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A STABLE ADAPTIVE OBSERVER FOR A CLASS OF NONLINEAR SECOND ORDER SYSTEMS

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ABSTRACT

We consider a class of single input single output second order nonlinear systems whose coefficients are bounded and have bounded time-variation. We describe an adaptive observer/identifier for these systems and derive sufficient conditions on the system and on the inputs that guarantee global stability of this adaptive observer. We present an application to a robot manipulator with two degrees of freedom.

1. STATEMENT OF THE PROBLEM

We consider the following class of single input single output (SISO) nonlinear systems:

$$\ddot{y}(t) + a_1(y,\dot{y},t)\dot{y}(t) + a_2(y,\dot{y},t)y(t) + f(y) = b(y,\dot{y},t)u(t) \quad (1.1)$$

where a_1, a_2 and b are parameters which may depend on y and $\dot{y}$, but which we shall consider as unknown functions of time, and where f(y) is a known function of y. Under the assumptions A.1 to A.5 on the parameters and the signals spelled out below, we present a stable adaptive observer for the on-line estimation of $\dot{y}(t)$ from measurements of u(t) and y(t), and we illustrate it with an application in robotics.

Our observer is an extension to nonlinear systems of an adaptive observer initially derived by Lüders and Narendra (1973) for linear time-invariant systems, and is a special case of an adaptive observer derived in Bastin and Gevers (1985) for a wide range of nonlinear systems. The reason for presenting this special case is that, for the system (1.1), we can present a complete set of assumptions that guarantee global stability of the observer. Before spelling out our assumptions, we need to make the following technical definitions.

Definition 1.1: C_Δ is a set $\{t_i\}$ of points in $[0,\infty)$ for which there exists a Δ such that for any $t_i, t_j \in C_\Delta$ with $t_i \neq t_j$, $|t_i - t_j| > \Delta$, i.e. C_Δ comprises points spaced at least Δ apart.

Definition 1.2: A function $f(\cdot)$ belongs to U_Δ if there exist Δ and C_Δ such that

i) $f(t)$ and $\dot{f}(t)$ are continuous on $\{[0,\infty) - C_\Delta\}$;

ii) there exist constants M_1 and M_2 such that $|f(t)| < M_1$ and $|\dot{f}(t)| < M_2$ $\forall t \in \{[0,\infty) - C_\Delta\}$;

iii) $\dot{f}(t)$ has finite limits as $t \downarrow t_i$ and $t \uparrow t_i$ for each $t_i \in C_\Delta$.

In other words, functions in U_Δ are smooth enough to have bounded continuous derivatives except at a countable number of switchings, which cannot occur too frequently.

We now make the following assumptions:

A.1: $a_1(\cdot)$, $a_2(\cdot)$ and $b(\cdot)$ are continuous w.r.t. y, $\dot{y}$ and t and differentiable w.r.t. t. They satisfy the inequalities

$$0 \leq \ell_1 \leq a_1 \leq \ell_2, \quad 0 \leq m_1 \leq a_2 \leq m_2, \quad 0 \leq n_1 \leq b \leq n_2$$

for some finite ℓ_2, m_2 and n_2, with

$$\ell_2 < \frac{m_2 + 2\sqrt{m_1 m_2} + 5m_1}{\sqrt{m_2} - \sqrt{m_1}}, \qquad \ell_1 > \sqrt{m_2} - \sqrt{m_1}$$

A.2: The derivatives $\dot{a}_1(\;)$, $\ddot{a}_1(\;)$, $\dot{a}_2(\cdot)$ and $\dot{b}(\cdot)$ are uniformly bounded, i.e. there exists a K, $0<K<\infty$, such that:

$$|\dot{a}_1(y,\dot{y},t)| \leq K, \quad |\dot{a}_2(y,\dot{y},t)| \leq K, \quad |\dot{b}(y,\dot{y},t)| \leq K$$

and $|\ddot{a}_1(y,\dot{y},t)| \leq K$ for all $t \in [0,\infty)$.

A.3: $u(t)$ and $y(t)$ belong to U_Δ, and $|u(t)| \leq M < \infty$ $\forall t \in [0,\infty)$

A.4: There exists $\delta>0$, $t_o>0$ and $\alpha_1>0$ such that $\forall t > t_o$

$$\int_t^{t+\delta} W(\tau)W^T(\tau)\,d\tau \geq \alpha_1 I$$

where

$$W^T(\tau) \triangleq \frac{1}{(s+\gamma)^3} [\; u \quad su \quad s^2u \quad s^3u \;]$$

for some arbitrary $\gamma>0$.

A.5: $f(y)$ is a known bounded function of y, and there exists N, $0<N<\infty$, such that $|f(y)| \leq N \quad \forall t\in[0,\infty)$ and all $u(\cdot)$.

Assumption A.1 will ensure that the system is BIBO stable, while A.2 will guarantee that the inputs of the error system of our adaptive observer remain bounded. A.3 provides that the signals $u(t)$ and $y(t)$ are smooth enough (see above), while A.4 is a persistence of excitation condition on the input that will be needed to guarantee the exponential convergence of the homogeneous part of the error system.

2. THE ADAPTIVE OBSERVER

The system (1.1) can be written in the following phase variable form:

$$\begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f(y) \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u \\ y = z_1 \end{cases} \tag{2.1}$$

Now consider the following transformation:

$$\begin{cases} x_1 = z_1 \\ x_2 = z_2 + (a_1 - c_2) z_1 \end{cases} \tag{2.2}$$

for some arbitrary positive constant $c_2>0$. Then (2.1) is equivalent with

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \theta_1 & 1 \\ \theta_2 & -c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f(y) \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u \\ y = x_1 \end{cases} \tag{2.3}$$

where

$$\begin{cases} \theta_1 = c_2 - a_1 \\ \theta_2 = \dot{a}_1 - a_2 + c_2 a_1 - c_2^2 \end{cases} \tag{2.4}$$

Notice that $\theta_1=\theta_1(t)$ and $\theta_2=\theta_2(t)$. The system (2.3) can now be rewritten in the following "adaptive observer canonical form" (see Bastin and Gevers, 1985):

$$\begin{cases} \dot{x}(t) = Rx(t) + \Omega(u,y)\theta(t) + \begin{bmatrix} 0 \\ f(y) \end{bmatrix} \\ y = x_1 \end{cases} \tag{2.5}$$

where

$$R = \begin{bmatrix} 0 & 1 \\ 0 & -c_2 \end{bmatrix}, \qquad \Omega(u,y) = \begin{bmatrix} y & 0 & 0 \\ 0 & y & u \end{bmatrix}, \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \tag{2.6}$$

with θ_1 and θ_2 as in (2.4) and θ_3=b. Notice that the form (2.5) is now linear in the unknown parameters $\theta_1(t)$ and $\theta_2(t)$, since $\Omega(u,y)$ contains only known quantities. Notice also that the transformation (2.2) is invertible, i.e. one can compute z_1 and z_2 (i.e. y and $\dot{y}$) from x_1 and x_2, provided a_1 can be estimated. It follows directly from (2.4) that the estimation (or tracking) of $a_1(t)$ is equivalent to the estimation of $\theta_1(t)$, given that c_2 is a known constant. As we shall see, c_2 will determine the dynamics of the adaptive observer.

We can now write the following adaptive observer for (2.5):

$$\dot{\hat{x}} = R\hat{x} + \Omega(u,y)\hat{\theta} + \begin{bmatrix} 0 \\ f(y) \end{bmatrix} + \begin{bmatrix} c_1\tilde{x}_1 \\ V^T\dot{\hat{\theta}} \end{bmatrix} \tag{2.7a}$$

where V is a vector $V^T = [0 \ v_2 \ v_3]$ which is the solution of

$$\begin{bmatrix} \dot{v}_2 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} -c_2 & 0 \\ 0 & -c_2 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} y \\ u \end{bmatrix}, \qquad V(0) = 0 \tag{2.7b}$$

$$\Phi = \begin{bmatrix} y \\ v_2 \\ v_3 \end{bmatrix} \tag{2.7c}$$

$$\dot{\hat{\theta}} = \Gamma\Phi\tilde{x}_1, \qquad \Gamma > 0 \tag{2.7d}$$

In (2.7a) c_1 is a positive constant, to be chosen by the designer and which, with c_2, will determine the dynamics of the state estimator, while $\tilde{x}_1 \triangleq x_1-\hat{x}_1$. Recall that $x_1 \triangleq y$ is measured. Equations (2.7b,c) constitute an auxiliary system which generates the regression vector $\Phi(t)$. Finally Γ in (2.7d) is a 3x3 positive definite matrix which will determine the dynamics of the parameter estimator; normally Γ will be chosen as $\mathrm{diag}(\gamma_1,\gamma_2,\gamma_3)$ with $\gamma_i>0$, i=1,2,3.

3. THE MAIN STABILITY RESULT

We first derive the error model for the adaptive observer. Define $\tilde{x} \triangleq x-\hat{x}$ and $\tilde{\theta} = \theta-\hat{\theta}$. It then follows from (2.5) - (2.7) that

$$\dot{\tilde{x}} = R^*\tilde{x} + \Omega(u,y)\tilde{\theta} - \begin{bmatrix} 0 \\ -VT\dot{\hat{\theta}} \end{bmatrix} \qquad (3.1)$$

where

$$R^* \triangleq \begin{bmatrix} -c_1 & 1 \\ 0 & -c_2 \end{bmatrix} \qquad (3.2)$$

We can now define

$$\tilde{x}^* \triangleq \tilde{x} - \begin{bmatrix} 0 \\ VT\tilde{\theta} \end{bmatrix} \qquad (3.3)$$

After lenghty manipulations the following error model is obtained

$$\begin{bmatrix} \dot{\tilde{x}}^* \\ \\ \dot{\tilde{\theta}} \end{bmatrix} = \left[\begin{array}{cc|ccc} \multicolumn{2}{c|}{R^*} & & \Phi^T & \\ & & 0 & 0 & 0 \\ \hline -\Gamma\Phi & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \tilde{x}^* \\ \\ \tilde{\theta} \end{bmatrix} + \left[\begin{array}{cc} 0 & 0 \\ \multicolumn{2}{c}{-VT} \\ \hline \multicolumn{2}{c}{I} \end{array}\right] \dot{\theta} \qquad (3.4)$$

where R^* is defined by (3.2), and $V(t)$ and $\Phi(t)$ are the solutions of (2.7b,c). Now (3.4) is a 5^{th} order linear time varying state equation driven by $\dot{\theta}$. Our main theorem states that this system is Bounded Input Bounded State (BIBS) stable. We shall denote

$$\varepsilon(t) \triangleq \begin{bmatrix} \tilde{x}^*(t) \\ \tilde{\theta}(t) \end{bmatrix} \qquad (3.5)$$

<u>Theorem 3.1</u>: If the assumptions A.1 to A.5 hold and if the constant K in A.2 is sufficiently small, then there exist finite positive constants K_1, K_2 and K_3 such that

$$\text{i)} \quad ||\epsilon(t)|| \leq K_1||\epsilon(0)|| + K_2 \tag{3.6a}$$

$$\text{ii)} \quad \lim_{t\to\infty} \sup||\epsilon(t)|| \leq K_3K \tag{3.6b}$$

<u>Proof</u>: We first prove that the homogeneous part of (3.4) is exponentially asymptotically stable (EAS); the result will then follow from the boundedness of $v^T\dot{\theta}$.

First note that by eliminating $\tilde{x}^*$ the homogeneous part can be rewritten as

$$\dot{\tilde{\theta}}(t) = -\Gamma\Phi(t)H(s)\{\Phi^T(t)\tilde{\theta}(t)\} \tag{3.7}$$

where $H(s) = e_1^T(sI - R^*)^{-1}e_1 = \frac{1}{s+c_1}$, with $e_1^T = (1\ 0)$. We notice that $H(\infty)$ 0 and that $H(s)$ is strictly positive real (SPR) when $c_1>0$. The exponential stability of the homogeneous part will follow (see Anderson et al. (1986), Theorem 2.3) if we can prove that $\Phi(t)$ is bounded, $\Phi(t)\in U_\Delta$ and $\Phi(t)$ is persistently exciting, i.e. if there exist positive constants T, t_0, α and β such that for all $t\geq t_0$:

$$0 < \alpha I \leq \int_t^{t+T}\Phi(\tau)\Phi^T(\tau)d\tau \leq \beta I < \infty \tag{3.8}$$

We note from (2.7b,c) that

$$\Phi^T(t) = [y(t)\ \frac{y(t)}{s+c_2}\ \frac{u(t)}{s+c_2}] \tag{3.9}$$

We first establish that $\Phi(t)$ is bounded. The conditions A.1 on $a_1(\cdot)$ and $a_2(\cdot)$ imply, by a result of Starzinskii (1952), that the homogeneous system $\ddot{y} + a_1\dot{y} + a_2y = 0$ is uniformly asymptotically stable with equilibrium $\dot{y}=y=0$. It then follows from Theorem 3.1 p. 105 of Willems (1970), and from the boundedness of b, u(t) and f(y) (assumptions A.2, A.3 and A.5) that y(t) is bounded on $(0,\infty)$. Therefore $\Phi(t)$ is bounded, since it is the output of a stable filter with inputs u(t) and y(t). In addition $\Phi(t)\in U_\Delta$ by A.3.

We now establish the lower bound in (3.8). Using A.2 and A.4, the BIBO stability of (1.1), and the boundedness of $u(t)$, it follows, using Theorem 6.2 of Mareels and Gevers (1986) that there exist $\alpha>0$, $t_0>0$ and $T>0$ such that $\forall t \geq t_0$

$$\alpha I \leq \int_t^{t+T} N(\tau)N^T(\tau)d\tau \tag{3.10}$$

where

$$N^T(\tau) \triangleq \left[y(\tau) \ \frac{y(\tau)}{s+\gamma} \ \frac{u(\tau)}{s+\gamma} \right]^T \tag{3.11}$$

with γ an arbitrary positive constant. The lower bound in (3.8) follows from the form of $\Phi(t)$: see (3.9). We conclude from Anderson et al. (1986) that the homogeneous part of (3.4) is EAS. Therefore there exist finite $K_1>0$ and $a>0$ such that

$$||\epsilon(t)|| \leq K_1 e^{-at}||\epsilon(0)|| + \int_0^t K_1 e^{-a(t-\tau)}||V^T(\tau)\dot{\theta}(\tau)||d\tau$$

The results (3.6a,b) follow immediately, using A.2, (2.4) and the fact that $V(t)$ is bounded because $u(t)$ and $y(t)$ are.

Comment 1: It follows immediately from (3.6) and (3.3) that there exist positive constants K_4, K_5 and K_6 such that

$$\text{i)} \quad ||e(t)|| \leq K_4||e(0)|| + K_5 \tag{3.12a}$$

$$\text{ii)} \quad \limsup_{t\to\infty} ||e(t)|| \leq K_6 K \tag{3.12b}$$

where K is defined in A.2 and

$$e(t) \triangleq \begin{bmatrix} \tilde{x}(t) \\ \tilde{\theta}(t) \end{bmatrix} \tag{3.13}$$

Hence all the internal variables of our adaptive observer remain bounded.

Comment 2: Notice from (3.12b) that the upper bound on the asymptotic accuracy of our observer is proportional to the rate K of the parameter variations. If the parameters of the system are constant, the errors (in $\tilde{x}$ and in $\tilde{\theta}$) converge to zero exponentially fast.

Comment 3: The boundedness assumption A.3 on u(t) can be replaced by the weaker condition that u(t) is locally integrable such that

$$\int_t^{t+T} |u(\tau)| d\tau \leq M < \infty \qquad \forall t \in [0,\infty)$$

and all finite T. However, this adds a number of technical complications in the proof.

The adaptive observer (2.7) estimates the state $\hat{x}(t)$ and the parameter vector $\hat{\theta}(t)$ of the system (2.3). It is closely related to an adaptive observer first derived by Lüders and Narendra (1973) for linear time invariant systems. Our contribution was to show that, with reasonable constraints on the parameters of the nonlinear system (1.1) and with bounds on their time-variation, the original state-estimation problem for (2.1) can be reformulated as a state observer + parameter estimator problem for (2.5) where the parameters θ(t) are now considered as time-varying parameters to be tracked. These parameters now multiply only the measured variables u(t) and y(t). From (2.7) the original state-estimates can now easily be reconstructed by inverting (2.2):

$$\begin{cases} \hat{z}_1 = \hat{x}_1 & (3.14a) \\ \hat{z}_2 \triangleq \hat{\dot{y}} = \hat{x}_2 + \hat{\theta}_1 x_1 & (3.14b) \end{cases}$$

The boundedness of $\hat{x}(t)$ and $\hat{\theta}(t)$ implies that of $\hat{z}(t)$.

Finally notice that the sufficient richness conditions A.4 on u(t) will be satisfied if u(t) is a linear combination of two sinusoids at different frequencies.

4. APPLICATION TO A ROBOT MANIPULATOR

We consider an application to a telescopic arm in a vertical plane, which performs a "pick and place" robot manipulation: see Fig. 1. We call M the mass of the load, ℓ(t) the length of the arm, y(t) the angle with the vertical axis, k the tension of the spring, α the viscous friction coefficient, u the applied torque, and we assume that the arm mass is negligible w.r.t. to the load. The equations of motion are:

$$M\ell^2\ddot{y} + M\dot{\ell}\ell\dot{y} + \alpha\dot{y} + ky + M\ell g \sin y = u \qquad (4.1)$$

In addition, we assume that the trajectory is imposed by a guiding device which imposes a fixed and known relationship between ℓ(t) and y(t):

$$\ell = g(y) \tag{4.2}$$

We consider an application where the angular position $y(t)$ and the torque $u(t)$ are measured, and where it is desired to estimate the angular speed $\dot{y}(t)$, the mass M and the friction coefficient α. From (4.1) and (4.2) we can write

$$\ddot{y} + \left[\frac{1}{\ell}\frac{dg}{dy}\dot{y} + \frac{\alpha}{M\ell^2} \right]\dot{y} + \frac{k}{M\ell^2} y + \frac{g}{\ell}\sin y = \frac{1}{M} \cdot \frac{u}{\ell^2} \tag{4.3}$$

Since $y(t)$ is measured, and $\ell = g(y)$ is known, it follows that $\ell(t)$ and $\frac{dg}{d\ell}$ are known. Now define

$$a_1(t) = \frac{1}{\ell}\frac{dg}{dy}\dot{y} + \frac{\alpha}{M\ell^2}, \quad a_2(t) = \frac{k}{M\ell^2}, \quad b = \frac{1}{M} \tag{4.4}$$

Using the transformation (2.2) and defining θ_1 and θ_2 as in (2.4) and $\theta_3 = b = \frac{1}{M}$, we obtain the representation (2.5) with

$$R = \begin{pmatrix} 0 & 1 \\ 0 & -c_2 \end{pmatrix}, \quad \Omega(u,y) = \begin{pmatrix} y & 0 & 0 \\ 0 & y & \frac{u}{\ell^2} \end{pmatrix}, \quad f(y) = -\frac{g}{\ell}\sin y \tag{4.5}$$

The adaptive observer (2.7) is directly applicable with u replaced by $\frac{u}{\ell^2}$ in (2.7b). Notice that this adaptive observer is also a parameter identifier, since from $\hat{x}$ and $\hat{\theta}$ one can estimate successively:

$$\hat{M} = \frac{1}{\hat{\theta}_3}, \quad \hat{\dot{y}} = \hat{x}_2 + \hat{\theta}_1 x_1, \quad \hat{\alpha} = \frac{\ell^2}{\hat{\theta}_3}\left[c_2 - \hat{\theta}_1 - \frac{1}{\ell}\frac{dg}{dy}\hat{\dot{y}}\right] \tag{4.6}$$

This is a common feature of the adaptive observers described in this paper.

5. SIMULATIONS

The robot application of Section 4 has been simulated on a Microvax II, using a NAG subroutine for the solution of the differential equations. Due to lack of time, only a very limited number of simulations have been performed at the time of printing this paper. The model (4.1)-(4.2) has been used with:

$$\ell(t) \triangleq g(y) = \ell_0 + \beta y(t)$$

and with the following parameters: α=0.3, M=1, ℓ_0=0.8, β=0.3, k=2. The input was a pseudo random binary sequence (PRBS) with mean 5 and amplitude ± 2. From these simulations, two conclusions can be drawn:

1) The adaptive observer/identifier performs very well as an observer, but not as well as an identifier. This means that the tracking of $\dot{y}$, computed as in (4.6), is excellent after an initial transient period, but that correct estimates of the parameters are much harder to obtain. The same observation was made in Narendra (1976) about the observer/identifier of Lüders and Narendra for time-invariant linear systems to which our own observer simplifies in the case of constant parameters.

2) The behaviour of the observer as a parameter identifier appears to be rather sensitive to an adequate choice of the gains $c_1, c_2, \gamma_1, \gamma_2$ and γ_3, and to the persistency of excitation of the input signal u(t). In particular, if the gains are too high, the parameter adaptation becomes exceedingly slow. This is in line with recent theory on robustness of adaptive systems and on averaging: see Anderson et al. (1986). On the other hand, the estimation of $\dot{y}(t)$ is rather insensitive to the gains, provided they are sufficiently small.

3) The observer for $\dot{y}$ performs almost as well for the nonlinear time-varying system (4.3) as for the time-invariant system to which (4.3) reduces in the case where β=0. In such case our observer is identical to that of Lüders and Narendra (1973). This is illustrated by Figures 2 and 3. These figures show the responses of the angular speed $\dot{y}$ (full line) and its estimate $\hat{\dot{y}}$ (dotted line) for the case of a constant system (β=0) in Fig. 2 and a time-varying nonlinear system (β=0.5) in Fig. 3, with the following observer gains: c_1=0.1, c_2=0.1, γ_1=0.2, γ_2=0.2, γ_3=0.2. In each case the input was a PRBS with mean 5 and amplitude 2.

6. CONCLUSIONS

We have shown how to construct a globally stable adaptive observer/identifier for a class of second order nonlinear systems with reasonable assumptions on the system parameters. Our adaptive observer is inspired by one that was originally derived for linear time-invariant systems. Our contribution has been to show that a class of second order nonlinear systems can be rewritten in a way that has some "linearity in the parameters" property, for which this adaptive observer can be readily derived, and to then prove global stability by combining a number of recent results on exponential stability and persistence of excitation. We believe that our results will find applications in the area of mechanical systems.

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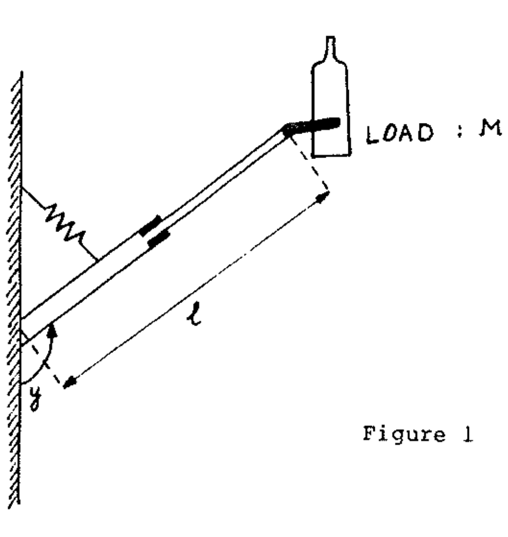

Figure 1

Figure 1 : A "pick and place" robot manipulator

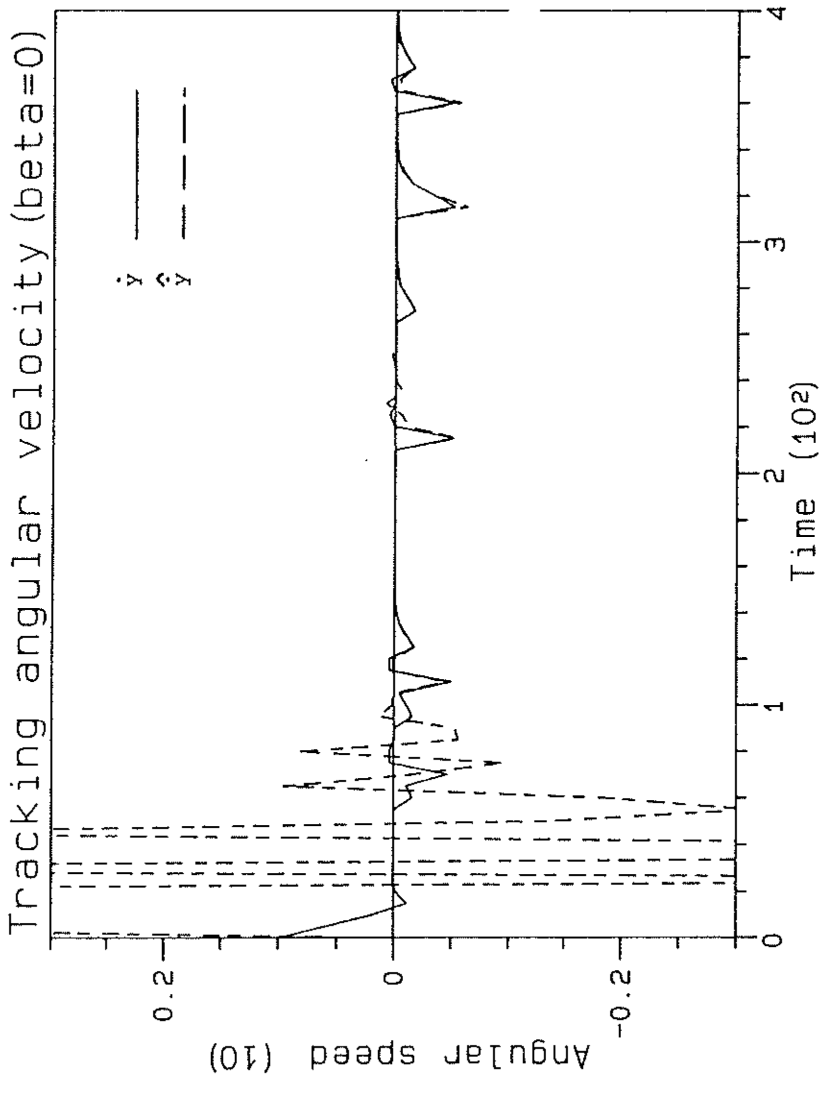
Tracking angular velocity (beta=0)
Angular speed (10)
Time (10²)
0.2
0
-0.2
0
1
2
3
4
$\dot{y}$
$\hat{\dot{y}}$

Figure 2

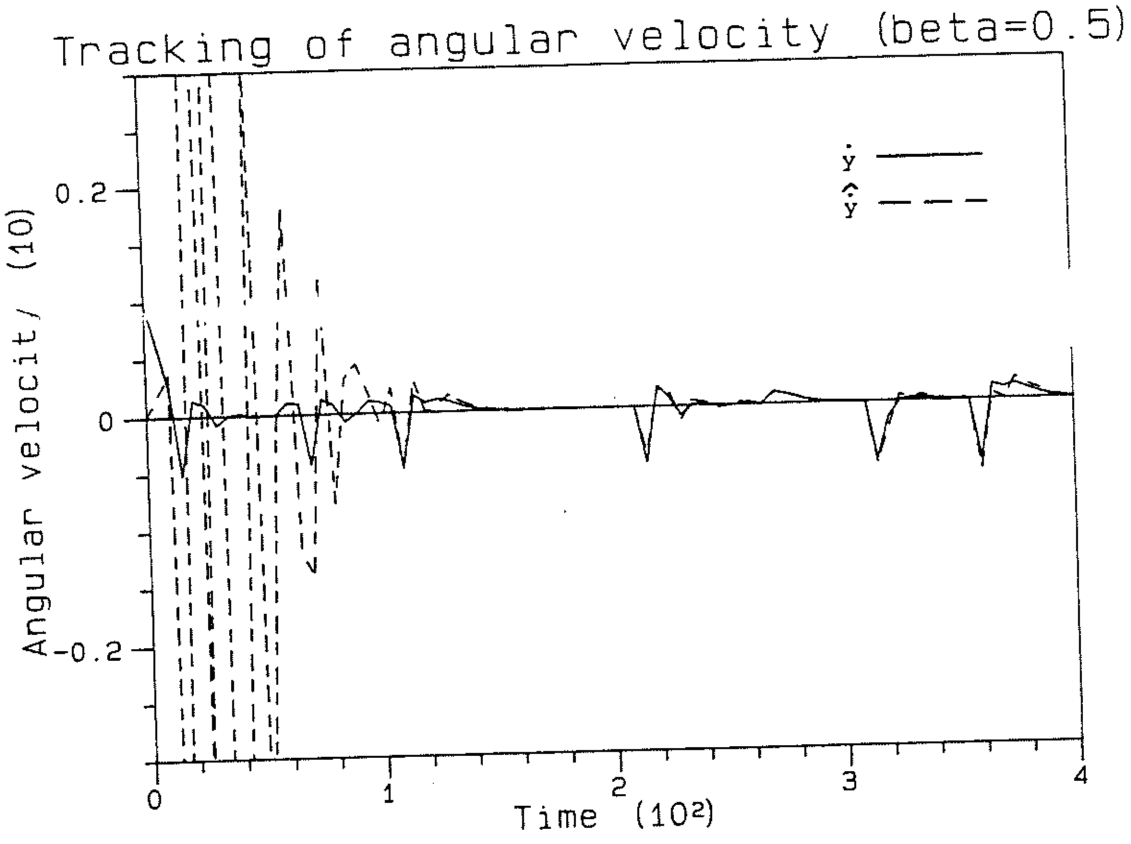
Tracking of angular velocity (beta=0.5)
$\dot{y}$
$\hat{\dot{y}}$
0.2
0
-0.2
Angular velocit/ (10)
0
1
2
3
4
Time (10^2)

Figure 3