Least costly identification experiment for control

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Extended Abstract

A controller for a real-life system G_0 is usually designed on the basis of a model $\hat{G}$ of G_0 identified using data collected from the true system. When designing the identification experiment, the control engineer often has to make a trade-off between her/his desire of obtaining an accurate model and the economical constraint of keeping the experimental costs low. Obtaining an accurate model requires a long identification experiment and a powerful input signal, while keeping the experimental costs low corresponds to a short experiment time and the excitation of G_0 with a low power signal.

The typical approach to this problem has been to maximize the accuracy of the identified model (possibly with a given, say, control-oriented objective in mind) for a given experiment time and under prespecified constraints on input power. In this paper, we address this tradeoff from the dual perspective; namely, we seek the least costly identification experiment leading to a required model accuracy, with a control-oriented objective in mind. More precisely, we assume that the experiment time is fixed, and we then define the least costly identification experiment for control as the experiment on G_0 whose input signal power $\mathcal{P}_u$ is minimized under the constraint that the controller $\hat{C}$ designed from the identified model $\hat{G}$ is guaranteed to stabilize and to achieve sufficient performance with the unknown true G_0. In this paper, the desired performance on G_0 is expressed by magnitude bounds on one (or several) closed-loop transfer functions of $[\hat{C}\ G_0]$ (H_∞ performance constraints). Even though our results focus on the design of the input signal $u(t)$ with minimum total power $\mathcal{P}_u$, for a fixed data length, they can easily be extended to the design of the shortest identification experiment for control, given a fixed input spectrum.

Our "least costly identification for control" approach to the experiment design problem is a novel way of addressing the above-mentioned tradeoff between low experiment cost and sufficient precision of the model (or the controller). Until now, this experiment design problem has been addressed from a different and/or converse angle. In [9, 15], model accuracy only is considered: the optimal identification experiment is defined as the one for which a weighted version of the covariance matrix of the identified parameter vector is minimized under a constraint on the maximal power of the input signal. In [8, 11, 6, 13, 10, 12], the experiment design problem is connected to control design; the optimal identification experiment is defined as the one which, under a constraint on the maximal power of the input signal, minimizes:

- either a measure of the performance degradation between the achieved loop $[\hat{C}\ G_0]$ and the ideal loop $[C_0\ G_0]$ ($\hat{C}$ is the controller designed with the identified model $\hat{G}$, and C_0 is the controller that would be designed with the unknown G_0). This measure of performance degradation is in fact $E|y(t) - \hat{y}(t)|_2^2$ where $y(t)$ is the output of the ideal loop and $\hat{y}(t)$ is the output of the achieved loop [8, 11, 6, 13].
- or some measure of the performance degradation between the achieved loop $[\hat{C}\ G_0]$ and the designed

loop $[\hat{C}\ \hat{G}]$ [10, 12]. In [10], this measure is the ν-gap between $\hat{G}$ and G_0 (a robust stability measure). In [12], this measure is the difference between $\hat{T} = (\hat{G}\hat{C})/(1+\hat{G}\hat{C})$ and $T_0 = (G_0\hat{C})/(1+G_0\hat{C})$.

To summarize, the main difference between the experiment design problem defined in the above references and the problem defined in this paper lies in the pursued objective. Indeed, our objective is to minimize the economical cost of the identification while guaranteeing the required performance level for the achieved loop $[\hat{C}\ G_0]$; the objective in the above references is somehow the dual objective i.e. obtaining the best performance for $[\hat{C}\ G_0]$ given a fixed cost for the identification experiment. In addition, this paper is the first one on experiment design which defines the performance objective for the model-based controller in an H_∞ control design framework (i.e. by magnitude bounds on the to-be-achieved closed-loop transfer functions).

Our new experiment design problem will be solved in the following context. We will assume that the identification experiment concerns a SISO LTI[1] true system G_0 and is performed in open loop with a full-order model structure [14]. We will further assume that the controller $\hat{C}$ we want to apply to the true system will be designed from the identified model $\hat{G} = G(z, \hat{\theta}_N)$ using a pre-defined H_∞ control design method with fixed weights [16]. In this particular context, we will also make use of the fact that, along with the model $\hat{G}$, an identification experiment delivers an uncertainty region centered at $\hat{G} = G(z, \hat{\theta}_N)$ and containing the true system $G_0 = G(z, \theta_0)$ at a user-chosen probability level [7]. In this paper, we will use an additive description of this uncertainty region $\mathcal{D}$. A reliable analytical expression of the size $r_u(\omega)$ of this estimated additive uncertainty region $\mathcal{D}$ can be obtained using a first-order approximation [14, 1]:

$$r_u(\omega) = \sqrt{\chi \lambda_1 \left(T(e^{j\omega}) P_\theta T(e^{j\omega})^T \right)} \tag{1}$$

where χ is a real constant dependent of the chosen probability level, $\lambda_1(A)$ denotes the largest eigenvalue of A, P_θ is the covariance matrix of $\hat{\theta}_N$ and

$$T(e^{j\omega}) \triangleq \begin{pmatrix} Re(\Lambda_G^T(e^{j\omega}, \theta_0)) \\ Im(\Lambda_G^T(e^{j\omega}, \theta_0)) \end{pmatrix} \quad \text{with} \quad \Lambda_G(z, \theta) = \frac{\partial G(z, \theta)}{\partial \theta}$$

Expression (1) shows that $r_u(\omega)$ is a function of the covariance matrix P_θ of the identified parameter vector $\hat{\theta}_N$ and, consequently, a function of the chosen input signal $u(t)$. Note also the dependence of $r_u(\omega)$ on the unknown true system. This dependence will occur at different stages of this presentation. Such a phenomenon is inherent for all experiment design problems and this difficulty will here be circumvented in a standard way by assuming that a preliminary estimate of the true system is known beforehand.

In the context presented above, we propose the following two-step methodology to solve the experiment design problem leading to the least costly identification for control. In a first step, we determine what is the largest additive uncertainty region that we can a-priori tolerate around the to-be-identified model $\hat{G}$ for the $\hat{G}$-based controller $\hat{C}$ to achieve the required H_∞ performance level with all systems in this uncertainty region. The size (i.e. the uncertainty radius) of this largest admissible uncertainty region for control is denoted $r_{adm}(\omega)$. In a second (identification design) step, we then deduce the least powerful stationary input signal $u(t)$ such that the size $r_u(\omega)$ of the identified uncertainty region $\mathcal{D}$ is at each frequency smaller than the largest admissible uncertainty radius $r_{adm}(\omega)$. By doing this, we ensure that the controller $\hat{C}$ designed with the model identified with such input signal, achieves the required H_∞ performance level with all systems in the identified $\mathcal{D}$ (and thus also with the true system G_0).

Technical aspects. In order to determine the largest admissible uncertainty radius $r_{adm}(\omega)$ for control, the main difficulty is the fact that the expression of the to-be-identified model $\hat{G}$ is not available for this computation. This difficulty will be circumvented by constructing a-priori a set to which we assume the model $\hat{G}$ will belong. We then show that the computation of $r_{adm}(\omega)$ boils down to a ν-analysis problem [4, 5].

In order to solve the second step, we will make use of results presented in [15] (and used in [10]). These results show that, for each stationary input signal $u(t)$, we can define a finite-sized vector x_u of moments of the input power spectrum $\Phi_u(\omega)$ weighted with a special weight depending on the true system G_0; and that

[1]single-input single-output linear time-invariant

this vector x_u is such that both the inverse P_θ^{-1} of the covariance matrix of the parameter vector identified with $u(t)$ and the power $\mathcal{P}_u$ of $u(t)$ are affine functions of x_u. Using these properties, we then show that the optimization of the power $\mathcal{P}_u$ of $u(t)$ under the constraint $r_u(\omega) \leq r_{adm}(\omega) \;\; \forall\omega$ can be reduced to a tractable LMI[2] optimization problem on the finite-sized vector x_u. A stationary input signal $u(t)$ for the identification can thereafter be constructed from the optimal moment vector x_u.

Note that the results presented in this paper clearly improve the preliminary results in [3] where a high-oder model approximation of $r_u(\omega)$ was used instead of (1).

As a last remark in this extended abstract, we would like to stress that the results on the least costly identification experiment for control can also be seen as a logical sequel to our earlier results on the connection between Prediction Error (PE) identification and Robust Control [2, 7]. Indeed, in these contributions, we have shown that, given a set of data collected from the true system, we are able to identify a model $\hat{G}$ along with its uncertainty description and to verify thereafter whether a $\hat{G}$-based controller stabilizes and achieves the required performance level with all systems in this uncertainty region. Due to the dependence of both the model and the uncertainty region on the chosen data set, this procedure may deliver a model with a too large uncertainty region for which it is impossible to design a controller meeting the performance requirements. By giving a procedure to choose the data set appropriately in order to avoid such a situation, this paper improves the connection between PE identification and robust control.

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