

# Identification for control : closing the loop gives more accurate controllers \*

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## Abstract

We compare open loop versus closed loop identification when the identified model is used for control design, and when the system itself belongs to the model class, so that only variance errors are relevant. For three different control design criteria (minimum variance, LQG and model reference control) we show that, under those conditions, a better performance is achieved by closing the loop during the identification. The measure of performance is the variance of the error between the output of the ideal closed loop system (with the ideal controller) and that of the actual closed loop system (with the controller computed from the identified model).

## 1 Introduction

Consider that a linear time invariant system, perturbed by noise, is to be controlled and that a control design criterion has been selected. The control design criteria that we study in this paper can be a Minimum Variance (MV) control design, or a Linear Quadratic Gaussian (LQG) design, or a Model Reference (MR) design. If the plant input-output transfer function,  $G(q)$ , and its noise model,  $H(q)$ , were known exactly, the solution of the control design problem would lead to the 'ideal' controller,  $C(q)$ .<sup>1</sup> The corresponding closed loop system is represented in Figure 1, which has been drawn in all generality for a two degree of freedom controller, and which we shall call the ideal closed loop system.

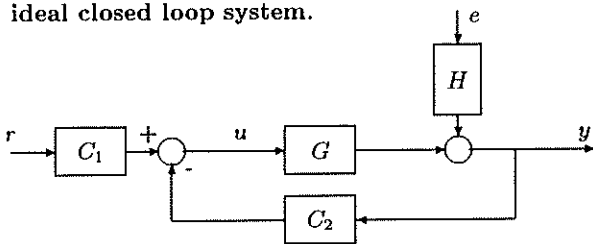


Figure 1: The ideal two degree of freedom control loop

We now consider the situation where the 'true system' of Figure 1 is unknown and where the controller is computed on the basis of the same criterion as above (either MV, or LQG, or MR) from a model  $[\hat{G}_N, \hat{H}_N]$  that is to be obtained by identification from a finite set of  $N$  input-output data collected on the true system. We further as-

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<sup>1</sup>In the case of a two degree of freedom controller,  $C(q)$  denotes the vector  $[C_1(q) \ C_2(q)]$ .

sume that the model structure used during identification is able to represent the true system; thus, 'the system is in the model set', the identified model will have no bias error, but only noise induced (i.e. variance) errors. Because the identified model,  $[\hat{G}_N, \hat{H}_N]$ , differs from the true system,  $[G, H]$ , the controller computed from  $[\hat{G}_N, \hat{H}_N]$  using the control design criterion, denoted  $\hat{C}_N$ , will differ from the ideal one,  $C$ , described above. We observe that  $\hat{C}_N = \hat{C}(\hat{G}_N, \hat{H}_N)$  is a random variable; thus, the error on the controller,  $C - \hat{C}_N$ , is also a random variable and by our assumptions it contains no bias. When the controller is applied to the actual system, the 'ideal closed loop' of Figure 1 is replaced by the actual - or achieved - closed loop system of Figure 2.

Since the objective of the identification of  $[\hat{G}_N, \hat{H}_N]$  is only to compute the controller, what really matters is not the error between the pairs  $[G, H]$  and  $[\hat{G}_N, \hat{H}_N]$ , but the error between  $C$  and  $\hat{C}_N$ , or - more importantly - the difference between the behaviour of the ideal closed loop system of Figure 1 and that of the actual closed loop system of Figure 2. We shall measure this error by the variance of the error between the output signals of these two loops, i.e. by  $J_V \triangleq E[y(t) - y_N(t)]^2$  when these loops are driven by the same signals  $r$  and  $e$ : see Figures 1 and 2. This will be our measure of the degradation that results from applying the suboptimal controller  $\hat{C}_N = \hat{C}(\hat{G}_N, \hat{H}_N)$  to the plant  $[G, H]$ , instead of the optimal controller  $C$ . Our results carry over easily to other error measures, such as any frequency weighted measure of  $E[C(e^{j\omega}) - \hat{C}_N(e^{j\omega})]^2$ , or a measure that includes the variance of the errors between both the input and the output signals of the two loops.

The problem addressed in this paper is that of identification design, namely the design of the identification experiment in such a way as to minimize the performance degradation between the ideal and the actual closed loops, as measured by  $J_V$ . We shall assume here that the number of data  $N$  that can be collected is fixed, and that least squares prediction error methods are used: see [1]. The

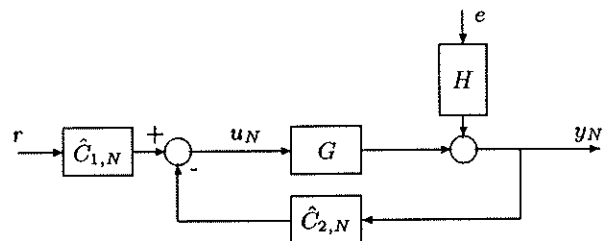


Figure 2: The actual two degree of freedom control loop

identification design problem involves questions such as:

- should one identify in open or closed loop?
- if closed loop identification is better, which controller should one apply during identification?
- what data filters should be used?

Our contribution is twofold. First we show that, when one uses either one of the three control design criteria mentioned earlier (MV, LQG or MR) and when only noise errors are involved, the ideal experimental conditions are to perform the identification in closed loop, with the ideal controller operating on the plant. This result holds independently of the level of external excitation or of possible constraints on input or output power. For minimum variance control, this result is well known (by those who know): see [2]. The problem with these results, as with most optimal experiment design results, is that the optimal identification design depends on the unknown system, since it requires knowledge of the ideal controller  $C = C(G, H)$ . An obvious question is whether applying a controller that is close to this optimal controller is any good, or is at least better than just doing open loop identification.

Our second contribution is to show that, under the conditions outlined above, one can obtain a better control performance on the actual system by computing the controller from a model that has been identified partly on closed loop data, provided  $N$  is large enough. More precisely, we shall compare two identification designs:

1. Identify the model  $[\hat{G}_N, \hat{H}_N]$  using  $N$  data collected in open loop, then compute the controller  $\hat{C}_N$  from this model.
2. Identify a model  $[\hat{G}_{N_1}, \hat{H}_{N_1}]$  from open loop data using a fraction  $N_1$  of the total experiment length; compute the corresponding controller  $\hat{C}_{N_1}$ ; apply it to the system and identify the model  $[\hat{G}_N, \hat{H}_N]$  using the remaining  $N - N_1$  data collected on the corresponding closed loop system. Compute the controller  $\hat{C}_N$  from this model.

We show that one can always obtain a better performance (i.e. a smaller degradation  $J_V$ ) with the second design than with the first one, provided  $N$  is large enough. This new result suggests a suboptimal but practically implementable identification design in which the experiment itself does not require knowledge of the unknown system.

A number of recent results on the interactions between identification and control design tend to support the idea that if a model is identified for the purpose of designing a controller, then closed loop identification is to be preferred over open loop identification: see [3], and the survey papers [4] and [5] with the many references therein. Most of this recent work focuses on bias errors, and the arguments are heuristic and supported by simulations. Our results are focused on variance errors only, and in that sense they can be criticized for requiring high order models. However, they are probably the first hard results that demonstrate that one can achieve better performance with a controller obtained from a model identified on closed loop

data than from a model identified on open loop data, with a practically feasible experiment design scheme.

The organization of the paper is as follows. In Section 2, we introduce the problem and we define the error criterion that is relevant in this context of identification for control with high order models. In Section 3, we show that, for any control design criterion leading to a one degree of freedom controller, the optimal experimental condition is to perform closed loop identification, and we compute the optimal controller for identification,  $C_{id}(q)$ , as a function of controller sensitivity functions. In Section 4 we show that, given enough data, an iterative closed loop identification design can always outperform open loop identification in the sense of leading to more accurate controllers for the same number of data. To establish this result we need to show that the optimal controller to be applied during identification,  $C_{id}(q)$ , is the ideal controller. This was shown in [2] for MV control design. The proofs for LQG and MR design are lengthy; for reasons of space, these results are not included here; they will appear in a journal version of this paper.

## 2 Statement of the problem

### The system

We consider that the task is to design a controller for some "true" linear time-invariant system described by

$$\mathcal{S} : y(t) = G(q)u(t) + H(q)e(t) \quad (1)$$

where  $G(q)$  and  $H(q)$  are scalar rational transfer function operators, with  $H(q)$  normalized such that  $\lim_{|q| \rightarrow \infty} H(q) = 1$ . Here  $q^{-1}$  is the delay operator ( $q^{-1}u(t) = u(t-1)$ ),  $u(t)$  is the control input signal,  $y(t)$  is the observed output signal and  $e(t)$  is white noise of zero mean and variance  $\sigma^2$ .

### The identification

The controller is to be designed on the basis of a model of the plant identified using a finite set of  $N$  input and output data  $\{y(t), u(t), t = 1, 2, \dots, N\}$  collected on the plant. A parametrized model set  $\mathcal{M} = \{M(\theta) : \theta \in D_\theta \subset \mathbb{R}^d\}$  is used, where  $M(\theta)$  is described by :

$$M(\theta) : y(t) = G(q, \theta)u(t) + H(q, \theta)e(t). \quad (2)$$

We make the important assumption that  $\mathcal{S} \in \mathcal{M}$ , i.e. there exists a  $\theta_0 \in D_\theta$  such that

$$G(q) = G(q, \theta_0), \quad H(q) = H(q, \theta_0). \quad (3)$$

In addition we assume throughout that Least Squares prediction error identification is used. It is further assumed the number of data  $N$  that can be collected for identification is fixed. The data collection can be done in open loop or in closed loop. In the case of closed loop identification, we denote by  $C_{id}(q)$ <sup>2</sup> the controller that operates during identification:

$$u(t) = C_{1,id}(q)r(t) - C_{2,id}(q)y(t), \quad (4)$$

where  $r(t)$  is the reference excitation signal used during identification.

<sup>2</sup>In the case of a two degree of freedom controller,  $C_{id} = [C_{1,id} \ C_{2,id}]$ .

Replacing  $C_{id}$  by  $C_{id}^{opt} + \Delta C_{id}$ , it then follows by some simple manipulations that (20) can be rewritten as:

$$E|\Delta C_N|^2 \cong \frac{n}{N}|H|^2|F_2|^2 \times \left(1 + \frac{\sigma^2}{|F_2|^2\phi_r}|F_1G + F_2H|^2|\Delta C_{id}|^2\right). \quad (31)$$

This expression holds for open loop and closed loop identification regardless of the control design criterion. The open loop formula (27) is recovered by setting  $|\Delta C_{id}| = |C_{id}^{opt}|$  and  $\phi_r = \phi_u$ . The following result follows immediately from (31).

**Theorem 2** *The designed controller error variance  $E|\Delta C_N|^2$  obtained by closed loop identification with an operating controller  $C_{id}(q)$  is smaller than that obtained by open loop identification at a frequency  $\omega$  if and only if:*

$$\left|\frac{C_{id}(e^{i\omega}) - C_{id}^{opt}(e^{i\omega})}{C_{id}^{opt}(e^{i\omega})}\right|^2 \leq \frac{\phi_r(\omega)}{\phi_u(\omega)} \quad (32)$$

This holds for any control design mechanism.

We now assume that  $C_{id}^{opt}(q) = C(q)$ , the ideal controller for the true system  $[G, H]$ , as is indeed the case for MV, LQG and MR control design.

We then consider the idea of splitting up the total data collection interval of length  $N$  into subintervals. In each interval (except possibly the first one) the identification is performed on closed loop data with the most recently estimated controller,  $\hat{C}_i$ , operating on the actual system. This experiment design setup is representative of the emerging iterative identification and control design schemes: see the references in [4] and [5]. Denoting by  $\Delta C_i = \hat{C}_i - C$  the error between the controller estimate  $\hat{C}_i$  at the end of the  $i$ -th subinterval of length  $N_i$  and the ideal controller  $C$ , we can then rewrite (31) in an iterative way :

$$E|\Delta C_i|^2 = \frac{1}{N_i}g(\omega)[1 + f(\omega)|\Delta C_{i-1}|^2], \quad (33)$$

where  $g(\omega) \triangleq n|H(e^{i\omega})F_2(e^{i\omega})|^2$  (34)

$$f(\omega) \cong \frac{\sigma^2}{|F_2(e^{i\omega})|^2\phi_r(\omega)} \times |F_1(e^{i\omega})G(e^{i\omega}) + F_2(e^{i\omega})H(e^{i\omega})|^2 \quad (35)$$

We assume that  $\phi_r(\omega)$  remains the same throughout the identification; therefore  $g(\omega)$  and  $f(\omega)$  depend on the true system and the control design mapping, but not on the specific experiment design during the  $i$ -th subinterval. The degradation measure  $J_V$  takes the form

$$J_V = \int_{-\pi}^{\pi} h(\omega)E|\Delta C|^2 d\omega \quad (36)$$

where  $h(\omega)$  depends only on the true system (see (16)) and  $\Delta C$  indicates the error between the ideal  $C$  and the estimated  $\hat{C}$  at the end of the last subinterval.

Consider first that a fixed controller (or no controller, i.e. open loop) is used during the total identification interval of length  $N$ . Then :

$$J_V^{(1)} = \frac{1}{N} \int_{-\pi}^{\pi} h(\omega)g(\omega)[1 + f(\omega)|\Delta C_0|^2] d\omega \quad (37)$$

where  $\Delta C_0 \triangleq C_{id} - C$  is the difference between the controller applied during identification and the ideal controller.

Consider next that the first  $(1-\alpha)N$  data ( $0 < \alpha < 1$ ) are collected on the plant operating with the same feedback controller as in the first experiment, and that the model estimated at the end of this first subinterval is used to compute a controller  $\hat{C}_1$  which is then applied to the plant for the remaining interval of length  $\alpha N$ . Let  $\hat{C}_2$  be the controller estimated from the model at the end of the total interval of length  $N$ . Then, at the end of the first subinterval, we have :

$$E|\Delta C_1|^2 \cong \frac{1}{(1-\alpha)N}g(\omega)(1 + f(\omega)|\Delta C_0|^2)$$

At the end of the total interval we have :

$$E|\Delta C_2|^2 \cong \frac{1}{\alpha N}g(\omega)(1 + f(\omega)|\Delta C_1|^2) \cong \frac{1}{\alpha N}g(\omega) \left[1 + \frac{1}{(1-\alpha)N}f(\omega)g(\omega)(1 + f(\omega)|\Delta C_0|^2)\right]$$

Denote  $\gamma_1 \triangleq \int_{-\pi}^{\pi} hg d\omega$ ,  $\gamma_2 \triangleq \int_{-\pi}^{\pi} hgf|\Delta C_0|^2 d\omega$ , and  $\gamma_3 \triangleq \int_{-\pi}^{\pi} fg^2h(1 + f|\Delta C_0|^2) d\omega$ . Then

$$J_V^{(1)} = \frac{1}{N}(\gamma_1 + \gamma_2), \quad J_V^{(2)} = \frac{1}{\alpha N} \left(\gamma_1 + \frac{\gamma_3}{(1-\alpha)N}\right).$$

Therefore,  $J_V^{(2)} < J_V^{(1)}$  if  $\gamma_1 + \frac{\gamma_3}{(1-\alpha)N} < \alpha(\gamma_1 + \gamma_2)$ , or equivalently if

$$N > \frac{\gamma_3}{(1-\alpha)[\alpha\gamma_2 - (1-\alpha)\gamma_1]} = N_0(\alpha) \quad (38)$$

Choose any  $\alpha$  such that  $\frac{\gamma_3}{\gamma_1 + \gamma_2} < \alpha < 1$ . This makes the denominator of (38) positive. Then for any  $N > N_0(\alpha)$ , we have  $J_V^{(2)} < J_V^{(1)}$ . The idea developed here with two subintervals can of course be repeated, leading to iterative identification and control design. We conclude that the iterative approach can always yield a smaller degradation provided enough data can be collected.

We make the final comment that the scheme presented here relies crucially on the iterative connection (33) between  $\Delta C_{i-1}$  and  $\Delta C_i$ . For this to hold, it is essential that  $C_{id}^{opt} = C$ . This was proved for MV in [2]. For LQG and MR control designs, a proof can be found in our journal version of this paper.

## 5 Simulations

In order to get a feeling for the performance improvements that can be achieved by using the iterative scheme suggested in Section 4, we have performed the following simulations.

We have taken a 'true system' that has the following ARX structure:

$$(1 - 1.5q^{-1} + 0.7q^{-2})y_t = q^{-1}(1 + 0.5q^{-1})u_t + e_t,$$

with  $e_t$  white noise of unit variance. The optimal MV controller (corresponding to the true system) is

$$u_t = -\frac{1.5 - 0.7q^{-1}}{1 + 0.5q^{-1}}y_t.$$

By the results of Section 3 (or by reading [2]) we know that the optimal, but unfeasible, identification conditions (in order to compute the best certainty equivalence MV controller) are to perform the identification in closed loop with this optimal controller in the loop. We have tested the feasible design scheme of Section 4 as follows.

With  $N = 1000$  data points allowed to be collected and a MV control design criterion, we have compared the following two experiments.

1. The identification is performed in open loop using 1000 data generated with  $\phi_u = 1$ .
2. The identification is performed in open loop first using  $(1 - \alpha)1000$  data (with  $0 < \alpha < 1$ ) generated with  $\phi_u = 1$ ; at the end of that first interval, the corresponding certainty equivalence MV controller  $\hat{C}_{(1-\alpha)1000}$  is applied to the plant, and the identification is performed on the closed loop system with  $\phi_r = 1$  for the remaining  $\alpha 1000$  data. This has been done for all values of  $\alpha$  between 0 and 1.

In each case, the certainty equivalence MV controller  $\hat{C}_{1000}$  is computed from the model  $[\hat{G}_{1000}, \hat{H}_{1000}]$  obtained at the end of the 1000 data and the corresponding performance degradation  $J_V$  vis-à-vis the optimal controller achieved on the 'true system' is computed. To do so, 200 Monte Carlo simulations have been run for the open loop identification experiment and for each of the iterative open/closed loop experiments in order to compute the experimental estimates of  $J_V$ . The results are shown in the top part of Figure 3.

The bottom line represents the performance degradation that results from identifying the system using 1000 data collected under the ideal experimental conditions, i.e. with the optimal controller in the loop. We get  $J_V^1 = 0.0015$ . The top value represents the performance degradation that results from identifying the system using 1000 data collected in open loop. We get  $J_V^2 = 0.0187$ , i.e. open loop identification performs more than 10 times worse than closed loop identification with the ideal controller. The broken line represents the performance degradation that results from using open loop followed by closed loop identification, for different values of the fraction  $\alpha$  of 'closed loop identification time'. We observe that for most values of  $\alpha$ , this feasible two-stage scheme performs almost as well as the unfeasible ideal experiment design.

A word of caution is in order, however. If  $\alpha$  is taken too large, that is if the open loop fraction of the total experiment time is too short, then the model obtained at the end of open loop identification may be so poor that the corresponding controller destabilizes the true system. This is indeed what happens. To give a honest representation of this phenomenon, we have represented in the bottom part of Figure 3 the total number of runs that produced unstable closed loops as a function of  $\alpha$ , until 200 stable runs were produced. The averages in the top part of the Figure are computed on the stable runs only.

## 6 Conclusion

We have presented two distinct contributions. We first showed that, when a model is identified with a view to

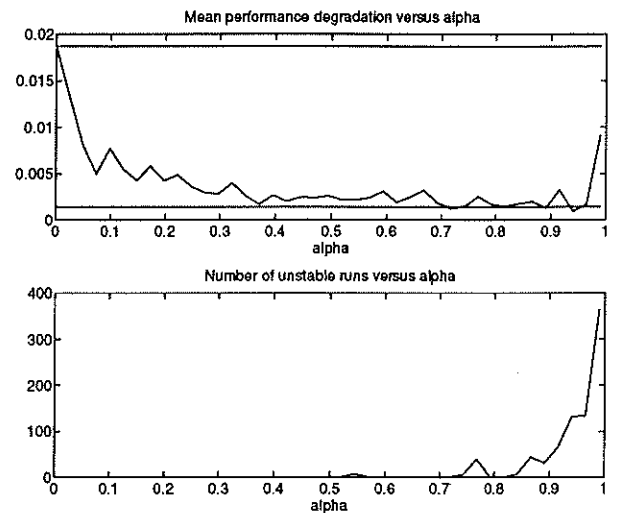


Figure 3: Mean performance degradation and number of unstable closed loop versus  $\alpha$

computing a controller and when the system is in the model set, the optimal experimental setup is to identify the system in closed loop with an ideal controller operating on the plant. This ideal controller depends on the unknown system.

Our second contribution has been to exploit this apparently useless result to establish that, provided the identification time is long enough, one can always obtain a more accurate controller estimate by performing at least part of the identification on closed loop data, with the current controller estimate operating on the plant. This gives theoretical support to the idea of iterative identification and control design.

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