

IDENTIFICATION FOR CONTROL: ACHIEVEMENTS AND OPEN PROBLEMS¹

Michel Gevers*

* *Center for Systems Engineering and Applied Mechanics
(CESAME)*

*Université Catholique de Louvain
Bâtiment Euler, 4 Av. Georges Lemaître,
B-1348 Louvain-la-Neuve, Belgium*

Phone: +32-10-472590

Email: gevers@csam.ucl.ac.be

Abstract: This paper presents the author's views on the development of identification for control. The paper reviews the emergence of this subject as a specific topic within the larger discipline of system identification, the major progress accomplished over the last 15 years and its practical impact, as well as the remaining open problems.

Keywords: Identification, identification for control, robust control, optimal design.

1. INTRODUCTION

Identification for control! Identification and control have initially emerged as two separate disciplines, developed by two distinct communities.

Identification has for a long time been the territory of mathematicians, statisticians, time-series analysts and econometricians. The history of system identification goes as far back as the work of Gauss and Legendre in the late 18th and early 19th century. An excellent presentation of the history of system identification can be found in (Deistler, 2002). Control theory has always been the territory of engineers. Until about 1960, most of control design was based on model-free methods, using Bode, Nyquist, or Ziegler-Nichols plots, combined with graphical design techniques. The introduction of state-space models in 1960, together with the solution of optimal control and optimal filtering problems in a Linear Quadratic Gaussian framework (Kalman, 1960a; Kalman,

1960b), gave birth to a tremendous development of model-based control design methods. Successful applications abounded, particularly in aerospace, where accurate models were readily available.

1965 saw the start of identification activity in the control community. The paper (Ho and Kalman, 1965) set the stage for state space realization theory which, 25 years later, became the founding block for what is now called subspace identification. The paper (Åström and Bohlin, 1965) introduced into the control community the Maximum Likelihood framework for the identification of input-output models. This gave rise to the celebrated Prediction Error (PE) framework (Ljung, 1999) that has since proven so successful. Undoubtedly the advent of identification theory was spurred by a desire to extend the applicability of model-based control design to broader and broader fields of applications, for which no reliable models could be obtained from first principles.

From 1965 to the late eighties, model-based control was applied to ever growing classes of dynamical systems and processes, with models obtained via the newly emerging identification techniques.

¹ This paper presents research results of the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office. The scientific responsibility rests with its author.

The prevailing habit at that time was to separate the identification step and the control design step. A model was identified first using the best available techniques; subsequently, a model-based control design was performed based on the “certainty equivalence principle”, i.e. the model was treated as if it represented the true system. Dual control and adaptive control were two early attempts to address the issue of parametric uncertainty and model-based control design in a synergistic way. In dual control the parameter estimation and the control design mechanism are obtained as the result of a single but complex optimization problem. In adaptive control, the parameter adjustment scheme is subsidiary to the control objective. Both schemes were developed for the case where the structure of the true system is assumed to be known, which severely limits their practical applicability. The solution of the dual control problem proved to be computationally intractable, even in the simplest cases. As for adaptive control, the major difficulty is that the parameters of the feedback control system change at every sampling instant, making the closed-loop dynamics nonlinear and their stability analysis extremely complex. After convergence mechanisms had been devised for the ideal case where the system is in the model set, attempts were made to robustify the adaptive control algorithms in order to take account of some modest degree of uncertainty. These attempts essentially consisted of introducing cautionary safeguards in the computation of the gain of the parameter adjustment scheme; see e.g. (Anderson *et al.*, 1986).

One of the main contributions of the control community to system identification theory was to consider identification as an exercise in estimating the *best possible approximate model* within some model set, rather than as a search for the *true system*. Together with this effort came the characterization of the approximate model in terms of bias error and variance error on the estimated transfer functions.

If the model of a system is exact, it is optimal for all applications. However, if the model is only an approximation of the “true system”, then the quality of the model should be dependent on the intended application. It thus makes sense to tune the identification towards the objective for which the model is to be used, i.e. to ensure that the distribution of model error is such that it does not deteriorate the objective too much. This gave rise to the paradigm of *goal-oriented identification* and it led one to view identification as a *design problem*. Identification for control has been the major outlet for this new paradigm. The reasons for this are many: (i) control is very often the main motivation for model building; (ii) high performance control can often be achieved with very

simple models, provided some basic dynamical features of the system are accurately reflected; (iii) a powerful robust control theory, based on nominal models and uncertainty sets, had been developed all through the eighties, but these models and uncertainty sets were not data-based for lack of a proper theory; (iv) identification for control research led to iterative model and controller tuning tools that were intuitive, practical and easy to implement by the process engineers.

One early approach to optimal identification design for control, that established a direct link between experimental conditions and controller performance, was obtained in (Gevers and Ljung, 1986) by considering variance errors only, i.e. the system was assumed to be in the model set. This approach consists, for a given certainty equivalence control design procedure, of computing the experimental conditions of the identification that minimize the average performance degradation that results from the fact that the controller is computed from an estimated (and hence random) model rather than from the exact true system. Like with all optimal experiment design methods, the optimal experiment depends on the unknown system: see Section 4. Hence, even though such results give useful insights, they do not provide an operational design method for the identification for control problem. In addition, they are based on a certainty equivalence controller design mechanism, rather than on a robust control design.

Except for the methods already mentioned (dual control, adaptive control, control-oriented optimal design), which are restricted to the case where the true system is in the model set, the first contributions in which identification and control design with restricted complexity models were looked upon as a combined design problem appeared only around 1990. The plenary (Gevers, 1991) at the 1991 IFAC Symposium on System Identification addressed many of the key issues; however, it was more an agenda for research than a presentation of solutions. Indeed, there was very little understanding at that time about the interplay between system identification and robust control. The two theories had been developed by separate communities with very little interaction. In the nineties, the activity in identification for control surged: in the plenary (Hjalmarsson, 2003), the author estimated that about 1,500 papers have appeared on the topic of identification for control after the plenary (Gevers, 1991).

When the application of a model is the design of a controller, then what really matters is the performance achieved by this model-based controller on the “true system”, not the intrinsic quality of the model. We illustrate this idea with a very simple example inspired by (Skelton, 1989). Let

a ‘true system’ be represented by $G_0(s) = \frac{1}{s+1}$. Then the ‘model’ $\hat{G}(s) = \frac{1}{s}$ would clearly be deemed to be absurd as a model for G_0 . However, if the objective is to design a high gain static output feedback controller, then \hat{G} would be a perfectly acceptable model. Indeed, with a high gain static output feedback $u = -Ky$, the closed loop transfer functions $\frac{K}{s+1+K}$ and $\frac{K}{s+K}$ become indistinguishable. Thus, whether or not a model is appropriate for control design depends as much on the controller that will be implemented as it depends on the plant/model mismatch.

In practice, the true system is unknown, the model is unknown at the identification design stage, and the controller that will be implemented is unknown because it depends on that model. What is typically known in control-oriented identification is the control performance objective. Some prior knowledge about the true system may also be available. Ideally, the design of a control-oriented identification procedure could then be formulated as follows: *Given a control performance objective, design the identification in such a way that the performance achieved by the model-based controller on the true system is as high as possible.* Identification design includes many choices: input data, feedback configuration (possibly), data length, model structure, identification criterion, validation criterion, etc. The identification design problem is impossible to solve in such generality. In order to get a handle on the problem, it has been customary to fix some of the choices; the number of data is usually taken to be fixed, and so is the model structure.

In the first half of the nineties, the research focused on the bias error distribution, assuming that low complexity models were being used for the design of the controller. It produced a string of results on the design of control-oriented nominal models (Schrama and Bosgra, 1993; Lee *et al.*, 1993; Åström and Nilsson, 1994; Zang *et al.*, 1995; de Callafon and Van den Hof, 1997). The first and rather obvious result was to establish that a model is good for control design if the closed loop system obtained by the feedback connection of that model with the designed controller is close to the system obtained by the feedback connection of the true system with that same controller. Since the ‘to be designed controller’ is not available at the identification stage, this led to the necessity of using an iterative scheme of model updates and controller updates.

A second important result that emerged from the understanding of control-oriented nominal models was that the experimental conditions of the identification should resemble as much as possible the experimental conditions obtained when the designed controller is applied to the true system

(Anderson and Gevers, 1998). This led to the observation that, for most control performance objectives, identification should be performed in closed loop (Hjalmarsson *et al.*, 1996; Forsell and Ljung, 2000). This observation triggered a revival of interest for closed loop identification, and the emergence of new identification methods specifically designed for this situation: see e.g. (Hansen *et al.*, 1989; Van den Hof and Schrama, 1993).

The iterative identification and control design schemes do not necessarily converge to a stationary point, corresponding to a stable closed loop system. Thus, they must be applied with caution, and a lot of work has gone into developing tools for safe model and controller updates. Even so, these iterative schemes have had a remarkably fast transfer into the world of applications. There are two main reasons for this:

- whereas much of the industrial world was still living with the belief that one should ‘open the loop’ to perform a valid identification experiment, here was a theory that showed the benefits of closed-loop identification; this came as welcome news to process control engineers who had never really liked the idea of opening the loop;
- in the process industry, thousands of measurements flow into the computer; here was a theory that showed how these data could be used for the design of a better controller.

As stated above, the work on control-oriented nominal models focused on the bias error distribution of the identified model, with the controller computed from the model in a certainty equivalence framework. Thus, that work did not incorporate the robust control concepts developed during the eighties. It focused on the design of identification criteria that minimize a (control-oriented) measure of the model error.

The second half of the nineties saw a shift towards the definition and estimation of *control-oriented uncertainty sets* (Kosut and Anderson, 1994; Mäkilä *et al.*, 1995; de Vries and Van den Hof, 1995; Bombois *et al.*, 2000) in order to put the control design into the framework of robust control design. The focus turned to shaping the distribution of the variance error of the identified models, i.e. on manipulating the shape of the uncertainty set. Indeed, the paradigm of robust control design is to compute a controller that achieves the best possible worst case performance, i.e. the best possible performance over all models in an uncertainty set. Such best worst case performance depends as much on the controller as it depends on the uncertainty set, and this set is directly dependent on the experimental conditions under which the identification is performed. The study of the interplay between experiment conditions

of the identification and properties of the robust controller has been split up into two questions:

- (1) what is the connection between a model uncertainty set and the properties of robust controllers computed from that set and, consequently, how should one define a control-oriented uncertainty set?
- (2) how should one design the identification experiment in such a way that the uncertainty set around the identified model has such ‘control-oriented’ property?

Even though many new insights have been gained on the first question, there is at this point no clear view as to the most operational definition of a ‘control-oriented uncertainty set’. We shall come back to this in Section 7.

To summarize this brief historical account, the work on identification of the last 15 years has been essentially developed in three directions: optimal control-oriented experiment design for identification, the definition and computation of control-oriented nominal models, and the connection between data-based uncertainty sets and the properties of robust controllers resulting from such sets. In addition, the demands of this highly visible field have generated lots of parallel work on estimation and validation of uncertainty sets from data, and on closed-loop identification. Some of the concepts that have emerged from these 15 years of work, such as the idea of improving the performance of an existing controller on the basis of closed loop data collected with the presently operating controller, have immediately found their way into practice. However, many questions remain unsolved, particularly on the optimal tuning of uncertainty sets, and we are still far from an automatic procedure that would go from the collection of experimental data to the design of a robust controller via a model and its uncertainty set.

In the rest of this paper, we explain in some more detail the specific achievements and the remaining unsolved questions in the different subtopics that we have sketched: what is optimal control-oriented experiment design? what is a control-oriented nominal model? why iterative design? what is a control-oriented uncertainty set? how can we match experiment design and uncertainty set? We have chosen to do the presentation of these ideas in a Prediction Error (PE) identification framework, because PE identification is by far the most successful and widely used identification method. In the next section, we first present the bare essentials of PE identification that are necessary to understand the remaining issues.

2. THE BARE ESSENTIALS OF PREDICTION ERROR IDENTIFICATION

The purpose of this paper is to present concepts rather than technicalities. Thus, to simplify the presentation, we assume that the unknown true system can be represented by a single-input single-output linear time-invariant system:

$$\mathcal{S} : y_t = G_0(z)u_t + v_t = G_0(z)u_t + H_0(z)e_t, \quad (1)$$

where $G_0(z)$ is a linear time-invariant causal operator, y is the measured output, u is the control input, and v is noise, assumed to be quasistationary, modelled as the output of a model $v_t = H_0(z)e_t$, where e is white noise. One considers a parametrized model set: $\mathcal{M} = \{(G(z, \theta), H(z, \theta), \theta \in D_\theta \subset \mathbb{R}^d)\}$ where $G(z, \theta)$ and $H(z, \theta)$ are typically rational transfer functions, and D_θ is a subset of admissible values for the parameter vector θ . To every θ corresponds a one-step ahead predictor:

$$\hat{y}_{t|t-1}(\theta) = H^{-1}(z, \theta)G(z, \theta)u_t + [1 - H^{-1}(z, \theta)]y_t, \quad (2)$$

and hence a one-step ahead prediction error:

$$\begin{aligned} \varepsilon_t(\theta) &\triangleq y_t - \hat{y}_{t|t-1}(\theta) \\ &= H^{-1}(z, \theta)[(G_0(z) - G(z, \theta))u_t + v_t] \end{aligned} \quad (3)$$

These prediction errors can, possibly, be filtered by a data filter $D(z)$, thus defining the filtered prediction errors $\varepsilon_t^f(\theta) = D(z)\varepsilon_t(\theta)$. The least squares PE estimate $\hat{\theta}_N$ based on N input-output data is then defined as

$$\hat{\theta}_N = \arg \min_{\theta \in D_\theta} V_N(\theta), \quad (4)$$

where the PE criterion is defined as

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^N [\varepsilon_t^f(\theta)]^2. \quad (5)$$

The estimate $\hat{\theta}_N$ defines the model $\hat{G} = G(z, \hat{\theta}_N)$, $\hat{H} = H(z, \hat{\theta}_N)$. Under reasonable conditions, $\hat{\theta}_N \xrightarrow{N \rightarrow \infty} \theta^*$, where $\theta^* \triangleq \arg \min_{\theta \in D_\theta} \bar{V}(\theta)$, with $\bar{V}(\theta) \triangleq E[\varepsilon_t^f(\theta)]^2$ (Ljung, 1999).

Two different situations need to be considered. The first one is when the model structure \mathcal{M} has been chosen sufficiently complex that the true system belongs to the model set. This is denoted $\mathcal{S} \in \mathcal{M}$, and means that there exists a value $\theta_0 \in D_\theta$ such that $G(z, \theta_0) = G_0(z)$ and $H(z, \theta_0) = H_0(z)$. In such case, under reasonable conditions, $\theta^* = \theta_0$, which means that the PE estimates of the transfer functions converge to the true transfer functions: $G(z, \hat{\theta}_N) \xrightarrow{N \rightarrow \infty} G_0(z)$, $H(z, \hat{\theta}_N) \xrightarrow{N \rightarrow \infty} H_0(z)$. When $\mathcal{S} \in \mathcal{M}$, the

parameter error converges to a Gaussian random variable:

$$\sqrt{N}(\hat{\theta}_N - \theta_0) \xrightarrow{N \rightarrow \infty} N(0, P_\theta). \quad (6)$$

The asymptotic parameter covariance P_θ can be estimated from the data, and the true parameter vector θ_0 belongs to an ellipsoid:

$$U_\theta = \{\theta | N(\theta - \hat{\theta}_N)^T P_\theta^{-1} (\theta - \hat{\theta}_N) < \chi^2\} \quad (7)$$

with probability $\alpha(d, \chi^2) = Pr(\chi^2(d) \leq \chi^2)$, where $\chi^2(d)$ denotes the χ^2 distribution with d degrees of freedom. Thus, when the system is in the model set, PE identification delivers a nominal model $G(z, \hat{\theta}_N), H(z, \hat{\theta}_N)$, together with an ellipsoidal confidence region in parameter space. This, in turn, defines an uncertainty region in the space of transfer functions:

$$\mathcal{D} = \{G(z, \theta) | \theta \in U_\theta\}. \quad (8)$$

In the more general situation where the system is not in the model set, $\lim_{N \rightarrow \infty} \hat{\theta}_N = \theta^* \neq \theta_0$. In such case, the transfer function error, $G_0(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N)$, at a given frequency ω , can be decomposed as:

$$G_0(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N) = \underbrace{G_0(e^{j\omega}) - G(e^{j\omega}, \theta^*)}_{\text{bias error}} + \underbrace{G(e^{j\omega}, \theta^*) - G(e^{j\omega}, \hat{\theta}_N)}_{\text{variance error}}. \quad (9)$$

The bias error arises when the model structure is unable to represent the true system. The variance error is caused by the noise and the finiteness of the data set; by definition of θ^* , it goes to zero asymptotically. In the mid-eighties, Ljung produced some important formulas for the characterization of bias and variance errors of identified transfer functions (Ljung, 1985; Wahlberg and Ljung, 1986). The bias was characterized implicitly by representing θ^* as the minimizing argument of a frequency integral. A variance error estimate for the estimated transfer functions was obtained under an assumption of model order going to infinity. More recent work has produced formulas for the estimation of an uncertainty set \mathcal{D} around \hat{G} , with the property that $G_0 \in \mathcal{D}$ with probability α , where α is any desired level close to 1 (e.g. $\alpha = 0.95$) even in the case where the system is not in the model set, and for finite model orders: see Section 6. Finally, we note that the results described in this section are valid for both open loop and closed loop identification.

3. THE GAME AND THE PLAYERS

In identification for control, a typical situation is that we can perform experiments on the true system (1) with the purpose of designing a feedback controller. The system may already be under

feedback control, in which case the task is to replace the present controller by one that achieves better performance. This situation is representative of many practical industrial situations. We then denote the present controller by C_{id} and the reference signal, if any, by r_t :

$$u_t = C_{id}(z)[r_t - y_t]. \quad (10)$$

Using N data collected on the system, in open loop or in closed loop, we can compute a model \hat{G} of the unknown G_0 , and possibly also a noise model \hat{H} of H_0 by PE identification. Since the complexity of a model-based controller is of the same order as that of the model, one often performs the identification with a low order model.

The traditional scenario in model-based robust control design was: *First estimate a model \hat{G} and an uncertainty set \mathcal{D} , then design a new controller $C(z)$ that achieves closed-loop stability and meets the required performance with all models in \mathcal{D} , and hence with the unknown true system G_0 .* For this scenario to be successful, a very accurate model \hat{G} was typically required.

The objective in *identification for control* is to replace that traditional scenario by the following. *On the basis of the required performance, and of any knowledge of the unknown system, design a control-oriented identification experiment that produces a model \hat{G} and an uncertainty set \mathcal{D} ; then design a new controller C that achieves closed-loop stability and meets the required performance with all models in \mathcal{D} , and hence with the unknown true system G_0 .* If necessary, repeat this design procedure, possibly with a more demanding performance criterion. In some scenarios, one first computes a class $\mathcal{C}(\hat{G}, \mathcal{D})$ of controllers, each of which achieves the required performance with all models in \mathcal{D} ; the controller C is then chosen within this class in such a way as to have some additional nice features (e.g. low complexity).

The goal of the new scenario is to achieve the same or better performance based on models (and hence controllers) of lower complexity. The class \mathcal{C} of controllers that achieve the required performance is larger if the model uncertainty set \mathcal{D} can be tuned towards that aim.

The players within this (iterative) identification and robust control design scenario are therefore:

- the unknown plant G_0
- the optimal controller C^{opt} for G_0
- the present controller C_{id} (if any)
- the present model \hat{G}_{init} (if any)
- the identified model \hat{G}
- the uncertainty set of models \mathcal{D} around \hat{G}
- the set \mathcal{C} of controllers that achieve the prescribed performance
- the new model-based controller $C \in \mathcal{C}$

Except for the unknown plant G_0 and its corresponding optimal controller C_0 , the designer has a handle on all other players. It is the complexity of the interplay between these players that makes the problem challenging and interesting. To illustrate the interplay, it is important to understand that one deals with five different feedback loops, which impact on one another: see Figures 1 to 5.

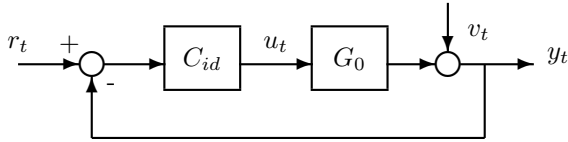


Fig. 1 : Experimental loop

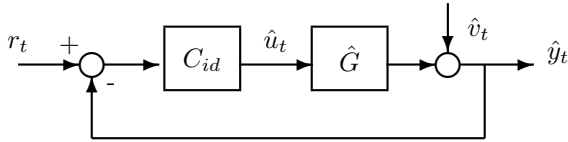


Fig. 2 : Identified system

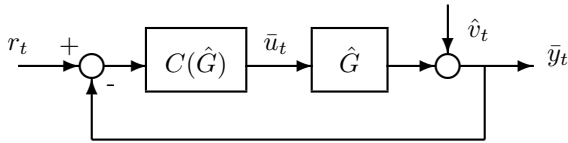


Fig. 3 : Design loop

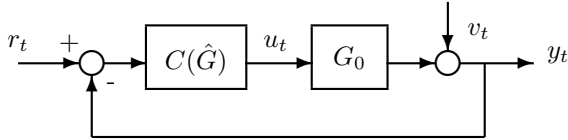


Fig. 4 : Achieved loop

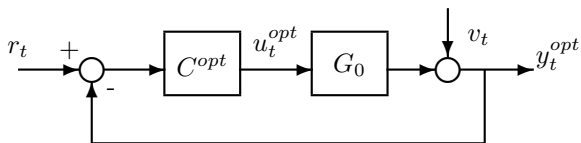


Fig. 5 : Optimal loop

In identification for control, the designer collects data on the experimental loop of Figure 1, and estimates a model \hat{G} such that the closed loop system of Figure 2 is “as close as possible” to the actual system of Figure 1. Sometimes, one of his design choices is the choice of a controller C_{id} and of a reference signal r_t in the experimental loop. On the basis of the identified model \hat{G} , possibly with an estimated uncertainty set \mathcal{D} , the designer then computes the new controller $C(\hat{G})$, or $C(\hat{G}, \mathcal{D})$: this generates the designed loop of Figure 3. However, what really matters is the performance achieved by this controller with the real system, i.e. the performance of the achieved loop of Figure 4. Thus, if the identifier performs a

good job, then the identified loop will be close to the experimental loop; but what is really desired is that the achieved performance on the loop of Figure 4 is close to the designed performance on the loop of Figure 3. If the experimental setup was such that C_{id} was identical (or at least close to) the “to-be-designed controller” C , then closeness of the loops of Figures 1 and 2 would entail closeness of the loops of Figures 3 and 4. This explains why it pays to have experimental conditions that closely match the conditions in which the “to-be-designed controller” will operate.

The first identification for control results, mentioned earlier, were based on a different approach. The aim was to find the optimal experiment conditions such that the output of the achieved loop of Figure 4 is as close as possible to the output of the optimal loop of Figure 5 that would be obtained if the true system were known exactly. Because of its historical precedence, and its intuitive appeal, we first present this approach in the next section.

4. OPTIMAL CONTROL-ORIENTED IDENTIFICATION DESIGN

In this approach an identification experiment design is called “optimal” if the controller computed from the estimated model is one that minimizes the average performance degradation vis-à-vis the performance that would be achieved with the ideal controller. The ideal controller is the controller that would be computed if the true system were known. We now explain this in some more detail.

We denote by $J(G, H, C)$ the control design criterion, and by $C = c(G, H)$ the certainty equivalence mapping that maps a model (G, H) into the corresponding optimal controller. In particular,

$$C^{opt} = c(G_0, H_0) = \arg \min_C J(G_0, H_0, C). \quad (11)$$

We consider PE identification of a parametric model from N data, and assume that $\mathcal{S} \in \mathcal{M}$. The control design mapping then defines a controller $\hat{C}_N = c(\hat{G}_N, \hat{H}_N)$ for each model (\hat{G}_N, \hat{H}_N) .² The controller \hat{C}_N is a random variable, because the estimated parameter vector $\hat{\theta}_N$ is random, and hence also the model. Applying \hat{C}_N (rather than C^{opt}) to the true system results in an achieved cost $J(G_0, H_0, \hat{C}_N) \geq J(G_0, H_0, C^{opt})$. This results in a “performance degradation” $J^{deg} = J(G_0, H_0, \hat{C}_N) - J(G_0, H_0, C^{opt})$, which is again a random variable.

The problem statement of optimal identification design for control is then phrased as follows: “Find the experimental conditions \mathcal{X} that minimize the

² Here \hat{G}_N is a shorthand notation for $G(z, \hat{\theta}_N)$, and similarly for \hat{H}_N .

average performance degradation”³. In view of what precedes, this can be formulated as:

$$\min_{\mathcal{X}} EJ(G_0, H_0, c(\hat{G}_N, \hat{H}_N)). \quad (12)$$

The expected value is taken with respect to the noise, which affects the model estimate, and hence the controller estimate.

In the context of certainty equivalence control design, this is probably the most logical (and ideal) problem formulation for an *optimal identification for control design*. However, there are several difficulties with this formulation (see Chapter 9 in (Albertos and Sala, 2002)), the main one being that the optimal experiment \mathcal{X} defined by (12) necessarily depends on the unknown system (G_0, H_0) . This is specific to all experiment design problems. It does not mean that such results are meaningless: they give useful guidelines for the identification design, and they may lead to iterative schemes that converge to the optimal experiment design: see e.g. (Hjalmarsson *et al.*, 1996).

The first application of this optimal experiment design concept to control-oriented identification was in (Gevers and Ljung, 1986), where an application to Minimum Variance control was treated. The results showed that the optimal experiment consists of performing closed-loop identification with the unknown optimal Minimum Variance controller in the loop. These results were later extended to other control performance criteria in (Hjalmarsson *et al.*, 1996; Forssell and Ljung, 2000).

5. ITERATIVE DESIGN FOR THE NOMINAL MODEL

In this section we discuss the control-oriented identification design of the nominal model, and we show why the pursuit of a control-oriented objective leads to iterative model and controller updates. This observation was made independently in the early nineties by several research teams, who were using different combinations of identification method and control design criterion. To understand the need for iterative design, consider the closed loop systems of Figures 3 and 4, and assume for simplicity that there is no noise ($v_t = 0$ and $\hat{v}_t = 0$), i.e. the control objective is a tracking performance objective.

³ \mathcal{X} denotes the set of all admissible experimental conditions that have an effect on the quality of the model estimates (\hat{G}_N, \hat{H}_N) , such as use of open-loop or closed-loop data, choice of input spectrum distribution, of regulator in the case of closed-loop identification, etc. By ‘admissible’ experimental conditions, we refer to conditions that obey possible constraints on signal powers or signal energies.

The controller is designed on the basis of the model \hat{G} and then applied to the system G_0 . The achieved performance will therefore be close to the desired performance if the two closed loop transfer functions, or a weighted version of these, are close to one another. Thus, we want the following error to be small⁴:

$$\frac{G_0 C}{1 + G_0 C} - \frac{\hat{G} C}{1 + \hat{G} C} = (G_0 - \hat{G}) C S_0 \hat{S} \quad (13)$$

where $S_0 = \frac{1}{1 + G_0 C}$ and $\hat{S} = \frac{1}{1 + \hat{G} C}$. Now, closed loop PE identification with a given model structure $\mathcal{M} = \{G(z, \theta) \mid \theta \in D_\theta\}$, and with a controller C_{id} in the loop, will asymptotically deliver a model $G(z, \theta^*)$, where $\theta^* = \arg \min_{\theta \in D_\theta} V(\theta)$, with $V(\theta)$ given by

$$V(\theta) = \int_{-\pi}^{\pi} |G_0 - \hat{G}(\theta)|^2 |C_{id} S_0|^2 |D|^2 \Phi_r d\omega. \quad (14)$$

Here $D(z)$ is the data filter that can be freely chosen by the user. Observe that, if $C_{id} = C$, and if the data filter is chosen such that $|D(e^{j\omega})|^2 \Phi_r(\omega) = |\hat{S}(e^{j\omega})|^2$, then the model $G(z, \theta^*)$ obtained asymptotically by such closed loop PE identification will make the error (13) small in an H_2 sense. By such design, the identification criterion would be matched to the control performance criterion. However, there are two difficulties with such design: (i) the controller C in the design loop of Figure 3 is a function of the identified model, $C = C(\hat{G})$, and it is therefore impossible to choose $C_{id} = C(\hat{G})$ at the identification design stage; (ii) the sensitivity function \hat{S} is also a function of the estimated model, $\hat{S} = \hat{S}(\hat{G})$. These observations have led to the concept of iterative design, where successive steps of closed loop identification and model-based controller design are performed. Thus, at iteration k , where a model \hat{G}_k has been obtained, select $C_{id,k} = C(\hat{G}_k)$, and $|D_k|^2 \Phi_r = |\hat{S}_k|^2$, where $\hat{S}_k = \frac{1}{1 + \hat{G}_k C(\hat{G}_k)}$.

The research work of the early nineties on the definition and computation of control-oriented nominal models led to several important conclusions, that can be summarized as follows.

- The identification criterion for the nominal model should aim at minimizing the distance between the achieved and the designed loop, where this distance is measured in a norm determined by the control performance criterion. An application of this principle to LQG control can be found in (Zang *et al.*, 1995), to H_∞ control in (Schrama and Bosgra, 1993) and to GPC in (Shook *et al.*, 1992), the latter result being based on dual control ideas.

⁴ For simplicity of notation, we omit all ω -dependent arguments whenever there is no risk of confusion.

- The identification must be performed in closed loop, with a specific data filter.
- The data filter is model-dependent (i.e. θ -dependent). Thus, one has to resort to iterative model/controller updates for the practical implementation of this design.

In summary, in identification for control, the control performance objective shapes the bias error distribution of the nominal model. This means that the nominal model has a bias error that is small in the frequency areas where it needs to be small for the design of a better controller, typically around the present cross-over frequency.

Iterative identification and control schemes flourished in the nineties, with various combinations of control criteria and identification criteria. The reader is referred to (Gevers, 1993; Bitmead, 1993; Van den Hof and Schrama, 1995) for details and for a survey on such iterative schemes. Unfortunately, it was found (Hjalmarsson *et al.*, 1995) that these iterative schemes do not generically converge to the achievable minimum (within the model/controller set) of the control performance cost.

Despite this, the concept of iterative identification and control design was rapidly adopted in process control applications: see e.g. (Partanen and Bitmead, 1995; Schrama and Bosgra, 1993; de Callafon *et al.*, 1993; Holmberg *et al.*, 2000; Cooley and Lee, 2001). One reason is that it is typical in such applications that large numbers of closed loop data are flowing into the control computer, and it then makes sense to use these data to replace the existing controller by one that achieves better performance. The practical impact of iterative model and controller redesign has been assessed in (Landau, 1999), where some interesting observations are made on the distinction between this batch-like mode of operation and the more classical methods of adaptive control.

6. MODEL UNCERTAINTY SETS AND THE ROBUST CONTROL PARADIGM

We have shown how control-oriented nominal models are obtained by minimization of an identification criterion that is determined by the overall control performance criterion. In establishing this result, one has assumed that the controller is computed from the estimated model \hat{G} using the certainty equivalence paradigm. At no point has the model uncertainty due to the noise been taken into account. Conversely, the control design step that has led to the iterative design schemes developed above is not based on robust control principles.

The paradigm of modern⁵ robust control design can be briefly summarized as follows.

One wants to design a controller for an unknown system G_0 . Some prior knowledge about G_0 allows one to assume that G_0 belongs to some *model uncertainty set* \mathcal{D} . Most often, a nominal model \hat{G} of G_0 is available, typically the centre of \mathcal{D} . Some performance objective is often given in the form of a criterion $J(G, C)$, to be minimized. If the true system were perfectly known, the optimal controller would then be defined as $C^{opt} = \arg \min J(G_0, C)$, where the minimization is performed over some predefined set of admissible controllers \mathcal{C} . In the robust control paradigm, the true system is unknown, but the information (\hat{G}, \mathcal{D}) is available. One then seeks a *robust controller* C with the following properties:

- (1) C must stabilize all models in \mathcal{D} ;
- (2) the worst performance achieved by C on any model in \mathcal{D} must be as high as possible.

The robust controller C is often selected as one that stabilizes all models in \mathcal{D} and achieves the best *worst-case performance* over \mathcal{D} , i.e. C is computed as

$$C = \arg \min_{C \in \mathcal{C}} \sup_{G \in \mathcal{D}} J(G, C). \quad (15)$$

The uncertainty model set \mathcal{D} plays a central role in this design strategy. In the robust control theory developed through the eighties and nineties, this set is god-given in that it is based on so-called “prior assumptions” about the model and its uncertainty. Many different descriptions of model uncertainty have been considered; most often they are expressed as frequency domain sets, containing both structured and unstructured components: see e.g. (Zhou *et al.*, 1995; Skogestad and Postlethwaite, 1996; Morari and Zafriou, 1989). The following are representative examples of commonly used uncertainty sets; for simplicity, we consider scalar transfer functions only.

Additive uncertainty set:

$$\mathcal{D}_A = \{G_\Delta(z) \mid G_\Delta(z) = G(z) + \Delta(z), \\ \text{with } |\Delta(e^{j\omega})| < W(e^{j\omega}) \quad \forall \omega\},$$

where $G(z)$ is a “nominal model” and $W(e^{j\omega})$ is a frequency weighting function.

Youla-Kucera uncertainty set:

$$\mathcal{D}_{YK} = \{G_\Delta(z) \mid G_\Delta(z) = \frac{N_x + D_c \Delta}{D_x - N_c \Delta}, \\ \text{with } |\Delta(e^{j\omega})| < W(e^{j\omega}) \quad \forall \omega\},$$

⁵ We refer to modern robust control design as the theory developed in the eighties, that is based on model uncertainty sets; in contrast, the classical robust design theory relies on robustness margins expressed in Bode, Nyquist or Nichols plots.

where $N_x, D_x, N_c, D_c, \Delta$ are stable, rational, proper transfer functions, $\hat{G} = \frac{N_x}{D_x}$ is a “nominal model”, and $C = \frac{N_c}{D_c}$ is any stabilizing controller of G_0 . This uncertainty set, dual of the Youla-Kucera set of all stabilizing controllers of a given system, was introduced for the description of model sets in (Hansen *et al.*, 1989).

The new approach, initiated around 1990, was to consider the estimation of uncertainty sets from data. Unfortunately, the available PE identification theory had rather little to offer to the existing robust control theory, for two reasons: (1) there were no adequate expressions for the estimation of the total error on an identified transfer function, or some upper bound on this error; the main difficulty was the estimation of the bias error, which could only be characterized by implicit integral expressions; (2) the available uncertainty descriptions were not given as frequency domain sets. As a result, a wide range of new identification techniques were developed. New model assumptions, noise assumptions and identification criteria were introduced whose main merit was to deliver computable error bounds on the estimated models. The survey paper (Ninness and Goodwin, 1995) is probably still one of the best presentations of these alternative methods. Some of these methods have been shown to lead to very conservative upper bounds on the uncertainty set. But clearly, this work has led to important advances in the characterization of bounds on transfer function error estimates.

One uncertainty set that was available in PE identification theory was the set (8) defined via the ellipsoidal set (7) in parameter space. For open loop identification, this set can be described as follows.

Prediction Error uncertainty set:

$$\mathcal{D}_{PE} = \{G(z, \theta) \mid G(z, \theta) = \frac{N(z, \theta)}{D(z, \theta)} \\ \text{with } (\theta - \hat{\theta}_N)^T R (\theta - \hat{\theta}_N) < 1\}$$

Here N and D are polynomials parametrized by θ , and R is proportional to the inverse of the estimated covariance matrix of $\hat{\theta}_N$. The difficulty with this PE uncertainty set is that it could be computed easily only in the case where the system is in the model set, and it did not connect with the robust control theory and design tools that were available in the early nineties. Thus, all through the nineties, a lot of research was produced by the identification community to

- (1) extend the use of uncertainty sets in order to also include the bias error; this was achieved by either embedding the bias error in a stochastic framework (Goodwin *et al.*, 1992; Hakvoort and Van den Hof, 1997), or by estimating the bias error through a

validation step that uses a full order model (Ljung, 2000; Gevers *et al.*, 2003).

- (2) develop a robust control stability and performance theory for PE uncertainty sets characterized by ellipsoids in parameter space (Bombois *et al.*, 2001).

Many important advances have thus been made in the characterization of bounds on transfer function error estimates, but this subject will undoubtedly remain an object of intense debate and activity for years to come.

7. TOWARDS CONTROL-ORIENTED UNCERTAINTY SETS?

In the second half of the nineties, one began to seriously study the interplay between model uncertainty sets and robust control objectives, in order to address the question of building *control-oriented uncertainty sets*. The motivation for this is based on the following two observations.

- It follows from the properties that define a robust controller (see the previous section) that satisfaction of these two properties hinges as much on the choice of the controller C as it does on the uncertainty set \mathcal{D} .
- The shape of a data-based uncertainty set depends very much on the experimental conditions under which it is estimated.

Combining these two observations leads to the idea of constructing control-oriented uncertainty sets by proper choice of experimental conditions.

To illustrate the connection between experiment conditions and model-based control properties, we consider a very simple gedankenexample, presented in (Gevers *et al.*, 1998), where we focus attention only on robust stability. Consider a “true system” G_0 described by the following simple ARX model:

$$(1 - 1.4z^{-1} + 0.45z^{-2})y_t = z^{-1}(1 + 0.25z^{-1})u_t + e_t,$$

where e is a unit variance white noise. With a constant gain feedback law $u_t = r_t - Cy_t$, the closed loop system is stable for $C < C_{max} = 2.2$. Consider now that we estimate the parameters of this ARX model by PE identification, using the exact structure. We can thus estimate an ellipsoid in parameter space, to which the true parameters belong with probability 95%, say. We can then compute the proportional output feedback controller with the highest gain, \hat{C}_{max} , that stabilizes all models \hat{G} whose parameters lie in that 95% uncertainty set. We call this controller the “optimal robust controller”. In order to show the effect of the experimental setup of the identification on the optimal robust controller, we have compared, by Monte Carlo simulations, two experimental conditions:

- (1) open loop identification with a unit variance white noise as input signal u ;
- (2) closed loop identification with a controller $u_t = r_t - y_t$ in the loop during data collection, and a white noise reference signal r with variance 18.38; this choice yields the same output variance as in the open loop experiment.

Each of these two experiments was run 100 times, each time with 1000 input-output data. For each run, the 95% confidence ellipsoid was computed in parameter space, and the gain \hat{C}_{max}^i of the i -th run was computed as the largest gain that would stabilize all models in the corresponding model set. On the basis of 100 runs, we then computed the average \hat{C}_{max} and its variance for each of the two experimental conditions. The following results were obtained for open loop (O.L.), respectively closed loop (C.L.), identification:

$$\text{O.L. : } \hat{C}_{max} = 1.36, \sigma_{\hat{C}_{max}}^2 = 0.12$$

$$\text{C.L. : } \hat{C}_{max} = 2.04, \sigma_{\hat{C}_{max}}^2 = 0.02$$

Remember that, for the true system G_0 , we have $C_{max} = 2.2$. This example clearly shows that the experimental conditions used in the feedback experiment are more control-oriented than those in the open loop experiment, since they lead to a much less conservative estimate of the limit gain for the robust controller. This example serves to illustrate the effect of the identification experiment on the set of admissible controllers, via the model uncertainty set. In a more realistic setup, the interplay between the design of the identification experiment, the corresponding model uncertainty set, and the set of admissible controllers, is a lot harder to understand and analyze.

In order to relate the identification design with the properties of a robust controller, one has resorted to splitting up the problem into its two components:

- understanding the effect of experimental conditions on model quality, or more precisely on the properties of the uncertainty set;
- understanding the interplay between the uncertainty set and the properties of the ensuing robust controller.

While the first problem is reasonably easy and has already yielded many results, the second problem is much harder and there is, as of today, no consensus on the best way to address it. We now elaborate on these two subproblems.

In PE identification, there has been a continuing activity aimed at characterizing the bias, variance, and total mean square error on the estimation of parametric transfer functions, dating back to the mid-eighties. The variance formulas of (Ljung, 1985) were based on an assumption of model order going to infinity; more accurate approximations have recently been obtained for

the variance formulas of finite order transfer function estimates (Ninness and Hjalmarsson, 2003). The bias formulas of (Wahlberg and Ljung, 1986) have given way to explicit expressions based on a stochastic embedding of the bias error, or on its estimation based on validation with full order models, as mentioned in the previous section.

All these formulas for the estimation of the bias, the variance or the total mean square error of identified transfer functions explicitly contain the effect of the experimental conditions (e.g. number of data, input spectrum, noise spectrum, feedback configuration, etc) on the error measure. This yields the possibility of optimizing over some relevant experimental design variable in order to minimize a particular measure of this error. In the 1970's, optimal input design for system identification was an active area of research, with different quality measures of the identified model being used for this optimal design (Zarrop, 1979; Goodwin and Payne, 1977). Thus, given a clear view as to what constitutes a *control-oriented uncertainty set*, one could certainly extend these optimal experiment design ideas to the construction of such sets.

So, what constitutes a *control-oriented uncertainty set*? There is no clear consensus yet on a good definition. One possible view is to say that a model uncertainty set \mathcal{D} is "control-oriented" if the corresponding set of admissible controllers, \mathcal{C} , is large. Such track has been pursued in e.g. (Douma *et al.*, 2003) where different uncertainty structures are compared, and in (Gevers *et al.*, 2003) where the worst-case ν -gap has been proposed as a control-oriented measure of size of \mathcal{D} , because it is related to the size of the corresponding set \mathcal{C} of stabilizing controllers. In (Hildebrand and Gevers, 2003) the corresponding optimal input design problem for the minimization of this worst-case ν -gap measure has been solved.

However, one could also define a control-oriented uncertainty set in a very different way as follows. Consider an uncertainty set \mathcal{D} with center \hat{G} containing the true G_0 , and a controller $C = C(\hat{G}, \mathcal{D})$ with nominal stability margin $b(\hat{G}, C)$ and nominal performance $J(\hat{G}, C)$. Then \mathcal{D} could be called control oriented if C stabilizes all models in \mathcal{D} , if the worst case stability margin $\sup_{G \in \mathcal{D}} b(G, C)$ of C with all models in \mathcal{D} is close to the nominal margin $b(\hat{G}, C)$, and if the worst case performance $\sup_{G \in \mathcal{D}} J(G, C)$ is close to the nominal performance $J(\hat{G}, C)$.

Thus, much work remains to be done on the definition of control-oriented uncertainty sets, and on the computation of the corresponding quality measures. This is certainly one area where one can expect a lot of research activity in the coming years.

8. CONCLUSIONS

We have attempted to explain the major issues related to the problem of identification for control, to present the successive solutions that have been brought to the overall problem, and to display the major remaining open problems. Most certainly, the major impact so far in terms of transfer of technology to the industrial world has been the iterative schemes of model and controller updates, which have provided a methodological background for iterative data-based controller performance enhancement. As for the synergy of robust control concepts and identification design concepts, we believe that most challenges are still ahead of us. The main reason is that a full understanding of the interplay between identification design and robust control analysis and synthesis cannot bypass the role of the uncertainty set, and that the analysis of this problem is difficult and involves techniques from several different subdisciplines.

9. REFERENCES

- Albertos, P. and A. Sala (2002). *Editors, Iterative Identification and Control*. Springer Verlag.
- Anderson, B.D.O. and M. Gevers (1998). Fundamental problems in adaptive control. *Perspectives in Control, a Tribute to Ioan Doré Landau* pp. 9–21.
- Anderson, B.D.O., R.R. Bitmead, C.R. Johnson Jr, P.V. Kokotovic, R.L. Kosut, I.M.Y. Mareels, L. Praly and B.D. Riedle (1986). *Stability of Adaptive Systems: Passivity and Averaging Analysis*. MIT Press. USA.
- Åström, K.J. and J. Nilsson (1994). Analysis of a scheme for iterated identification and control. In: *Proc. IFAC Symp. on Identification*. Copenhagen, Denmark. pp. 171–176.
- Åström, K.J. and T. Bohlin (1965). Numerical identification of linear dynamic systems from normal operating records. In: *Proc. IFAC Symposium on Self-Adaptive Systems*. Teddington, UK.
- Bitmead, R.R. (1993). Iterative control design approaches. In: *Proc. 12th IFAC World Congress*. Vol. 9. Sydney, Australia. pp. 381–384.
- Bombois, X., M. Gevers and G. Scorletti (2000). A measure of robust stability for an identified set of parametrized transfer functions. *IEEE Transactions on Automatic Control* **45**(11), 2141–2145.
- Bombois, X., M. Gevers, G. Scorletti and B.D.O. Anderson (2001). Robustness analysis tools for an uncertainty set obtained by prediction error identification. *Automatica* **37**(10), 1629–1636.
- Cooley, Brian L. and Jay H. Lee (2001). Control-relevant experiment design for multivariable systems described by expansions in orthonormal bases. *Automatica* **37**(2), 273–281.
- de Callafon, R.A. and P.M.J. Van den Hof (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems* **3**(1), 77–101.
- de Callafon, R.A., P.M.J. Van den Hof and M. Steinbuch (1993). Control-relevant identification of a compact disc pick-up mechanism. In: *Proc. 1993 IEEE Conf. on Decision and Control*. San Antonio, Texas. pp. 2050–2055.
- de Vries, D.K. and P.M.J. Van den Hof (1995). Quantification of uncertainty in transfer function estimation: a mixed probabilistic - worst-case approach. *Automatica* **31**, 543–558.
- Deistler, M. (2002). System identification and time series analysis: Past, present, and future. In: *‘Stochastic Theory and Control’, Festschrift for Tyrone Duncan*. Springer. Kansas, USA. pp. 97–108.
- Douma, S.G., P.M.J. Van den Hof and O.H. Bosgra (2003). Controller tuning freedom under plant identification uncertainty: double youla beats gap in robust stability. *Automatica* **39**, 325–333.
- Forssell, U. and L. Ljung (2000). Some results on optimal experiment design. *Automatica* **36**, 749–756.
- Gevers, M. (1991). Connecting identification and robust control: A new challenge. In: *Proc. IFAC/IFORS Symposium on Identification and System Parameter Estimation*. Budapest, Hungary. pp. 1–10.
- Gevers, M. (1993). Towards a joint design of identification and control?. In: *Essays on Control: Perspectives in the Theory and its Applications, Birkhauser, New York* (H.L. Trentelman and J.C. Willems, Eds.). Birkhauser. New York. pp. 111–151.
- Gevers, M. and L. Ljung (1986). Optimal experiment designs with respect to the intended model application. *Automatica* **22**, 543–554.
- Gevers, M., B. D.O. Anderson and B. Codrons (1998). Issues in modeling for control. In: *Proc. American Control Conference*. Philadelphia, USA. pp. 1615–1619.
- Gevers, M., X. Bombois, B. Codrons, G. Scorletti and B.D.O. Anderson (2003). Model validation for control and controller validation in a prediction error identification framework - Part I: theory. *Automatica* **39**(3), 403–415.
- Goodwin, G. and R.L. Payne (1977). *Dynamic System Identification: Experiment Design and Data Analysis*. Academic Press. New York.
- Goodwin, G.C., M. Gevers and B. Ninness (1992). Quantifying the error in estimated transfer functions with application to model or-

- der selection. *IEEE Trans. Automatic Control* **37**, 913–928.
- Hakvoort, R.G. and P.M.J. Van den Hof (1997). Identification of probabilistic system uncertainty regions by explicit evaluation of bias and variance errors. *IEEE Trans. Automatic Control* **42**(11), 1516–1528.
- Hansen, F., G. Franklin and R. Kosut (1989). Closed-loop identification via the fractional representation: Experiment design. *Proc. American Control Conference* pp. 1422–1427.
- Hildebrand, R. and M. Gevers (2003). Identification for control: optimal input design with respect to a worst-case ν -gap cost function. *SIAM Journal on Control and Optimization* **41**(5), 1586–1608.
- Hjalmarsson, H. (2003). From experiments to closed-loop control (plenary lecture). In: *CD-ROM of 13th IFAC Symposium on System Identification*. Rotterdam, The Netherlands.
- Hjalmarsson, H., M. Gevers and F. De Bruyne (1996). For model-based control design, closed-loop identification gives better performance. *Automatica* **32**, 1659–1673.
- Hjalmarsson, H., S. Gunnarsson and M. Gevers (1995). Optimality and sub-optimality of iterative identification and control design schemes. In: *Proc. American Control Conference 95*. Vol. 4. Seattle, Washington. pp. 2559–2563.
- Ho, B.L. and R.E. Kalman (1965). Effective construction of linear state-variable models from input-output functions. *Regelungstechnik* **12**, 545–548.
- Holmberg, U., S. Valentinotti and D. Bonvin (2000). An identification-for-control procedure with robust performance. *Control Engineering Practice* **8**, 1107–1117.
- Kalman, R.E. (1960a). Contributions to the theory of optimal control. *Boletín de la Sociedad Matemática Mexicana* **5**, 102–119.
- Kalman, R.E. (1960b). A new approach to linear filtering and prediction problems. *Transactions ASME, Series D, J. Basic Eng.* **82**, 34–45.
- Kosut, R.L. and B.D.O. Anderson (1994). Least-squares parameter set estimation for robust control design. In: *Proc. American Control Conference*. Baltimore, Maryland. pp. 3002–3006.
- Landau, I.D. (1999). From robust control to adaptive control. *Control Engineering Practice* **7**(9), 1113–1124.
- Lee, W.S., B.D.O. Anderson, R.L. Kosut and I.M.Y. Mareels (1993). A new approach to adaptive robust control. *Int. Journal of Adaptive Control and Signal Processing* **7**, 183–211.
- Ljung, L. (1985). Asymptotic variance expressions for identified black-box transfer function models. *IEEE Trans. Automatic Control* **AC-30**, 834–844.
- Ljung, L. (1999). *System Identification: Theory for the User, 2nd Edition*. Prentice-Hall. Englewood Cliffs, NJ.
- Ljung, L. (2000). Model error modeling and control design. In: *CD-Rom Proc. of IFAC Symposium on System Identification, paper WeAM1-3*. Santa Barbara, California.
- Mäkilä, P.M., J.R. Partington and T.K. Gustafsson (1995). Worst-case control-relevant identification. *Automatica* **31**(12), 1799–1819.
- Morari, M. and E. Zafriou (1989). *Robust Process Control*. Prentice-Hall. Englewood Cliffs, NJ.
- Ninness, B. and G.C. Goodwin (1995). Estimation of model quality. *Automatica* **31**(12), 32–74.
- Ninness, B. and H. Hjalmarsson (2003). Variance error quantifications that are exact for finite model order. *submitted for publication*.
- Partanen, A.G. and R.R. Bitmead (1995). The application of an iterative identification and controller design to a sugar cane crushing mill. *Automatica* **31**, 1547–1563.
- Schrama, R.J.P. and O.H. Bosgra (1993). Adaptive performance enhancement by iterative identification and control design. *Int. Journal of Adaptive Control and Signal Processing* **7**(5), 475–487.
- Shook, D.S., C. Mohtadi and S.L. Shah (1992). A control-relevant identification strategy for gpc. *IEEE Trans. Automatic Control* **AC-37**(7), 975–980.
- Skelton, R.E. (1989). Model error concepts in control design. *Int. Journal of Control* **49**(5), 1725–1753.
- Skogestad, S. and I. Postlethwaite (1996). *Multi-variable Feedback Control: Analysis and Design*. J. Wiley. Chichester, U.K.
- Van den Hof, P.M.J. and R.J.P. Schrama (1993). An indirect method for transfer function estimation from closed loop data. *Automatica* **29**, 1523–1527.
- Van den Hof, P.M.J. and R.J.P. Schrama (1995). Identification and control - closed-loop issues. *Automatica* **31**, 1751–1770.
- Wahlberg, B. and L. Ljung (1986). Design variables for bias distribution in transfer function estimation. *IEEE Trans. Automatic Control* **AC-31**, 134–144.
- Zang, Z., R.R. Bitmead and M. Gevers (1995). Iterative weighted least-squares identification and weighted LQG control design. *Automatica* **31**(11), 1577–1594.
- Zarrop, M. (1979). *Optimal experiment design for dynamic system identification*. Lecture Notes in Control and Information Sciences, Vol. 21, Springer Verlag. Berlin, New York.
- Zhou, K., J.C. Doyle and K. Glover (1995). *Robust and Optimal Control*. Prentice Hall, New Jersey.