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TRACKING CAPABILITIES OF ADAPTIVE OBSERVERS FOR LINEAR TIME-VARYING SYSTEMS.

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ABSTRACT

We present several adaptive observers which are inspired by well known adaptive observers initially derived for linear-time-invariant systems. We study the tracking capabilities of these observers when the parameters of the linear system are time-varying. We show that under mild assumptions global stability of the error systems is obtained, and we relate the asymptotic tracking error to the speed of parameter variation.

I. INTRODUCTION

We are concerned, in this paper, with the analysis of the global stability of adaptive observers which have been designed for time invariant systems, when they are applied to time varying systems.

In section 2, it is shown how a globally stable adaptive observer, identical to that proposed by Kreisselmeier [1], is "naturally" suitable for time varying systems given in "regressor from" :

$$y(t) = \varphi^T(t)\theta(t).$$

We remark that the global stability of the same observer is also established when it is applied to time varying systems given in "input/output" form.

In section 3, we prove the global stability of the same adaptive observer when it is applied to time varying systems given in observer canonical form. The same is done in section 4 for a reduced order observer proposed by Lüders and Narendra [2] for time-invariant systems.

In each case, in the line of the pioneering work of Anderson and Johnstone [3], the global stability is established using standard BIBO stability theory (Willems, [4]) and allows to relate the asymptotic estimation error to the speed of parameter variation.

Finally sections 5 and 6 contain additional comments and conclusions.

II. SYSTEMS DESCRIBED IN REGRESSOR FORM

Consider first that the system is described in the following "regressor form"

$$y(t) = \varphi^T(t)\ \theta(t) \tag{2.1}$$

where

$$\varphi^T(t) = \frac{s^{n-1}}{F(s)}\,y, \ \ldots, \frac{1}{F(s)}y, \frac{s^{n-1}}{F(s)}\,u, \ \ldots \frac{1}{F(s)}\,u, \ \ldots, \frac{1}{F(s)}\,u] \tag{2.2}$$

$$\theta^T(t) = [\theta_1(t)\ \ldots\theta_n(t)\ \theta_{n+1}(t)\ \ldots\theta_{2n}(t)] \tag{2.3}$$

$F(s)$ is an exponentially stable polynomial :

$$F(s) = s^n + f_1 s^{n-1} + \ldots + f_n \tag{2.4}$$

We assume throughout that the vector $\theta(t)$ has a bounded and continuous derivative (w.r.t. time) and that the system (2.1) - (2.4) is bounded input bounded output (BIBO) stable.

For linear time-invariant systems, such a model (or a slight variant thereof) arises naturally from a transfer function model : see e.g. [5]. Assume now a time-varying linear plant.

$$y^{(n)}(t) + a_1(t)y^{(n-1)}(t) + \ldots + a_n(t)y(t) =$$
$$b_1(t)u^{(n-1)}(t) + \ldots + b_n(t)u(t) \tag{2.5}$$

Operating on (2.5) by $\frac{1}{F(s)}$ gives a model

$$y(t) = \varphi^T(t)\ \theta(t) + \eta(t) + dec(t) \tag{2.6}$$

where

$$\theta^T(t) = [f_1 - a_1(t)\ \ldots f_n - a_n(t)\ b_1(t)\ldots b_n(t)] \tag{2.7}$$

$dec(t)$ are decaying exponential terms, and $\eta(t)$ is an error term arising from commuting time-varying operators:

$$\eta(t) = \frac{1}{F(s)}\ (\zeta^T(t)\theta(t)) - \frac{1}{F(s)}(\zeta^T(t)).\theta(t) \tag{2.8}$$

and

$$\zeta^T(t) = [y^{(n-1)}(t)\ldots y(t)\ \ u^{(n-1)}(t)\ldots u(t)] \tag{2.9}$$

The model (2.6) can be viewed as a generalization of (2.1), where $\eta(t)$ represents unmodelled dynamics. Such approach has been taken in [2], where most results are derived under the assumption $\eta(t) = 0$. We shall comment later that, when (2.6) arises from (2.5), $\|\eta(t)\|$ can be bounded by $\|\dot\theta\|_\infty$. As far as we are concerned, we consider that both models are two different but natural time-varying generalizations of time-invariant models.

State space model

We shall now describe an adaptive observer for (2.1). First we construct a state variable form for (2.1), which is inspired by Kreisselmeier [1].
Straightforward calculations show that (2.1) can be rewritten as

$$\begin{cases} \dot x(t) = Fx(t) + \Omega(t)\theta(t) + \psi(t)\dot\theta(t) \\ y(t) = C^T x(t) \end{cases} \tag{2.10}$$

where

$$F = \left[\begin{array}{c|c} -f_1 & \\ \vdots & I_{n-1} \\ \vdots & \\ \hline -f_n & 0 \ldots 0 \end{array}\right] \qquad C^T = [1\ 0 \ldots 0] \tag{2.11}$$

$$\underset{(nx2n)}{\Omega(t)} = \begin{bmatrix} y & & 0 & u & & 0 \\ & \ddots & & & \ddots & \\ 0 & & y & 0 & & u \end{bmatrix} \tag{2.12}$$

nx2n and $\psi(t)$ is the ~~2nxn~~ matrix state of the following auxiliary filter

$$\begin{aligned} \dot{\psi}(t) &= F\psi(t) + \Omega(t) \\ \cancel{\psi}^T(t) &= C^T\psi(t) \end{aligned} \tag{2.13}$$

φ

$\varphi(t)$ was defined in (2.2). Note that $x(t) = \psi(t)\theta(t)$.

Comment 2.1.

The state space model (2.10) is a modification of a state space model for time-invariant systems derived by Kreisselmeier [1]. Our model contains the added term $\psi(t)\dot{\theta}(t)$, which accounts for the parameter variations.

Adaptive observer

The following is a natural adaptive observer for the model (2.10)

$$\dot{\hat{x}}(t) = F\hat{x}(t) + \Omega(t)\hat{\theta}(t) + \psi(t)\Gamma\varphi(t)[y(t) - C^T\hat{x}(t)] \tag{2.14.a}$$

$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)[y(t) - C^T\hat{x}(t)] \tag{2.14.b}$$

where $\Gamma = \Gamma^T > 0$ is a gain maxtrix. We consider this observer to be natural, because the parameter update has the classical form $\Gamma\varphi e$, while the state-estimate is obtained by replacing x, θ, $\dot{x}$ and $\dot{\theta}$ by their estimates in the state equation. Notice that it is a full order observer.

Error system

Define $\tilde{x} = x - \hat{x}$ and $\tilde{\theta} = \theta - \hat{\theta}$.
Then the error system is as follows.

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F - \psi\Gamma\varphi C^T & \Omega \\ -\Gamma\varphi C^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} \psi\dot{\theta} \\ \dot{\theta} \end{bmatrix} \tag{2.15}$$

Simplified adaptive observer

Notice that $\hat{x}(t)$ converges exponentially fast to $\psi(t)\hat{\theta}(t)$. Indeed it is easy to check that

$$\frac{d}{dt}(\hat{x} - \psi\hat{\theta}) = F(\hat{x} - \psi\hat{\theta}) \tag{2.16}$$

This suggests the following simplified adaptive observer :

$$\hat{x}(t) = \psi(t)\hat{\theta}(t) \tag{2.17a}$$

$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}(t)] \tag{2.17b}$$

[1]

together with the auxiliary filter (2.13). The error system is identical to (2.15). The two observers differ only through initial conditions effects on $\hat{x}(t)$. The observer (2.17) was proposed for time-invariant systems by Kreisselmeier [1].

We have the following stability theorem.

Theorem 2.1.

Assume that :

A1) the system (2.1) - (2.4) is BIBO stable

A2) $u(t)$ is continuous and bounded :

$$\sup_t |u(t)| \leq U < \infty$$

A3) the parameter vector θ has bounded continuous derivative

$$\sup_t |\dot{\theta}(t)| \leq M < \infty$$

A4) $u(t)$ is such that the regressor vector $\varphi(t)$ is persistently exciting, i.e. $\exists T, \alpha, \beta > 0$ such that

$$\alpha I \leq \frac{1}{T}\int_s^{s+T} \varphi(t)\varphi^T(t)dt \leq \beta I \quad \forall s$$

Then

P1) the state $\psi(t)$ of the auxiliary filter (2.13) is uniformly bounded :

$$\lim_{t\to\infty} \sup \|\psi(t)\| \leq K_1 U \quad \text{for some } K_1 > 0$$

P2) the estimation errors of the observers (2.14) or (2.17) are bounded and

$$\lim_{t\to\infty} \sup \left\| \begin{matrix} \tilde{x}(t) \\ \tilde{\theta}(t) \end{matrix} \right\| \leq C(U)\ M$$

where $C(U)$ is a strictly increasing, positive function of U.

Proof : A1 and A2 imply that $y(t)$ is bounded with :

U

$$\lim_{t\to\infty} \sup |y(t)| \leq C_1 U \quad \text{for some } C_1 > 0$$

Since F is a stability matrix, P1 follows

By defining $e \triangleq \tilde{x} - \psi\tilde{\theta}$, the error system can be rewritten as :

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F & 0 \\ -\Gamma\varphi C^T & -\Gamma\varphi\varphi^T \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta} \end{bmatrix} \tag{2.18}$$

The uniform asymptotic stability of the homogeneous part of (2.18) follows from the stability of F and from A.4. It also follows that $e(t) \to 0$ and that $t \to \infty$

$$\lim_{t\to\infty} \sup \|\tilde{\theta}(t)\| \leq C_1(U)M \tag{2.19}$$

q

P2 follows easily.

Comment 2.2.

We have shown that the Kreisselmeier observer [1] derived for time-invariant systems is robust to parameter variations. This is due to the uniform asymptotic stability of the homogeneous part of (2.15). The error system (2.15) is analyzed in some detail in [6] where some robustness properties to multiplicative errors are also demonstrated.
Notice that convergence of $\hat{x}$ to x and $\hat{\theta}$ to θ is achieved in the case of a time invariant system ($\dot{\theta}=0$).

Comment 2.3.

If the same observer (2.14) or (2.17) is applied to the model (2.6) instead of (2.1), the error system will be (2.15) again, but with the driving term replaced by

$$\begin{bmatrix} \psi(\dot{\theta} - \Gamma\varphi\eta) \\ \dot{\theta} - \Gamma\varphi\eta \end{bmatrix} + \mathrm{dec}(t) \tag{2.20}$$

In addition, if the unmodelled dynamics $\eta(t)$ are as in (2.8), then it can be shown (using Morse's swapping lemma [7]) that

$$\sup|\eta(t)| \leqslant \frac{K_2}{a} \|\psi\|_\infty M$$

where $K2 > 1$ and $a > 0$ is such that $\mathrm{Re}\ \lambda_i(F) \leqslant -a$. Therefore, the result P2 holds in this case with a slightly modified bound on the right hand side.

III. SYSTEM DESCRIBED IN CANONICAL OBSERVER FORM

We now consider linear time varying systems represented in canonical observer form

$$\dot{x}(t) = A(t)x(t) + b(t)u(t)$$
$$y(t) = (1\ 0\ \ldots 0)\ x(t) = c^T x(t) \qquad (3.1)$$

where c^T is as in (2.11),

$$A(t) = \left[\begin{array}{c|c} a(t) & \begin{array}{c} I_{n-1} \\ \hline 0 \ldots 0 \end{array} \end{array}\right] \qquad (3.2)$$

$$a^T(t) = (-a_1(t)\ \ldots\ldots -a_n(t)) \qquad (3.3.a)$$

$$b^T(t) = (b_1(t)\ \ldots\ldots\ b_n(t)) \qquad (3.3.b)$$

We assume again that $a(t)$ and $b(t)$ have bounded and continuous derivatives and that (3.1) is BIBO stable. A large family of linear time-varying systems can be transformed to (3.1) by a Lyapunov transformation.

Comment 3.1

An equivalent input-output representation for (3.1) exists provided the parameter coefficients $a_i(t)$, $b_i(t)$ have bounded continuous derivatives of order $n + 1 - i$, $i = 1, \ldots, n$. Indeed, (3.1) is equivalent with

$$y^{(n)}(t) + (a_1 y)^{(n-1)}(t) + \ldots + (a_n y)(t) =$$
$$(b_1 u)^{(n-1)}(t) + \ldots + (b_n u)(t) \qquad (3.4)$$

This can be represented as

$$y^{(n)}(t) + k_1(t)y^{(n-1)}(t) + \ldots + k_n(t)y(t) =$$
$$g_1(t)u^{(n-1)}(t) + \ldots + g_n(t)u(t) \qquad (3.5)$$

where $k_i(t)$, $g_i(t)$ have bounded continuous derivatives. Notice also that (3.5) can be put in the form (3.1) iff k_i, g_i have bounded continuous derivatives of order $n+1-i$, $i = 1, \ldots, n$.

We now transform the system (3.1) to a model that is identical to the model (2.10) except for the term $\psi(t)\dot{\theta}(t)$, and we show that if the observer (2.14) or (2.17) of the previous section is used, global stability is again achieved.

The system (3.1) can be rewritten as

$$\dot{x}(t) = Fx(t) + \Omega(t)\theta(t)$$
$$y(t) = c^T x(t) \qquad (3.6)$$

where F is as in (2.11), $\Omega(t)$ as in (2.12) and $\theta(t)$ as in (2.7). The coefficients $f_1, \ldots, f_n$ in F can be chosen arbitrarily; they are chosen such that F is a stability matrix. Note that (3.6) is identical to (2.10) except for the term $\psi(t)\dot{\theta}(t)$. If we apply the same adaptive observer (2.14) or (2.17) together with the auxiliary filter (2.13), the error system becomes:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F - \psi\Gamma\varphi C^T & \Omega \\ -\Gamma\varphi C^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta} \end{bmatrix} \qquad (3.7)$$

Notice that it is identical to (2.15) except for the driving term. With $e = \tilde{x} - \psi\tilde{\theta}$ as in section 2, the error system becomes :

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F & 0 \\ -\Gamma\varphi C^T & -\Gamma\varphi\varphi^T \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} -\psi\dot{\theta} \\ \dot{\theta} \end{bmatrix} \qquad (3.8)$$

Compare with (2.18)

Stability analysis

Since the error systems (3.8) and (2.18) are identical, except for a slight modification of the driving term, the results of Theorem 2.1 apply identically the models of this section, with a slightly modified expression for C(U) in result P2.

IV. REDUCED ORDER OBSERVERS

Since the first component of $x(t)$ is measured in the models (2.10) and (3.6), a reduced order observer can be used instead of (2.14a). To do this requires a further state transformation and leads to an adaptive observer initially proposed by Lüders and Narendra [2] for time-invariant systems. We present the transformation for the time-varying model (3.6) and show that the Lüders and Narendra observer yields global stability also in this time-varying case. First we choose the arbitrary coefficients $f_1, \ldots, f_n$ such that

$$\det(SI - F) = s^n + f_1 s^{n-1} + \ldots + f_{n-1}s + f_n$$
$$= s(s+c_2)\ \ldots\ (s+c_n) \qquad (4.1)$$

In particular $f_n = 0$

where $c_2, \ldots, c_n$ are any positive but mutually different constants. We define

$$G = \begin{bmatrix} 0 & 1 \ldots\ldots 1 \\ . & -c_2 \qquad 0 \\ . & \\ 0 & 0 \qquad -c_n \end{bmatrix} \qquad (4.2)$$

$\Rightarrow \det(sI - G) = \det(sI - F)$

Now let Q_1, Q_2 be, respectively the observability matrices of the pairs (C, F) and (C, G) with C as in (2.11), and define

$$z = Tx, \quad T = Q_2^{-1} Q_1 \qquad (4.3)$$

Then (3.6) is equivalent with

$$\begin{cases} \dot{z}(t) = Gz(t) + \Omega(t)\ \theta(t) \\ y(t) = C^T z(t) \end{cases} \qquad (4.4)$$

where

$$\theta^T(t) = [(T(f-a(t)))^T\ \ (Tb(t))^T] \qquad (4.5)$$

with

$$f^T = (f_1\ \ldots\ldots\ f_n) \qquad (4.6)$$

Recall that, in the representation (3.6), $\theta(t)$ was defined by (2.7). A detailed derivation of this transformation can be found in [8]. See also [2].

For the representation (4.4), consider now the following reduced order observer :

$$\dot{\hat{z}}(t) = G\hat{z}(t) + \Omega(t)\hat{\theta}(t) + \begin{bmatrix} c_1 \\ \psi(t)\Gamma\varphi(t) \end{bmatrix} (y(t) - \hat{z}_1(t))$$
$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)[y(t) - \hat{z}_1(t)] \qquad (4.7)$$

where c_1 is an arbitrary positive constant, $\Gamma = \Gamma^T > 0$ is a gain matrix and ψ (of dimension (n-1)x2n) and φ are defined via the auxiliary filter :

$$\dot{\psi}(t) = G_2\psi(t) + \Omega_2(t)$$

$$\varphi^T(t) = (1\ldots.1)\ \psi(t) + \Omega_1(t) \tag{4.8}$$

$$G_2 = \text{diag}\ (c_2, \ \ldots, \ c_n) \in \mathbb{R}^{(n-1)\times(n-1)} \tag{4.9}$$

$$\Omega(t) = \begin{bmatrix} \Omega_1(t) \\ \Omega_2(t) \end{bmatrix} \text{ with } \Omega_1(t) \in \mathbb{R}^{1\times 2n},\ \Omega_2(t) \in \mathbb{R}^{(n-1)\times 2n} \tag{4.10}$$

Comment 4.1.

This adaptive observer was initally proposed by Lüders and Narendra [2] for linear time-invariant systems; it was recently extended by Bastin and Gevers [8] to several classes of nonlinear and time-varying systems. It is called reduced order here because the auxiliary filter has fewer states than for the previous observers.

Stability analysis

We define the following error variables :

$$\tilde{z}(t) \triangleq z(t) - \hat{z}(t), \quad \tilde{\theta}(t) = \theta(t) - \hat{\theta}(t)$$

$$e(t) \triangleq \tilde{z}(t) - \begin{bmatrix} 0 \\ \psi(t)\tilde{\theta}(t) \end{bmatrix} \tag{4.11}$$

The following error system is then obtained

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \left[\begin{array}{c|c} G_1 & \begin{matrix} \varphi^T \\ 0_{(n-1)\times 2n} \end{matrix} \\ \hline -\Gamma\varphi \quad 0_{2n\times(n-1)} & 0_{2n\times 2n} \end{array}\right] \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} + \left[\begin{array}{c} 0 \\ -\psi \\ \hline I \end{array}\right] \dot{\theta}$$

where

$$G_1 = \begin{bmatrix} -c_1 & 1 \ \ldots\ldots & 1 \\ & -c_2 & 0 \\ 0 & & -c_n \end{bmatrix} \tag{4.13}$$

The stability and boundedness results of theorem 2.1 apply almost identically in this case. More precisely:

Theorem 4.1.

Assume that

B1) the system (3.1) is BIBO stable

B2) u(t) has continuous and bounded derivatives and

$$\sup_t |u(t)| \leq U < \infty$$

B3) A3 and A4 hold (see theorem 2.1)

Then

P1′) $\lim\limits_{t\to\infty} \sup\|\psi(t)\| \leq K_1 V$ for some $K_1 > 0$

with $\psi(t)$ defined by (4.8)

P2′) $\lim\limits_{t\to\infty} \sup\left\|\begin{matrix}\tilde{z}(t)\\ \tilde{\theta}(t)\end{matrix}\right\| \leq C(U)M$

where C(U) is a strictly increasing positive function of U.

Proof : see [8]

Comment 4.2

Neglecting exponentially decaying terms due to initial conditions, the homogeneous part of the error system (4.12) can be described in shorthand notation as :

$$\dot{\tilde{\theta}}(t) = -\Gamma\varphi(t)\left[\frac{1}{s+c_1}(\varphi^T\tilde{\theta})\right](t) \tag{4.14}$$

The regressor vector $\varphi(t)$ has a very simple form :

$$\varphi^T = (y \ \frac{1}{s+c_2}y \ \ldots \frac{1}{s+c_n}y \quad u \ \frac{1}{s+c_2}u \ \ldots \frac{1}{s+c_n}u) \tag{4.15}$$

V. FURTHER COMMENTS AND GENERALIZATIONS

1. An interesting feature of the various adaptive observers presented in this paper is that stability of the error systems and boundedness of the variables is guaranteed without any SPR condition or without any requirement of small adaptation gains.

2. Establishing persistency of excitation (PE) of the regressor vector $\varphi(t)$ in the presence of a nonstationary linear system behaviour is a nontrivial issue; yet this is a crucial assumption for our stability results : assumption A.4. We have dealt with this issue at length in [9]. The available results indicate that it possible to ensure a persistently exciting regression vector by using an input u(t) which has a sufficiently rich spectrum, provided that the time variations θ in the system parameters are slow and/or integral small (i.e. sufficiently fast) as compared to the system dynamics excited by the input. See [9] for details.

3. The adaptive observers presented here can easily be extended to nonlinear systems which can be represented (possibly after some transformations) to the form :

$$\dot{x}(t) = Fx(t) + \Omega(y,u,t)\ \theta(t) + f(y,u,t)$$

$$y(t) = c^T x(t)$$

where F and c are as in (2.11), $\theta(t)$ is an unknown time-varying parameter vector, and $\Omega(y,u,t)$ and $f(y,u,t)$ are known bounded functions of the output and input.
In particular, all bilinear time-invariant observable systems can be bought into this format. See [8] for details. The same stability properties (cfr Theorem 2.1) will hold for these nonlinear systems. However, in the nonlinear situation the PE condition becomes hard to verify a priori, except in very sepcial cases. Some specific results have been obtained for biochemical processes in [10].

VI. CONCLUSIONS

Starting from an input-output model described in time-varying regression form, we have transformed this model into the special state-space form (2.10)-(2.13). This form is closely related to one initially described by Kreisselmeier [1] but it has an added term that accounts for the parameter variations. A full order adaptive observer derives almost naturally from this state-space form. Global stability is easy to prove, and in addition we have shown that the asymptotic estimation error bounds are proportional to the bounds on the parameter variations.

We have then shown that, if one starts from a time-varying system described in canonical observer form, it can be transformed to the state-space form (3.6) (identical to 2.10) up to the $\psi\theta$ and the same full order observer can be used with the same stability and boundedness properties.

Finally we have shown that the state-space model (3.6) can be further transformed to another special state-space model (4.4), from which a reduced order observer can be derived which is identical to one derived by Lüders and Narendra [2] for time-invariant systems. Again, the same stability and boundedness properties can be proved.

Our aim has been to study the tracking capabilities of a class of observers that are, for the most part, closely connected to observers that were initially derived for time-invariant systems. We have shown that these observers are robust under parameter variations.

The main message to be derived is that, if the parameter variations are slow enough so that the regressors remain persistently exciting, then the global stability of the error system is preserved, and the estimation errors will be asymptotically proportional to the speed of the parameter variations.

What is not clear yet is whether a particular one of our observers should be chosen depending on the particular model in which the system is initially described, and how the asymptotic error bounds can be influenced by the design parameters.

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