) = H(q, \theta_0)$, then we say that 'the system is in the model set': $\mathcal{S} \in \mathcal{M}$.

The data collection can be done in open loop or in closed loop. In the case of closed loop identification, we denote by $C_{id}(q)$ the controller that operates during identification:

$$u_t = C_{id}(q)[r_t - y_t], \tag{1.3}$$

where $r_t$ is the reference excitation signal used during identification.

The Least Squares prediction error method applied to $N$ input-output data delivers an estimate $\hat{\theta}_N$ of $\theta$:

$$\hat{\theta}_N = \arg\min_{\theta \in D_\theta} V_N(\theta) \tag{1.4}$$

where

$$\begin{aligned} V_N(\theta) &= \sum_{t=1}^{N} [\epsilon_t^f(\theta)]^2 && (1.5) \\ \epsilon_t^f(\theta) &= D(q)\epsilon_t(\theta) = \frac{D(q)}{H(q,\theta)}[y_t - P(q,\theta)u_t]. && (1.6) \end{aligned}$$

Here $\epsilon_t(\theta) \triangleq y_t - \hat{y}_{t|t-1}(\theta)$ is the one-step-ahead prediction error for the model $M(\theta)$, while $D(q)$ is a data filter. In turn, this produces a model:

$$\hat{P}_N = P(e^{j\omega}, \hat{\theta}_N), \quad \hat{H}_N = H(e^{j\omega}, \hat{\theta}_N). \tag{1.7}$$

Under reasonable conditions on the data and the model structure [13], $\hat{\theta}_N$ converges as $N \to \infty$ to

$$\theta^* = \arg\min_{\theta \in D_\theta} \bar{V}(\theta) \tag{1.8}$$

where

$$\bar{V}(\theta) = \lim_{N \to \infty} E V_N(\theta) = E[\epsilon_t^f(\theta)]^2. \tag{1.9}$$

If identification is performed using closed loop data (as is often the case in identification for control), then the asymptotic expression for the cost criterion becomes[2]:

$$\bar{V}(\theta) = \frac{1}{2\pi}\int_{\pi}^{\pi} \left\{ \left| \frac{(P - P(\theta))}{1 + PC_{id}} \right|^2 |C_{id}|^2 \Phi_r + \left| \frac{1 + P(\theta)C_{id}}{1 + PC_{id}} \right|^2 \Phi_v \right\} \frac{|D|^2}{|H(\theta)|^2} d\omega \tag{1.10}$$

This expression gives an implicit characterization of the model to which $P(e^{j\omega}, \hat{\theta}_N)$, $H(e^{j\omega}, \hat{\theta}_N)$ converge if the number of data tends to infinity. Thus, it plays a key role in understanding the *bias error* distribution obtained with closed loop identification. An approximate expression for the *variance error* on the transfer function estimates can also be obtained: see e.g. [13].

[2]For reasons of space, we have deleted the $\omega$-dependence in all arguments.

# 3 THE DUAL CONTROL APPROACH

In this section we present the key ideas of *dual control*, as well as some recent developments. Dual control can be seen as one way (perhaps even the optimal way) of solving the joint identification and control design problem when the system is in the model set. Thus, all the ideas and results of this section are limited to the case where $\mathcal{S} \in \mathcal{M}$. Our presentation borrows heavily from the recent PhD thesis of C. Kulcsár [12].

The concept of dual control was introduced by Fel'dbaum [6] who understood that, when one wants to minimize a control performance criterion for a system with unknown parameters, the control has the dual role of maintaining the state close to its desired value while at the same time learning the unknown parameters. These two roles are conflicting.

Thus, consider now that the parameter vector $\theta$ has a prior probability distribution, say $\Pi(\theta)$, and that the task is to design an optimal control sequence $U_0^N \triangleq \{u_0, u_1, \cdots, u_{N-1}\}$ that minimizes the following cost:

$$J_{0,N}(U_0^{N-1}) = E[\sum_{t=0}^{N-1} c_t(u_t, \theta) \mid I^0], \tag{1.11}$$

where

$$c_t(u_t, \theta) = (y_{t+1} - y_{t+1}^*)^2 + \lambda_t u_t^2, \tag{1.12}$$

with $\lambda_t$ nonnegative scalars, and where $I^0$ contains all prior information about the system, i.e. the noise distribution and the prior distribution on $\theta$. The minimum of (1.11) is obtained by solving the following succession of nested optimization problems:

$$\begin{aligned} J_{0,N}^* = \min_{u_0} E[c_0(u_0, \theta) + \min_{u_1} E[c_1(u_1, \theta) + \ldots \\ + \min_{u_{N-1}} E[c_{N-1}(u_{N-1}, \theta) \mid I^{N-1}] \ldots \mid I^1 \mid I^0] \end{aligned}$$

Define

$$J_{t,N}(u_t) = E[c_t(u_t, \theta) + J_{t+1,N}^* \mid I^t] \tag{1.13}$$

and

$$J_{t,N}^* = \min_{u_t} J_{t,N}(u_t). \tag{1.14}$$

Notice that the 'cost-to-go', $J_{t,N}(u_t)$, can be written as

$$J_{t,N}(u_t) = \int \left[c_t(u_t, \theta) + J_{t+1,N}^*\right] \times \Pi(y \mid \theta, u_t)\, \Pi(\theta \mid I^t)\, dy\, d\theta \tag{1.15}$$

Observe that the distribution of the random vector $\theta$ influences the distribution of all future input and output signals, while it is itself influenced by these signals through the parameter estimation procedure. This puts the exact solution to the dual optimal control problem beyond the reach of present-day computer technology except in the simplest of cases.

For this reason, a lot of effort has been spent on computing suboptimal solutions. One particularly simple suboptimal solution is to assume, at time $t$, that $\hat{\theta}_t$ is exact, i.e. to replace the probability distribution of $\theta$ in (1.15) by a probability density centered at $\hat{\theta}_t$ and with zero variance. This idea is called *Certainty Equivalence* (CE) control. It does not take into account the effect of the control on the precision of future estimates of $\theta$.

Interesting progress has been accomplished in the PhD thesis of C. Kulcsár [12]. For Finite Impulse Response (FIR) models, and assuming a Gaussian distribution for all random variables, she observed that the expected values of all future covariances of $\theta$ depend on the future only through their dependence on the future control signals. She then proposed a suboptimal solution in which, at time $t$, the mean of the future $\{\hat{\theta}_k, k \geq t\}$ is frozen at $\hat{\theta}_t$, while the expected values of the covariances $\{\Sigma_k, k \geq t\}$ are calculated exactly. These future covariances depend on all future controls $\{u_t, \cdots, u_{N-1}\}$, that need to be optimized. This suboptimal problem can now be solved using CPU times that are entirely reasonable, yielding solutions that are vastly superior to Certainty Equivalence solutions. Illustrative examples can be found in [12].

# 4 OPTIMAL IDENTIFICATION DESIGN FOR CONTROL

The optimal identification design is based on the minimization of a quality criterion that compares the *optimal closed loop system* with the *actual closed loop system*: see Figures 2 and 3. These two loops are assumed driven by the same external signals, the reference $r_t$ with spectrum $\phi_r(\omega)$, and the white noise $e_t$ with variance $\sigma^2$. Their outputs are denoted $y_t^o$ and $y_t$, their inputs $u_t^o$ and $u_t$, respectively. In the optimal closed loop system of Figure 2, the controller $C$ is computed from the true plant $[P, H]$ using the chosen control design criterion: $C = c(P, H)$. In the actual closed loop system of Figure 3, the controller $\hat{C}$ is the Certainty Equivalence controller that results from a model $[\hat{P}_N, \hat{H}_N]$ identified using $N$ data: $\hat{C} = \hat{C}_N \triangleq c(\hat{P}_N, \hat{H}_N)$.

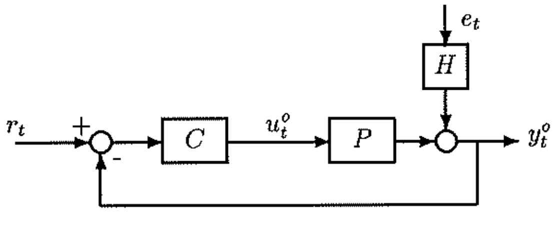


**Figure 2** Optimal closed loop system

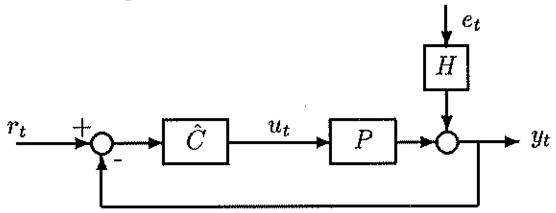


**Figure 3** Actual closed loop system

Our *identification design criterion* will be

$$J_V = E[y_t^o - y_t]^2. \tag{1.16}$$

It is a measure of the degradation that results from using the estimated $\hat{C}_N$ on the plant instead of the optimal $C$. The use of this measure as an identification design criterion was first proposed in [8].

Assuming $\Delta C_N \triangleq \hat{C}_N - C$ to be small and using a Taylor series expansion, one arrives, after some manipulations, at:

$$J_V \cong \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|P|^2\phi_{y^o}}{|1+PC|^2} E|\Delta C_N|^2 d\omega \tag{1.17}$$

Here, the expected value is taken with respect to the probability distribution of the noise during the identification experiment, which produces the random variable $\hat{C}_N = c(\hat{P}_N, \hat{H}_N)$. With $\Delta C_N$ small, assume again that we can write :

$$\Delta C_N = [F_1 \;\; F_2] \begin{bmatrix} \Delta P_N \\ \Delta H_N \end{bmatrix}, \tag{1.18}$$

where $\Delta P_N \triangleq P - \hat{P}_N$, $\Delta H_N \triangleq H - \hat{H}_N$, $F_1 = \frac{\partial C}{\partial P}$ and $F_2 = \frac{\partial C}{\partial H}$. Inserting the standard covariance formula for $[\Delta P_N, \Delta H_N]$ [13] in (1.18) yields, after some further manipulations, the following expression for the variance of the controller error at a frequency $\omega$ [9]:

$$E|\Delta C_N(e^{j\omega})|^2 \cong \frac{n}{N}|H|^2\{|F_2|^2 + \frac{\sigma^2}{\phi_r}|F_1 + (F_1G + F_2H)C_{id}|^2\} \tag{1.19}$$

It follows immediately that the controller variance $E|\Delta C_N(e^{j\omega})|^2$ is minimized, at every frequency, by performing the identification *in closed loop* with an operating controller:

$$C_{id}^{opt}(q) = -\frac{F_1(q)}{F_1(q)P(q) + F_2(q)H(q)}. \tag{1.20}$$

This optimal choice of course also minimizes the criterion $J_V$.

COMMENTS

- As is typical of optimal experiment design results, this optimal design depends on the unknown system $[P, H]$ and is therefore not feasible.
- It has been shown in [9] that, for Model Reference Control, $C_{id}^{opt}(q) = C(q)$. The same result had been shown for Minimum Variance Control in [8].
- If identification is performed under the ideal closed loop condition with $C_{id} = C_{id}^{opt}(q)$, the control error variance becomes:

$$E|\Delta C_N(e^{j\omega})|_{idcl}^2 \cong \frac{n}{N}|H|^2|F_2|^2. \tag{1.21}$$

With open loop identification, we get

$$E|\Delta C_N(e^{j\omega})|_{ol}^2 \cong \frac{n}{N}|H|^2\{|F_2|^2 + \frac{\sigma^2|F_1|^2}{\phi_u}\} \tag{1.22}$$

Even though this result is of obvious theoretical interest, it might not appear very useful given that the optimal identification design depends on the unknown system. However, it is shown in [9] that, when $C_{id}^{opt}(q) = C(q)$ (as is the case for Minimum Variance Control and Model Reference Control), an iterative identification and control design leads to a better controller than open loop identification. By iterative design is meant that identification is performed first in open loop for a fraction of the total data collection interval; the model estimated at the end of that interval is used to design a certainty equivalence controller, which is applied to the plant; the identification is continued in closed loop during a second time interval with this controller operating on the plant; at the end of this second interval a new certainty equivalence controller is computed from the present model and applied to the plant again; etc. A simulation example illustrating this iterative scheme is presented in [9].

# 5 MATCHING IDENTIFICATION AND CONTROL CRITERION

The main drawback of the *optimal identification design for control*, as we have noted, is that the design depends upon the unknown system, because the design criterion $J_V$ is based on a comparison between the *optimal* and the *achieved* closed loop systems of Figures 2 and 3. An alternative, but suboptimal, formulation of the identification criterion is based on a comparison between the *achieved* closed loop system of Figure 3 and the *designed or nominal* closed loop system of Figure 4.

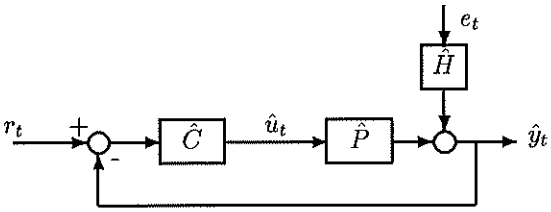


**Figure 4** Designed (or nominal) closed loop system

Comparing these two loops is classical in robust control theory; thus, we call this third approach to identification design for control the *robust control approach.* However, a key difference (and complication) in our problem formulation with respect to classical robust control thinking is that now both the nominal model $\hat{P}$ and the controller $\hat{C}$ are objects to be designed, as opposed to just $\hat{C}$ in classical robust control design. Thus, one must attempt to perform both the identification design and the control design in such a way that the nominal performance is high and that the two loops of Figures 3 and 4 are 'close to one another' in a sense to be defined.

If nothing is fixed, then such problem formulation might not make sense. To simplify matters somewhat, we make the following assumptions.

- A control criterion has been selected.
- At some stage of the design, a model structure has been chosen, typically of lower complexity than the 'true system'.

With these assumptions, one can show that every control criterion induces an identification criterion that 'matches' that control criterion. In addition these identification criteria take the form of *closed loop identification* criteria. The idea that an identification criterion can be made to match a control criterion was initially advanced for LQG control by Zang et al. [16] and for $H_\infty$ control by Schrama [14]. It was applied to pole placement control and analysed by Åström [1] and Åström and Nilsson [2].

To explain the matching of the control and identification criteria, we take the simplest control design problem, namely the pole placement control problem without disturbances analysed by Åström. Thus, consider the two loops of Figures 3 and 4, and consider that the control design problem is to design $\hat{C}$ such that the designed closed loop transfer function from $r_t$ to $\hat{y}_t$ is a given reference model, i.e. compute $\hat{C}$ from $\hat{P}$ such that

$$\frac{1}{1+\hat{P}(q)\hat{C}(q)} = S(q), \tag{1.23}$$

where $S(q)$ is some admissible reference model.

It follows from Figures 3 and 4 that the 'control performance error', defined as the error between the actual and the designed outputs, is given by:

$$y_t - \hat{y}_t = S\left[\frac{P\hat{C}}{1+P\hat{C}}r_t - \frac{\hat{P}\hat{C}}{1+P\hat{C}}r_t\right] = S[y_t - \hat{P}u_t]. \tag{1.24}$$

Equation (1.24) can be seen as an equality between a control performance error on the left hand side (LHS) and a filtered identification error on the right hand side (RHS). Indeed, the RHS is a filtered (by $S(q)$) version of the output error $y_t - P(q,\theta)u_t$, where $u_t$ and $y_t$ are collected on the actual closed loop system of Figure 3 with $\hat{C}$ operating. Thus, it appears that the control performance error can be minimized by performing *identification in closed loop with a data filter* $S(q)$. However, in the RHS of (1.24) the closed loop signals $y$ and $u$ are functions of the controller, and hence of the model parameter vector $\theta$. In addition, the sensitivity function $S(q)$, which is used as a reference model in the present design, may also be a function of $\theta$ as is the case when the model contains nonminimum phase zeros that need to be preserved in the closed loop system. Hence, a more suggestive way to write (1.24) is as follows:

$$y_t - \hat{y}_t = S(q,\theta)[y_t(\theta) - P(q,\theta)u_t(\theta)]. \tag{1.25}$$

Even though the RHS of (1.25) looks like a filtered closed loop prediction error, it cannot be minimized by standard identification techniques, because $\theta$ appears everywhere and not just in $P(\theta)$. As a consequence, the approach suggested in all known 'identification for control' schemes is to perform identification and control design steps in an iterative way, whereby the $i$-th identification step is performed on filtered closed loop data collected on the actual closed loop system with the $(i-1)$-th controller operating in the loop. This corresponds to an $i$-th identification step in which the following filtered prediction error is minimized with respect to $\theta$:

$$y_t - \hat{y}_t = S(q, \hat{\theta}_{i-1})[y_t(\hat{\theta}_{i-1}) - P(q,\theta)u_t(\hat{\theta}_{i-1})]. \tag{1.26}$$

We refer the reader to [7] and [15] for details on such iterative schemes.

**Note:** The equality (1.25) between a control error and an apparent 'identification error' has been derived here on the basis of a model reference control desgin scheme. For optimization-based control design criteria, such as e.g. LQG, or $H_\infty$, one does not arrive at an equality such as (1.25), but one can upper-bound the mismatch between the achieved and the designed control criteria by the norm of the RHS of (1.25). Thus, one is also led to iterative identification and control schemes in which the appropriate norm of the RHS of (1.26) is minimized by an identification step.

An interesting question is whether these iterative identification and control schemes converge to the minimum of the achieved cost over the set $\mathcal{C} \triangleq \{\hat{C}(P(\theta)) \;\; \forall\theta \in D_\theta\}$ of all certainty equivalence controllers. For the example given above, this corresponds to asking whether, by successively minimizing over $\theta$ the mean square of the prediction errors defined by (1.26), one will converge to the minimum of

$$J(\theta) \triangleq E\{S(q,\theta)[y_t(\theta) - P(q,\theta)u_t(\theta)]\}^2. \tag{1.27}$$

This question has been analyzed in [11], where it has been shown that the answer is in general negative: the iterative identification and control schemes do not generically converge to the minimum of the achieved cost. In fact, it has been shown in [4] that the optimal controller within this reduced order controller set $\mathcal{C}$ is not always the Certainty Equivalence controller of a model $\hat{P}$ that can be obtained as a result of an identification experiment on the true system, whether the data are collected in open loop or in closed loop.

The last observation raises the question whether one could not minimize the criterion (1.27) directly by some optimization method, without resorting to identification. In fact, minimizing (1.27) over all possible $\theta$ corresponds to a direct minimization over the controller set $\mathcal{C}$, since every model parameter vector $\theta$ defines a controller parameter vector, say $\rho = \rho(\theta)$, via the mapping $C(\rho) = C(P(\theta))$. In particular, returning to the model reference problem above, we note that $y_t - \hat{y}_t$ can also be written as:

$$y_t - \hat{y}_t = \frac{P\hat{C}}{1 + P\hat{C}} r_t - Tr_t = y_t(\rho) - Tr_t. \tag{1.28}$$

where $T = 1 - S$ and is a fixed design quantity. Thus, minimizing $J(\theta)$ is equivalent to minimizing the control criterion $E[y_t(\rho) - Tr_t]^2$ with respect to the controller parameters $\rho$. Recent work of Hjalmarsson et al. [10] has shown that this is indeed possible using an iterative scheme that avoids any identification step.

## 6 CONCLUSIONS

We have given some motivation for the idea of performing control-oriented identification. This research activity, though very recent, is now in full bloom. Although the work so far is essentially theoretical, successful industrial applications have already been reported and the rewards in terms of productivity improvements appear to be promising.

The problem of identification design for control, and more generally of the combined design of identification and control, has been addressed from a wide range of angles. In this chapter we have presented three different approaches to the problem. These reflect more the author's perspective on the problem than they cover the very wide range of viewpoints and results that have been presented in the fast growing literature on the subject. We refer to the special issues of journals and the symposia, workshops and numerous invited sessions on this subject for a fuller coverage.

Our presentation has gone from the very idealised 'dual control approach' through the 'optimal design approach' to the more realistic 'robust control approach'. This last approach leads to the much publicised iterative identification and control design schemes. Finally we have raised the question of whether these iterative identification for control approaches and methods should not, in the end, give way to a more direct controller parameter tuning approach, particularly now that such a scheme is available that requires almost no prior knowledge on the real plant.

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