Håkan - icd/ CDC94 - hakan - icd/maint.ps C78 CDC 94 Final - 30/8/94

A Convergent Iterative Restricted Complexity Control Design Scheme [1]

Håkan Hjalmarsson#[2], Svante Gunnarsson# and Michel Gevers$
#Dept. of Electrical Engineering, Linköping University, S-581 83 Linköping, Sweden
$CESAME, Louvain University, B-1348 Louvain-la-Neuve, Belgium

Abstract

In this contribution we propose an optimization approach to the design of a restricted complexity controller. The design criterion is of LQG type containing two terms. The first term is the quadratic norm of the error between the output of the true closed loop and a desired response. The second term is the quadratic norm of the input signal. It is shown that the minimization of this criterion do not require a model of the system. Closed loop experimental data can be used instead. The result is an iterative scheme of closed loop experiments and controller updates which converges to a local minimum of the design criterion under the condition of bounded signals.

Keywords: Control Design, Optimization

1. Introduction

Many control objectives can be expressed in terms of a criterion function, LQG and H_∞ control being the standard examples. Generally, explicit solutions to such optimization problems require full knowledge of the plant and disturbances and complete freedom in the complexity of the controller. In practice, neither of these conditions are satisfied. Recently, so called iterative identification and control design schemes have been proposed in order to overcome these problems, see e.g. [1], [2] and [3]. These schemes iteratively perform plant model identification and model-based controller update in the closed loop. Behind these schemes is the notion that closed loop experiments with the best available controller should generate data that are "good" for identification of models suited for a new and improved control design. See [4] for a survey.

[1]This paper presents research results of the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. The scientific responsibility rests with its authors.

[2]The ideas in this paper were developed during Sept 1993-March 1994 when the first author was on postdoctoral leave at CESAME, Louvain University.

However, no one seems to have explored the fact that the objective is to minimize a control design criterion. In the case of a known plant, Edmunds [5] has suggested an optimization based reduced order control design procedure. Here we consider the completely different situation where the plant is assumed to be unknown, and we connect the idea of iterative design with that of numerical optimization. This naturally leads to a Gauss-Newton scheme that tends to a descent direction of the control design criterion. An important ingredient of the method is that the gradient of the criterion is based directly on closed-loop experiments. No modeling procedure is involved. This is in stark contrast with the optimization based approaches that previously have appeared in an adaptive control context [6] and [7]. There the gradient of the criterion is obtained through the estimation of a full order model of the plant and disturbance characteristics.

The scheme lends itself both to direct optimization of controller parameters or to indirect methods where a (reduced order) model is used in an intermediate step. However, in both cases it is the control design criterion that is minimized, either directly with respect to the controller parameters, or indirectly with respect to the model parameters which can be viewed as a reparametrization of the controller, given a model based control design criterion.

The problem of optimization based iterative control design offers an interesting possibility compared with usual optimization problems, namely the possibility of adapting the criterion. If the initial controller gives bad performance, it can be quite tricky to find the optimal controller, *i.e.* the surface of the criterion can be very rough, thus allowing only small steps in each iteration. However, it is the authors' experience that the problem is simplified by starting with an objective that is easier to achieve (lower bandwidth) and then successively increasing the bandwidth as the performance is increased. This has close ties with the so called windsurfing approach [3] to iterative control design.

This contribution is disclosed as follows. In Section 2 we present the design criterion and in Section 3 we discuss how this criterion can be minimized using experimental data. The convergence result is established in Section 4 while engineering aspects are considered in Section 5. An example is presented in Section 6.

2. The design criterion

Let the true system be given by

$$y(t) = G_0(q)u(t) + v(t) \tag{1}$$

where $\{v(t)\}$ is a (process) disturbance. The output, $\{y(t)\}$, from the true system will be called the achieved response. We will use the following two degrees of freedom controller:

$$u(t) = C_r(q,\rho)r(t) - C_y(q,\rho)y(t) \tag{2}$$

where $\{r(t)\}$ is an external reference signal. The parameter ρ represents the parametrization of the controller pair $C = \{C_r, C_y\}$. This parametrization can be direct, *e.g.* $C_r(\rho) = \rho_1 + \rho_2 q^{-1}$, or indirect by means of a model $G(\theta)$ of the true system. In the latter case, the controller parameters are functions of the model parameters, $\rho = \rho(\theta)$, with the function resulting from some control design criterion. It is possible for $C_r(\rho)$ and $C_y(\rho)$ to have common parameters. To ease the notation somewhat we will from now on omit the time argument of the signals. In addition, whenever signals are obtained from the closed loop system with the controller $\{C_r(\rho), C_y(\rho)\}$ operating, we will indicate this by using the ρ-argument; thus, $y(\rho)$ will denote the output of the system (1) in feedback with the controller (2).

Let T_d be a desired stable closed loop response from reference signal to output signal

$$y_d = T_d r. \tag{3}$$

The error between the achieved and desired response is

$$\begin{aligned}\tilde{y}(\rho) &= y(\rho) - y_d \\ &= \frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} r - T_d r + \frac{1}{1 + C_y(\rho)G_0} v.\end{aligned} \tag{4}$$

It is natural to formulate the design objective as a minimization of some norm of $\tilde{y}(\rho)$. Although it is not necessary from a procedural viewpoint we shall restrict the attention to the quadratic criterion:

$$\rho^* = \arg\min_{\rho} J(\rho), \tag{5}$$

where

$$J(\rho) = \frac{1}{2}\mathrm{E}\left[(L_y\tilde{y}(\rho))^2\right] + \frac{\lambda}{2}\mathrm{E}\left[(L_u u(\rho))^2\right]. \tag{6}$$

Here E denotes expectation over v and r which we assume to be realizations of stationary stochastic processes. In this criterion the first term is the, by L_y, frequency weighted norm of the error between the desired response and the achieved response. The second term is the penalty on the control effort which is frequency weighted by the filter L_u. As formulated, this is a model reference problem with an additional penalty on the control effort. Notice though that with $T_d \equiv 1$ this becomes an LQG problem with tracking.

With $T_0(\rho)$ and $S_0(\rho)$ denoting the achieved closed loop response and the sensitivity function with the controller $\{C_r(\rho), C_y(\rho)\}$, and given the statistical independence of r and v, $J(\rho)$ can be written as

$$\begin{aligned}J(\rho) &= \frac{1}{2}\mathrm{E}\left[\{L_y(T_d - T_0(\rho))r\}^2 + \{L_y S_0(\rho)v\}^2\right] \\ &\quad + \frac{\lambda}{2}\mathrm{E}\left[(L_u u(\rho))^2\right].\end{aligned} \tag{7}$$

The first term is the tracking error, the second term is the contribution due to the disturbance and the last term is the penalty on the control effort.

3. Criterion minimization

We now address the minimization of $J(\rho)$ given by (6). To facilitate the notation we shall in this section assume that $L_y = L_u = 1$. It is straightforward to include these. It is evident from (4) that $J(\rho)$ depends in a fairly complicated way on ρ. Furthermore, the true system G_0 and the spectrum of $\{v\}$ are unknown.

The problem we would like to solve is to find a solution for ρ to the equation

$$0 = J'(\rho) = \mathrm{E}\left[\tilde{y}(\rho)\tilde{y}'(\rho)\right] + \lambda\mathrm{E}\left[u(\rho)u'(\rho)\right]. \tag{8}$$

This is done by taking repeated steps in a descent direction

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} J'(\rho_i). \tag{9}$$

Here R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of J, such as a Gauss-Newton approximation of this Hessian. As stated this problem is intractable since it involves taking expectation. It is, however, exactly a problem that can be attacked with stochastic approximation procedures such as suggested by Robbins and Monro [8]. One replaces J' with an approximation based on the current samples. In order to do this, the signals $\tilde{y}(\rho_i)$ and $u(\rho_i)$ and their gradients $\tilde{y}'(\rho_i)$ and $u'(\rho_i)$

are required and in what follows we examine how to generate estimates of these signals.

From (4) it is clear that $\tilde{y}(\rho_i)$ is obtained by taking the difference between the achieved response from the system operating with the controller $C(\rho_i)$ and the desired response. Regarding $\tilde{y}'(\rho)$, we have the following expression

$$\tilde{y}'(\rho) = \frac{1}{C_r(\rho)} C_r'(\rho) T_0(\rho) r - \frac{1}{C_r(\rho)} C_y'(\rho) \left(T_0^2(\rho) r + T_0(\rho) S_0(\rho) v \right). \quad (10)$$

In this expression the quantities $C_r(\rho)$, $C_r'(\rho)$ and $C_y'(\rho)$ are known functions of ρ which depend on the parametrization of the restricted complexity controller, while the quantities $T_0(\rho)$ and $S_0(\rho)$ depend on the unknown system. Thus, $\tilde{y}'(\rho)$ can only be obtained by running experiments on the actual closed loop system. We note also that the last two terms in (10) involve double filtering of the signals r and v through the closed loop system. Thus, to obtain these signals more than one experiment is needed. In [6] this problem is avoided by using an estimation procedure of the full order plant and then a synthetic generation of the gradient. Here we shall use experiments for the computation of $\tilde{y}'(\rho_i)$ in each iteration. To proceed, notice that

$$T_0 y = T_0^2 r + T_0 S_0 v.$$

Thus the last two terms in (10) can be obtained by taking the output signal from one experiment on the closed loop system and using it as reference signal in a separate experiment. In each iteration i we will use three experiments, each of duration N say, with the fixed controller $C(\rho_i)$ operating on the actual plant; we denote the corresponding output signals by $\{y_i^j\}$, $j = 1, 2, 3$. The corresponding reference signals are

$$r_i^1 = r; \quad r_i^2 = y_i^1; \quad r_i^3 = r. \quad (11)$$

This gives the following expressions for the output signals

$$\begin{aligned} y_i^1 &= T_0(\rho_i) r + S_0(\rho_i) v_i^1 && (12) \\ y_i^2 &= T_0^2(\rho_i) r + T_0(\rho_i) S_0(\rho_i) v_i^1 + S_0(\rho_i) v_i^2 && (13) \\ y_i^3 &= T_0(\rho_i) r + S_0(\rho_i) v_i^3 && (14) \end{aligned}$$

where v_i^j denotes the disturbance acting on the system at iteration i and experiment j. Notice that these disturbances are mutually independent since they come from different experiments. With these experiments

$$\tilde{y}_i = y_i^1 - y_d \quad (15)$$

is a perfect realization of $\tilde{y}(\rho_i)$ and

$$\hat{\tilde{y}}_i' = \frac{1}{C_r(\rho_i)} \left(C_r'(\rho_i) y_i^3 - C_y'(\rho_i) y_i^2 \right) \quad (16)$$

is a perturbed version (by the disturbances v_i^2 and v_i^3) of $\tilde{y}'(\rho_i)$ since

$$\hat{\tilde{y}}_i' = \tilde{y}'(\rho_i) + \frac{S_0(\rho_i)}{C_r(\rho_i)} \left(C_r'(\rho_i) v_i^3 - C_y'(\rho_i) v_i^2 \right). \quad (17)$$

There are several things to observe here. Firstly, the disturbance that is generated in the first experiment is not a nuisance. The output of the first experiment is used in (15) to create an exact version of the signal $\tilde{y}(\rho_i)$ which is used in the criterion J: see (4). Secondly, the output of the first experiment (with the disturbance) is exactly what one wants to use as reference signal in the second experiment. The only nuisances that are introduced are the disturbance contributions from the second and third experiments.

An estimate of the gradient

With the signals defined in the preceding subsections, an estimate of the gradient of J can be formed by taking

$$\hat{J}_i' = \frac{1}{N} \sum_{t=1}^{N} \left(\tilde{y}_i(t) \hat{\tilde{y}}_i'(t) + \lambda u_i(t) \hat{u}_i'(t) \right). \quad (18)$$

Notice here that

$$\mathrm{E}\left[\hat{J}_i' \right] = J'(\rho_i), \quad (19)$$

which basically is what is needed for a stochastic approximation algorithm to work. The requirement (19) is the motivation for the third experiment. Otherwise, it would be tempting to use the data from the first experiment instead of the third one in (16), but then (19) would not hold because the error between $\hat{\tilde{y}}_i'$ and $\tilde{y}_i'$ would be correlated with $\tilde{y}_i$ and the error between $\hat{u}_i'$ and u_i' would be correlated with u_i. When one is far away from the minimum this bias is negligible and one can omit the third experiment. However, as soon as the algorithm seems to converge the third experiment should be included.

Modification of search direction

There are many possible choices for the update direction R_i in the iteration (9). The identity matrix gives the negative gradient direction. Another interesting choice is

$$R_i = \frac{1}{N} \sum_{t=1}^{N} \left(\hat{\tilde{y}}_i'(t) \left[\hat{\tilde{y}}_i'(t) \right]^T + \lambda \hat{u}_i'(t) \left[\hat{u}_i'(t) \right]^T \right), \quad (20)$$

for which the signals are available from the experiments described above. This will give a biased (due to the disturbance in the second experiment) approximation of the Gauss-Newton direction. It is the authors' experience that this choice is superior to the pure gradient.

Implementation

Notice that the computation of $\hat{\tilde{y}}_i'$ in (16) requires the filtering with the inverse of C_r. If C_r is non-minimum phase, as may happen, this is not feasible. A similar problem occurs if the gradients of C_y and/or C_r are unstable. These problems can be overcome by extending L_y and L_u with an all-pass frequency weighting filter L_a, which leaves the objective function $J(\rho)$ of (6) unchanged. We omit the details due to lack of space. We now summarize the algorithm.

Algorithm 3.1
With a controller $C(\rho_i) = \{C_r(\rho_i), C_y(\rho_i)\}$ operating on the plant, generate the signals y_i^1, y_i^2, y_i^3 of (12)-(14) and compute $\tilde{y}_i$, $\hat{\tilde{y}}_i'$, u_i and $\hat{u}_i'$ using (15), (16) and the corresponding expressions for the input signal. Let the next controller parameters be computed by:

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \hat{J}_i' \tag{21}$$

where $\hat{J}_i'$ is given by (18), where $\{\gamma_i\}$ is a sequence of positive real numbers that determines the step size and where $\{R_i\}$ is a sequence of positive definite matrices that are, for example, given by (20). Repeat this step, replacing i by $i + 1$.

4. Convergence analysis

The convergence properties of the algorithm are given in the next theorem.

Theorem 4.1 *Consider Algorithm 3.1 with $R_i \geq \delta I$ $\forall i$, for some $\delta > 0$. Assume that the procedure is complemented with the all-pass filtering procedure described above included if necessary so that we have an unconstrained minimization problem, i.e. $\rho \in \mathbb{R}^d$ for some integer d. Assume that the $\{\gamma_i\}$ satisfy the usual conditions for convergence [9]:*

$$\sum_{i=1}^{\infty} \gamma_i = \infty, \quad \sum_{i=1}^{\infty} \gamma_i^2 < \infty.$$

Let the reference signal $\{r\}$ and the disturbances in each experiment $\{v_i^j\}$ be realizations of bounded stationary stochastic processes where these processes are mutually independent.

Then, provided that the signals $\{y_i^j\}$; $j = 1,2,3$; $i = 1,2,\ldots$ stay bounded,

$$\rho_i \to \{\rho : J'(\rho) = 0\} \qquad w.p.1. \tag{22}$$

Proof: The theorem follows from Theorem 1 p. 77 in [10] (which originally appeared in [9]) by the use of $f(x) = \frac{1}{2}\mathrm{E}\left[\tilde{y}^2(x)\right] + \frac{\lambda}{2}\mathrm{E}\left[u^2(x)\right]$. ■

This appears to be the first convergence result for an iterative restricted complexity control design scheme.

5. Engineering aspects

We believe that this scheme has the potential of becoming a useful tool for the tuning of controller parameters in practical use. The method is simple and applies to tuning of simple PID controllers as well as more complex controllers. A special feature is that there is no need to select an appropriate input spectrum. The engineer is supplied with design tools that are easy to grasp: the step size which controls how much the controller is allowed to change in an iteration; the frequency weighting filters L_y and L_u which can be used to emphasize certain frequency regions, and the possibility to modify the desired bandwidth of the system. We now go through these tools in more detail.

Interactive smooth controller update

The engineer can use the step size to control how much a controller changes from one iteration to another. Before actually implementing a controller it is possible to compare the Bode plots of the new controller with the previous ones to see whether they are reasonably consistent. If one doubts whether it will work or not one has the possibility of decreasing the step size and/or of extending the experiment so as to reduce the effects of the disturbances in the gradient calculation. The situation is quite comforting: one is backed up by the knowledge that for a small enough step size and large enough data set one will always go in a descent direction of the criterion.

Criterion modification

There is full freedom in modifying the criterion at any iteration. At an initial stage one may not know how large a bandwidth it is possible to achieve with the chosen complexity of the controller. Thus, one may in the beginning make a conservative choice and as the performance is improved increase the requirements. This is much in the spirit of the windsurfer approach [3].

On-line considerations

Even though the second experiment uses a different reference signal than the desired one, the scheme is essentially one that functions with data collected during normal operating conditions. It is possible to show that the the response of the second experiment, which uses the reference signal $r_i^2 = y_i^1$, is not so much different from the response of the two other experiments that are driven by the desired signal r.

6. A numerical illustration

To illustrate the Algorithm 3.1 we have used a sampled version of the following system

$$G(s) = \frac{9}{(s+1)(s^2+0.06s+9)}$$

which has also been used in [3]. The sampling frequency used was $T_s = 0.1$ s. The Bode diagram is depicted in Figure 1. The sampled data were corrupted by a zero mean white disturbance with variance $\sigma^2 = 0.02$. The design objective was to have a step response corresponding to a second order system with a double pole corresponding to a (continuous time) -6dB bandwidth of 4 rads^{-1}, *i.e.* the desired reference model T_d is of the type

$$T_d = \frac{(1-\alpha)^2}{(1-\alpha q^{-1})^2}$$

where $\alpha = e^{-0.1\omega_B}$ where ω_B is the desired continuous time bandwidth. The criterion was taken to be (6) with $\lambda = 0$. The controller has one degree of freedom and is a simple extension of a PID controller:

$$C(\rho) = \frac{\rho_1 + \rho_2 q^{-1} + \rho_3 q^{-2} + \rho_4 q^{-3}}{1-q^{-1}}.$$

The reference signal was chosen to be a square wave with a period time of $10s$. Each experiment used $30s$ of data and since only a one degree of freedom controller was used only two experiments were required in each iteration (rather than three), thus $60s$ of data was used in each iteration. The initial controller gave the closed loop response shown in Figure 4a. Since it was fairly obvious from an initial step response that the system contains a poorly damped mode around 3 rads^{-1} and the reference signal contained little energy at that frequency, a frequency weighting filter L_y was used to alert the scheme of this undesirable and potentially destabilizing mode. The filter was taken to be

$$L_y = \frac{1}{(1-0.95e^{0.3i}q^{-1})(1-0.95e^{-0.3i}q^{-1})}$$

which emphasizes the region around $\omega = 3$ rads^{-1}. In Figure 2 the first updated controller is shown with and without this filter. When the filter is used, the method is clearly aware of this mode and decreases the gain in that region while it clearly neglects it when no special emphasis is given to this frequency region.

During the first eight iterations the bandwidth of T_d was restricted to 1 rads^{-1}. Then it was gradually increased by 1 rads^{-1} after each second iteration. For each iteration the choice of step size was based on a comparison between controllers using different step sizes and experience from previous iterations. Initially, due to the large values of the gradient that are obtained when a badly tuned controller such as the initial one is updated, the step size was kept at 0.25. As seen in Figure 3, the controllers quickly formed a notch around the resonance which indicates stability enhancement. Due to that, the step size was increased to 1 after iteration 6. The desired and achieved responses with the controllers obtained after iteration 8,10 and 14 are shown in Figure 4b), c) and d). Figure 5 displays the closed loop transfer functions for the initial and final closed loops. Figure 6 is the corresponding plot for the sensitivity functions.

7. Final discussion

In this paper we have examined an optimization approach to iterative control design. The important ingredient is that the gradient of the design criterion is computed from measured closed loop data. The approach is thus not model-based. The scheme has been shown to converge to a local minimum of the design criterion under the assumption of boundedness of the signals in the loop.

From a practical viewpoint, the scheme offers several advantages. It is straightforward to apply. No sophisticated tools are necessary. It is possible to control the rate of change of the controller in each iteration. The objective can be manipulated between iterations in order to tighten or loosen performance requirements. Certain frequency regions can be emphasized if desired.

The method presented in this paper can be viewed as a local approach. By this we mean that the method only tries to estimate the shape, *i.e.* the gradient, of the criterion at the current operating point at each iteration. This is very different from model based approaches where the experiments are used to estimate models relevant for the control design. This means that the information in the data is extrapolated in some sense to form a global model. These methods could thus be called global. In view of this one could suspect that the local approach is more robust. It takes the data for what they are. This is, however, another story.

References

[1] Z. Zang, R.R. Bitmead, and M. Gevers. H_2 iterative model refinement and control robustness enhancement. In *Proc. of the 30th Conference on Decision and Control*, pages 279–284, Brighton, 1991.

[2] R.J.P. Schrama and P.M.J. Van den Hof. Iterative identification and control design: A three step procedure

with robustness analysis. In *Proc. ECC*, pages 237–241, Groningen, 1993.

[3] W.S. Lee, B.D.O. Anderson, R.L. Kosut, and I.M.Y Mareels. A new approach to adaptive robust control. *Int. Journal of Adaptive Control and Signal Processing*, 7:183–211, 1993.

[4] M. Gevers. Towards a joint design of identification and control? In H. L. Trentelman and J. C. Willems, editors, *Essays on Control: Perspectives in the Theory and its Applications.* Birkhäuser, 1993.

[5] J.M. Edmunds. Control system design and analysis using closed-loop nyquist and bode arrays. *International Journal of Control*, 30:773–802, 1979.

[6] E. Trulsson and L. Ljung. Adaptive control based on explicit criterion minimization. *Automatica*, vol. 21:385–399, 1985.

[7] G.G. Goodwin and P.J. Ramadge. Design of restricted complexity adaptive regulators. *IEEE Trans. Automatic Control*, 24(4):584–588, 1979.

[8] H. Robbins and S. Monro. A stochastic approximation method. *Ann. Math. Stat.*, 22:400-407, 1951.

[9] J.R. Blum. Multidimensional stochastic approximation method. *Ann. Math. Stat.*, 25:737–744, 1954.

[10] M.T. Wasan. *Stochastic Approximation.* Cambridge University Press, Cambridge, Great Britain, 1969.

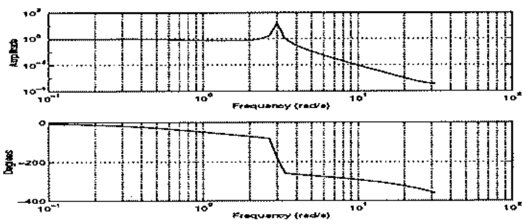

Figure 1: Bode diagram of the true system.

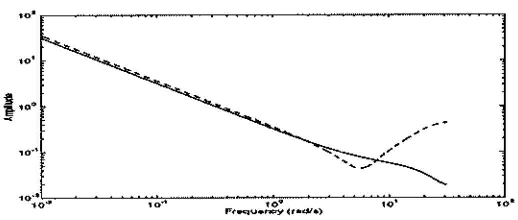

Figure 2: Controller after first iteration. solid line: without frequency weighting filter; dashed line: with filter.

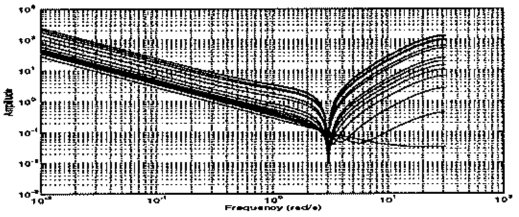

Figure 3: Evolution of the controller. The initial one has the lowest low frequency gain. This gain increases in each iteration.

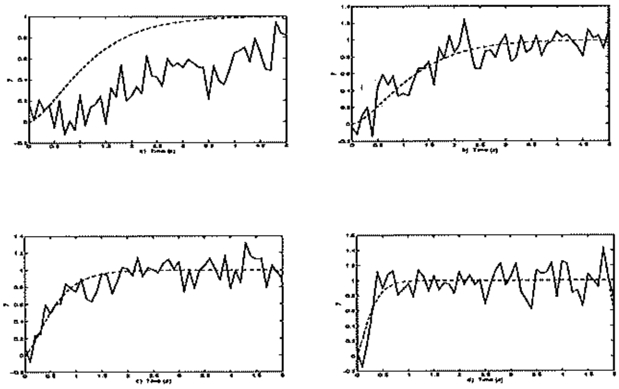

Figure 4: Output response together with ideal responses $T_d r$ (dashed lines). ω_B is the -6dB bandwidth of T_d. a) Initial. b) After 8 iterations, $\omega_B = 1\text{rads}^{-1}$. c) After 10 iterations, $\omega_B = 2\text{rads}^{-1}$, d) After 14 iterations, $\omega_B = 4\text{rads}^{-1}$

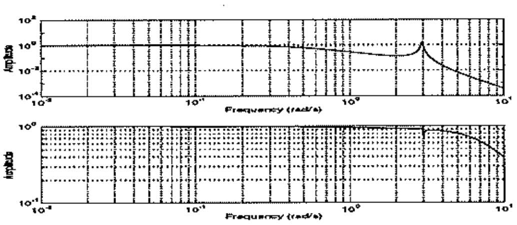

Figure 5: Initial and final closed loop frequency functions

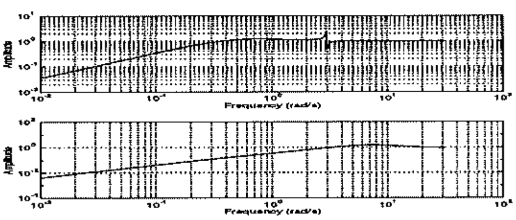

Figure 6: Initial and final sensitivity functions