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Disturbance Rejection: on-line refinement of controllers by closed loop modelling

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Abstract

Many practical applications of control system design based on input-output measurements permit the repeated application of a system identification procedure operating on closed loop data together with successive refinement of the designed controller. Recently several iterative schemes for mutually enhanced plant identification and robust control design have been proposed [1]-[3]. In this paper we shall analyze the methodology of [1] from the viewpoint of closed loop signal conditioning and investigate the effect of the noise modelling error and plant modelling error on the closed loop performance.

1 Introduction

Recently increasing attention has been paid among the control and identification communities to the investigation of practically feasible methods for mutually enhanced plant identification and robust control design. Several such iterative schemes for implementing combined plant identification and robust control design have been proposed [1-3]. In these schemes, models are fitted in closed loop with the current controller dictating the emphasis given to different frequency bands. In [1] this is coupled to the control design phase in which frequency weighting is injected from the modelling experiment. The incorporation of specific frequency weightings into the control law design process essentially forces the designed controller to "carry" the unmodelled uncertainty information between the model used for control design and the actual plant to be controlled . This control law when applied to the real system, will result in a better closed loop performance in terms of signal tracking and disturbance rejection. The key feature of this work was to specify a *global* performance criterion for the true plant and to recognise that control design was performed using an identified model derived based on closed loop experimental data. Compared with normal robust control paradigms, here one is permitted to perform closed loop data collection in order to redesign the controller to ameliorate performance. In this paper we shall analyse the methodology of [1] from the viewpoint of closed loop signal conditioning and investigate the effect of both noise modelling error and plant modelling error on closed loop performance.

This paper is organized as follows: In Section 2 we begin by summarizing the scheme proposed in [1] for iterative plant identification and robust control design. Section 3 presents the standard *LQG* control design from the viewpoint of input output approach. This involves in solving a factorization equation derived from the return difference equality (or algebraic Riccati equation) to specify the closed loop charateristic polynomial and solving a Diophantine equation which results in the corresponding optimal *LQG* control law. Section 4 is devoted to the development of an iterative scheme for implementing plant identification and robust control design. In Section 5 we shall analyze the effect of noise modelling error and plant modelling error on the whole process of iterative plant identification and control design. Section 6 gives some concluding remarks of the paper. Since the linear quadratic tracking problem can be recast as an equivalent regulation problem, throughout this paper, we shall mainly consider the *LQG* regulation problem. Also, for clarity, all the results will be derived for single input single output systems.

2 Preliminaries

Suppose that we have a true plant system with input-output relationship described by

$$y_t = P(z)u_t + v_t \tag{1}$$

where $P(z)$ is a strictly proper rational transfer function, u_t is the input, v_t is an unmeasurable disturbance acting on the output y_t. Also we are given a parametrized model set

$$\mathcal{M} = \{\widehat{P}(z,\theta), \quad \theta \in D_\theta \subset R^d\}, \tag{2}$$

together with (possibly) a noise model v_t'. A particular model in that model set, driven by an input u_t^c, will produce an output signal described by

$$y_t^c(\theta) = \widehat{P}(z,\theta)u_t^c + v_t' \tag{3}$$

for a particular value of the parameter θ, where $\widehat{P}$ is a strictly proper transfer function. Associated with the model (3) is the following one step ahead predictor:

$$\widehat{y}_t(\theta) = \widehat{H}^{-1}(z)\widehat{P}(z,\theta)u_t + (1 - \widehat{H}^{-1}(z,\theta)y_t \qquad (4)$$

see Ljung [5], where v_t' is modelled as $v_t' = \widehat{H}(z)e_t$ with e_t white noise. The iterative scheme of [1] is characterised by prescribing a single global objective

$$J^* = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N}[(y_t - r_t)^2 + \lambda u_t^2]\right\} \qquad (5)$$

and deriving a controller from LQG methods, where r_t is a reference signal which we desire the plant output, y_t, to track . The quadratic criterion for control is coupled with the following *local* Least Squares identification criterion,

$$V(\theta) = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N}[D(z)(y_t - \widehat{y}_t(\theta))]^2\right\} \qquad (6)$$

where the stable filter $D(z)$ is used to affect frequency weighting as described in [4].

The iterative scheme proposed in [1] proceeds by successive steps of

- plant experiment under fixed controller generating a finite length data set;
- fitting of approximate plant model $\widehat{P}(z,\theta)$ and closed loop signal spectra Φ_y, Φ_u;
- design of a feedback controller based on $\widehat{P}(z,\theta)$ and signal spectra.

The identification provides both the plant model, $\hat{P}(z,\theta)$, and signal-based measures of model error derived from closed loop spectra:

$$F_1 = \left(\frac{\Phi_{y-r}}{\Phi_{y^c-r}}\right)^{1/2}, \quad F_2 = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}. \qquad (7)$$

where Φ_{y-r}, Φ_{y^c-r}, Φ_u, Φ_{u^c} are the spectra of the corresponding signals. All these signals are readily available from the closed loops of Fig.1 and 2. In the actual implementation of the iterative scheme, the above spectra are replaced by spectral estimates obtained from data collected on the real plant and on the local model, both operating in closed loop with the presently active local controller. These spectra are then used to formulate a frequency weighted control problem,

$$J^c = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N}\{[F_1(y_t^c - r_t)]^2 + \lambda[F_2 u_t^c]^2\}\right\}.$$

The identification filter $D(z)$ is , in turn, chosen from the closed loop control:

$$D(z) = G(z)(1 + \hat{P}(z)C_2(z))^{-1} \qquad (8)$$

with

$$G(z)G^*(z^{-1}) = 1 + \lambda C_2(z)C_2^*(z^{-1}). \qquad (9)$$

where $C_2(z)$ is the feedback component of the LQG controller of the form $u_t = C_1(z)r_t - C_2(z)y_t$, see Fig.1 and 2. As was proved and discussed in [1], the choice of these specific filters have the essential effect that modelling and control law design are performed in a mutually supportive way.

The development of these methods in [1] proceeded from the basis of forcing the frequency domain specification of the local control and identification objectives to reflect the global criterion (5). In the following sections we shall derive a more signal-centred approach in which the Return Difference Equalities, see [4] associated with the *global* and *local* LQ formulations are interpreted to show that the iterations of control design affect the conditioning of closed loop signals for the true plant. For example, the LQ control design for the global problem in the case of minimum variance control of the true plant may be seen as a prescription for the whitening of the tracking error signal $y_t - r_t$. The frequency weightings F_1 and F_2 above then are easily viewed (via the appropriate local Return Difference Equality) as modifying the local criterion to resemble the global whiteness objective.

The advantage of this signal conditioning approach is that it helps us to focus on the description of closed loop objectives in terms of data measurable by experiment, rather than, say, by more esoteric presumptions of worst case errors in transfer functions etc. This interpretation of the control objective in a signal vein is the converse of the frequency domain consideration of the identification criterion, à la Ljung [5], or of the LQG objective via the frequency domain Return Difference Equality.

3 Weighted LQG control design

3.1 Standard *LQ* regulation problem

To begin with, we consider the standard *LQG* regulator design problem for a given plant model of the form (3). We assume that the transfer functions for the plant model and the disturbance model are of the form

$$\widehat{P}(z) = \frac{\widehat{B}(z)}{\widehat{A}(z)}, \quad \widehat{H}(z) = \frac{\widehat{C}(z)}{\widehat{A}(z)}.$$

where for simplicity we have suppressed the dependence of the transfer functions on the parameter θ. This corresponds to an *ARMAX* model structure in identification jargon. Consider the following infinite horizon *LQG* regulation problem: minimize over admissible $\{u_t^c\}$.

$$\widehat{J} = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N}[(y_t^c)^2 + \lambda(u_t^c)^2]\right\} \qquad (10)$$

Suppose we write the plant model in state space form

$$x_{t+1} = Fx_t + Gu_t + w_t \qquad (11)$$
$$y_t^c = Hx_t + q_t \qquad (12)$$

and minimize the *LQ* regulation criterion

$$\lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N}[x_t^T Q_c x_t + u_t^{cT} R_c u_t^c]\right\} \qquad (13)$$

The solution $u = Kx$ may be found by spectral factorization of the following control Return Difference Equality (*RDE* for short),

$$R_c + G^T(z^{-1}I - F)^{-T}Q_c(zI - F)^{-1}G$$
$$= [I - K(z^{-1}I - F)^{-1}G]^T(G^TPG + R_c)[I - K(zI - F)^{-1}G]$$

where P is the solution of the corresponding Algebraic Riccati Equation (ARE for short). In polynomial form with $Q_c = H^TH$ and $R_c = \lambda I$, this takes the form

$$\gamma\widehat{G}(z)\widehat{G}^*(z^{-1}) = \lambda\widehat{A}(z)\widehat{A}^*(z^{-1}) + \widehat{B}(z)\widehat{B}^*(z^{-1}) \tag{14}$$

where γ is a constant which depends on the design parameter λ. It is easy to see that in the above equality $\widehat{G}(z)$ is the resulting closed loop characteristic polynomial corresponding to the optimal *LQ* optimal controller. Instead of solving the *ARE* we can directly solve the above factorization problem to get $\widehat{G}(z)$. Since the plant model we used is supposed to be strictly proper *i.e.*, $deg(\widehat{A}) > deg(\widehat{B})$, and the control parameter λ is strictly positive, the solution to the above factorization problem is unique, see [12]. In order to obtain the optimal dynamic output feedback controller, we solve the following Diophantine (Van Compernolle) equation for $R(z)$ and $S(z)$,

$$\widehat{G}(z)\widehat{C}(z) = \widehat{A}(z)R(z) + \widehat{B}(z)S(z) \tag{15}$$

Then the unique control law which minimizes the performance criterion (10) is

$$u_t^c = -\frac{S(z)}{R(z)}y_t^c. \tag{16}$$

For more detail see [12]. This solution embodies implicitly the Kalman filter (via the polynomial $\widehat{C}(z)$ in (15)) and couples the noise model into the control design. One might equally well have proceeded by solving the filtering ARE/spectral factorization and then a Diophantine equation for the controller. Applying the above optimal controller to the plant model we have

$$y_t^c = \frac{R(z)}{\widehat{G}(z)}e_t, \quad u_t^c = -\frac{S(z)}{\widehat{G}(z)}e_t. \tag{17}$$

So the *designed* optimal cost in the frequency domain is

$$\widehat{J} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{|R|^2 + \lambda|S|^2}{|\widehat{G}|^2}\right]d\omega. \tag{18}$$

Assume that the true plant system can be written as

$$y_t = \frac{B(z)}{A(z)}u_t + \frac{C(z)}{A(z)}e_t \tag{19}$$

i.e., $P(z) = \frac{B(z)}{A(z)}$, $v_t = \frac{C(z)}{A(z)}e_t$ in (1), as opposed to the model

$$y_t^c = \frac{\widehat{B}(z)}{\widehat{A}(z)}u_t^c + \frac{\widehat{C}(z)}{\widehat{A}(z)}e_t \tag{20}$$

Applying the controller to the true plant, we have

$$y_t = -\frac{B(z)}{A(z)}\frac{S(z)}{R(z)}e_t + \frac{C(z)}{A(z)}e_t \tag{21}$$

So the *achieved* closed loop transfer functions from e_t to y_t and from e_t to u_t are, respectively,

$$y_t = \frac{C(z)R(z)}{A(z)R(z) + B(z)S(z)}e_t \tag{22}$$

$$u_t = -\frac{C(z)S(z)}{A(z)R(z) + B(z)S(z)}e_t \tag{23}$$

and the achieved cost in the frequency domain is

$$J^* = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{|C\widehat{G}|^2}{|AR + BS|^2}\left(\left|\frac{R}{\widehat{G}}\right|^2 + \lambda\left|\frac{S}{\widehat{G}}\right|^2\right)\right]d\omega \tag{24}$$

3.2 Weighted LQG Design

In this section, we shall analyze the design scheme proposed in [1] by using the above input output approach. Now we consider the following weighted *LQG* regulation problem

$$J^c = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\{[F_1y_t^c]^2 + \lambda[F_2u_t^c]^2\}. \tag{25}$$

By using (17) we can write the above criterion in the frequency domain as

$$J^c = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{|F_1R|^2 + \lambda|F_2S|^2}{|\widehat{G}|^2}\right]d\omega. \tag{26}$$

A direct comparison between (26) and (24) suggests that in order to make the minimization of (26), which is at our disposal, to be equivalent to that of (24), which is not at our disposal, we should choose $F_1 = F_2$ as follows (in fact, it is easy to verify from (17), (22) and (23))

$$F_1 = \frac{\Phi_y}{\Phi_{y^c}} = \left|\frac{\widehat{G}C}{AR + BS}\right|^2 = \frac{\Phi_u}{\Phi_{u^c}} = F_2, \tag{27}$$

where Φ_y, Φ_{y^c}, Φ_u, and Φ_{u^c} are the the closed loop spectra of the related signals y_t, y_t^c, u_t, and u_t^c. As indicated in [1], this choice of the weightings will force the minimization of (25) to be equivalent to the minimization of the global objective (5) with $r_t = 0$. Setting $F_1 = F_2 = \bar{\phi}$, we now consider the weighted *LQG* control problem.

$$J^c = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}[(\bar{\phi}y_t^c)^2 + \lambda(\bar{\phi}u_t^c)^2] \tag{28}$$

From the input output viewpoint, the above weighted LQG control design problem can be solved first by solving the factorization problem (14), set

$$\bar{G} = \widehat{G}\bar{\phi},$$

and then solving the following Diophantine equation,

$$\bar{G}(z)\widehat{C}(z) = \widehat{A}(z)\bar{R}(z) + \widehat{B}(z)\bar{S}(z) \tag{29}$$

For the weighted *LQG* control problem we obtain the corresponding controller transfer function as $\bar{S}(z)/\bar{R}(z)$. Applying this controller to the real system we have the achieved performance objective as

$$J^* = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{|C\widehat{G}|^2}{|A\bar{R} + B\bar{S}|^2}\left(\left|\frac{\bar{R}}{\bar{G}}\right|^2 + \lambda\left|\frac{\bar{S}}{\bar{G}}\right|^2\right)\right]d\omega \tag{30}$$

4 Iterative Design Scheme

Now we propose the following algorithm to perform our combined plant identification and control design.

Experiment : Suppose there exists a controller $K_0(z) = \frac{S_0(z)}{R_0(z)}$, which stabilizes the real closed loop system depicted in Fig. 3. In the case when the plant itself is stable, we choose $K_0(z) = 1$.

step 1 - Initial Closed Loop Identification : We use controller $K_0(z)$ to begin our closed loop identification. That is, we choose a plant model $\hat{P}_0$ from the model set (2) to minimize $V(\theta)$ with $D_0(z) = 1$. The identifier, together with $K_0(z)$, provides $\hat{P}_0(z)$, $\bar{\phi}_0(z)$, where $\bar{\phi}_0(z)$ is given by

$$\bar{\phi}_0 = \left(\frac{\Phi_y}{\Phi_{y^c}}\right)^{1/2} = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2},$$

where Φ_y, Φ_{y^c}, etc., are the related closed loop spectra corresponding to the initial controller $K_0(z)$. Their estimates can be obtained by using (17), (22), (23), with $S(z)$, $R(z)$, replaced by $S_0(z)$, $R_0(z)$, there.

Step 2 - Initial LQ Control Design : From Step 1 we get $\hat{P}_0 = \frac{\hat{B}_0}{\hat{A}_0}$, $\bar{\phi}_0$. We then use this information to solve the following factorization equation for $\widehat{G}_0$

$$\gamma\widehat{G}_0(z)\widehat{G}_0^*(z^{-1}) = \lambda\widehat{A}_0(z)\widehat{A}_0^*(z^{-1}) + \widehat{B}_0(z)\widehat{B}_0^*(z^{-1}) \tag{31}$$

Set $\bar{G}_0 = \widehat{G}_0\bar{\phi}$, we then solve the following Diophantine equation for $\bar{S}(z)$ and $\bar{R}(z)$,

$$\bar{G}_0(z)\widehat{C}(z) = \widehat{A}_0(z)\bar{R}_1(z) + \widehat{B}_0(z)\bar{S}_1(z) \tag{32}$$

for $\bar{S}_1(z)$ and $\bar{R}_1(z)$. This defines a new controller $\bar{K}_1(z) = \frac{\bar{S}_1(z)}{\bar{R}_1(z)}$.

Step 3 - Identification : Using the newly obtained controller $\bar{K}_1(z)$ we solve the following factorization equation

$$G_1(z)G_1^*(z^{-1}) = 1 + \lambda\bar{K}_1(z)\bar{K}_1^*(z^{-1})$$

for stable minimum phase $G_1(z)$, and we select the identifier filter

$$D_1(z) = G_1(z)(1 + \hat{P}_0(z)\bar{K}_1(z))^{-1}$$

and perform the identification stage to minimize $V(\theta)$. This, together with $\bar{K}_1(z)$, provides us with $\hat{P}_1(z)$, $\bar{\phi}_1(z)$.

Step 4 : Continue as in Step 2.

Step 5 : Continue as in Step 3.

Remarks

- In practical control design, the spectra used in step 1 is replaced by spectral estimates obtained from data collected on the plant and model. It is worth noting that all quantities required to estimate these spectra are available and approximation can easily be made based on finite data records. For more detail, see [1].
- Since the scheme is based on the input output approach, the advantage of the scheme over that proposed in [1] is its simplicity to implement. Also it will be more robust against digital approximation error.

5 Noise model in the iterative design

In this section we shall investigate the effect of the plant modelling error and the disturbance modelling error on the frequency weighted local LQG regulator design problem and the global LQG regulation. First we recall that the weighting $\bar{\phi}$ used in the local control objective (28) is given as

$$\bar{\phi}^2 = \left|\frac{\widehat{G}C}{AR+BS}\right|^2.$$

Note that

$$\begin{aligned}\frac{\widehat{G}C}{AR+BS} &= \frac{\frac{C}{A}}{\frac{R}{G}+\frac{B}{A}\frac{S}{G}} \\ &= \frac{H}{\frac{R}{G}+\frac{\widehat{B}}{A}\frac{S}{G}+\left(\frac{B}{A}-\frac{\widehat{B}}{A}\right)\frac{S}{G}} \\ &= \frac{H}{\widehat{H}+(P-\widehat{P})\frac{S}{G}}\end{aligned} \tag{33}$$

From (33) we offer the following observations:

- In the case when a correct model is used in the control design, *i.e.* $P = \widehat{P}$ (or equivalently, suppose that the model set (2) is large enough to contain the true plant transfer function $P(z)$), the frequency weighting $\bar{\phi}$ in the local LQG control design criterion is actually the ratio between the true plant noise spectrum and the modelled noise spectrum. From [1] we know that the weighted LQG control problem can be recast as an unweighted LQG control design problem with the plant model $\widehat{P}$ unchanged and with a new noise model transfer function as $\bar{\phi}\widehat{H}$. Of course, this new noise model is a better approximation of the true system noise. So the LQG controller designed by minimizing the local criterion (28) will result in a better achieved closed loop disturbance rejection.
- If we model the noise v_t approximately correctly, *i.e.*, $H \approx \widehat{H}$, then approximately we have

$$|\bar{\phi}|^2 = \left|\frac{1+K\widehat{P}}{1+KP}\right|^2. \tag{34}$$

Although the noise model is correct, due to the unmodelled uncertainty $L = P - \widehat{P}$, the closed loop performance in terms of disturbance rejection will be different for the true system and the nominal system if the controller used is derived from the minimization of the unweighted LQG performance criterion (10). Hence, the incorporation of the weighting $\bar{\phi}(z)$ as defined by (34) into the weighted LQG control design performance criterion (28) has the effect that the resulting controller will achieve approximately the same level of disturbance rejection for the true system and the nominal system with the new noise model transfer function $\hat{\phi}H$, since we have

$$y_t = \frac{1}{1+KP}v_t \quad y_t^c = \frac{1}{1+K\hat{P}}v_t'$$

now with $v_t = H(z)e_t$ and $v_t' = \hat{\phi}(z)H(z)e_t$.

- In the general case, *i.e.*, when both correct plant model and noise model are not available to designer, the incorporation of the frequency weighting into the local control criterion can be interpreted as to produce a controller with the capability that when it is applied to the true system, a satisfactory trade-off can be achieved in terms of diminishing the effect of the plant modelling error and the noise modelling error on the true system performance.

6 Concluding remarks

In this paper, we have analyzed the scheme proposed in [1] for combined plant identification and robust control design from the viewpoint of closed loop signal conditioning and discussed the effect of the plant modelling error and noise modelling error on the designed and achieved closed loop systems. An algorithm for iterative plant identification and control design was proposed. This simply involves in solving some specific factorization equations and Diophantine equation.

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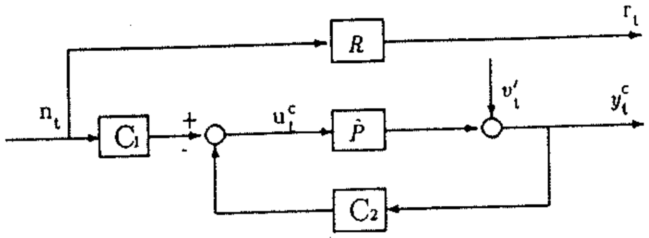

Figure 1

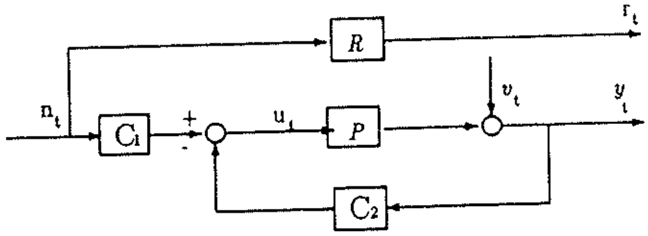

Figure 2

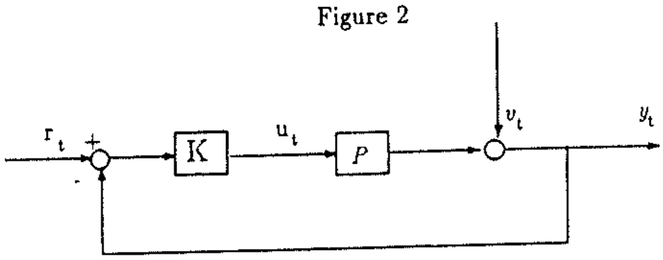

Figure 3

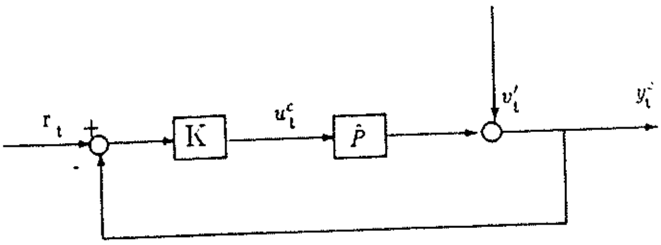

Figure 4