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CLOSED-LOOP IDENTIFICATION WITH AN UNSTABLE OR NONMINIMUM PHASE CONTROLLER

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Abstract: In many practical cases, the identification of a system is done using data measured on the plant in closed loop with some controller. In this paper, we show that the internal stability of the resulting model, in closed loop with the same controller, cannot always be guaranteed if this controller is unstable and/or nonminimum phase, and that the direct, indirect and joint input-output approaches of closed-loop prediction-error identification present different properties regarding this stability issue. We give some guidelines to avoid the emergence of this problem; these guidelines concern both the experiment design and the identification method.

Keywords: Closed-loop identification, stability, unstable or nonminimum phase controller.

1. INTRODUCTION

In many practical applications, the identification of a system is carried out in closed loop with some (possibly to be replaced) controller. There are several reasons for performing the identification in closed loop: besides possible physical or technical constraints, a major reason is the fact that closed-loop identification generally yields modelling errors that are better tuned for control design (see e.g. Hjalmarsson et al., 1996).

In this paper, we address some identifiability problems, generally ignored by the practitioner, which are related to the presence of unstable poles or nonminimum phase zeros in the controller used during the identification. We show that their presence may lead to a model which is guaranteed to be destabilised by this controller, although the true system is stabilised.

In Section 2, we define the closed-loop setup and the notion of closed-loop stability that we consider in this paper. Section 3 describes the effect of using an unstable or nonminimum phase controller during the identification experiment on the closed-loop stability of the resulting model for various closed-loop identification techniques. Some experiment design guidelines are given in order to avoid this problem. A realistic numerical simulation is given as an illustration in Section 4. Finally, conclusions are drawn in Section 5.

2. CLOSED-LOOP SETUP AND STABILITY

We consider a 'true' plant P_0 in closed loop with some stabilising controller K, as depicted in Fig. 1, where $u(t)$ is the input of the plant, $y(t)$ its output, and $v(t)$ an output disturbance that can be described by $v(t) = H_0(z)e(t)$ where $e(t)$ is a white noise sequence and $H_0(z)$ some stable, minimum phase, monic transfer function. $r_1(t)$ and $r_2(t)$ are two possible sources of exogenous signals (references). We make the assumption that the plant and the controller are linear and time invariant. For the sake of simplifying the notations, we only consider Single-Input-Single-Output transfer functions, although the same derivations can be done in the multivariable case.

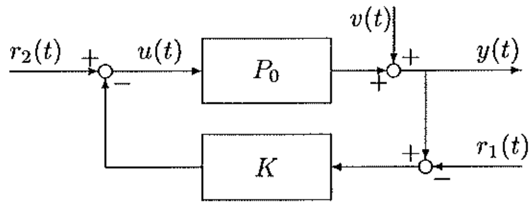

Fig. 1. Closed-loop system

In mainstream robust control, the following generalised transfer matrix is often considered[1]:

$$T(P_0, K) = \begin{bmatrix} \frac{P_0 K}{1+P_0 K} & \frac{P_0}{1+P_0 K} \\ \frac{K}{1+P_0 K} & \frac{1}{1+P_0 K} \end{bmatrix} \triangleq \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}. \quad (1)$$

The entries of $T(P_0, K)$ are the transfer functions between the exogenous reference signals and the input and output signals of the plant:

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = T(P_0, K) \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} + \begin{bmatrix} T_{22} \\ -T_{21} \end{bmatrix} v(t). \quad (2)$$

The closed-loop system of Fig. 1 is said to be *internally stable* if all four entries of $T(P_0, K)$ are stable. A measure of this stability is the *generalised stability margin* defined as

$$b_{P_0,K} \triangleq \begin{cases} \|T(P_0, K)\|_\infty^{-1} & \text{if } (P_0, K) \text{ is stable} \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Note that $0 \leq b_{P_0,K} \leq \sup_{\tilde{K}} b_{P_0,\tilde{K}} < 1$; the upper bound $\sup_{\tilde{K}} b_{P_0,\tilde{K}}$ depends on the system P_0.

3. DESTRUCTION OF THE NOMINAL STABILITY WITH AN UNSTABLE AND/OR NONMINIMUM PHASE CONTROLLER

Let us consider the identification of an unknown plant P_0 in closed loop with some known stabilising controller K as in Fig. 1. We now show how identifiability and nominal stability problems may occur with each of the indirect, joint input-output and direct methods (for details, see Söderström and Stoica, 1989), in case the controller present in the loop during identification is nonminimum phase or unstable. Our analysis will lead to experiment design guidelines for such cases.

3.1 *The indirect approach*

In this approach, a single entry of the generalised closed-loop transfer matrix $T(P_0, K)$ of (1) is identified, and used to derive a model $\hat{P}$ of the plant using knowledge of the controller K. We can thus consider four different cases, depending on which of the four entries is identified. Remember that K is assumed to stabilise P_0, so that all four entries of $T(P_0, K)$ are stable.

Case 1: estimate T_{11}. A stable estimate $\hat{T}_{11}$ of T_{11} in (1) is obtained from measurements of $r_1(t)$ and $y(t)$. A model $\hat{P}$ of the plant P_0 is then reconstructed according to

$$\hat{P} = \frac{\hat{T}_{11}}{K - K\hat{T}_{11}}. \quad (4)$$

The nominal generalised closed-loop transfer matrix can be expressed as a function of $\hat{T}_{11}$:

$$T\left(\hat{P}, K\right) = \begin{bmatrix} \hat{T}_{11} & \frac{\hat{T}_{11}}{K} \\ K(1-\hat{T}_{11}) & 1-\hat{T}_{11} \end{bmatrix}. \quad (5)$$

This shows that the closed loop made of $\hat{P}$ and K will be internally unstable if K has nonminimum phase zeros or unstable poles. Indeed,

- *if K has nonminimum phase zeros*, $\frac{\hat{T}_{11}}{K}$ will be unstable since, in practice, there will never be a perfect matching of these zeros in $\hat{T}_{11}$ due to the bias and/or variance error in $\hat{T}_{11}$. Furthermore, if such a zero is located on the unit circle, i.e. if $K(e^{j\omega_0}) = 0$ at some frequency ω_0, then K will not transmit the component of $r_1(t)$ at that frequency: $\phi_y^{r_1}(\omega_0) = |T_{11}(e^{j\omega_0})|^2 \phi_{r_1}(\omega_0) = 0$. Hence, the output Signal-to-Noise Ratio will be 0 at ω_0 and very low around it, yielding a bad estimate of T_{11} around ω_0 (we call such a zero a *blocking zero* with respect to $r_1(t)$);
- *if K has unstable poles*, $K(1-\hat{T}_{11})$ will be unstable for the same reason of imperfect identification.

Case 2: estimate T_{12}. A stable estimate $\hat{T}_{12}$ of T_{12} in (1) is obtained from measurements of $r_2(t)$ and $y(t)$. $\hat{P}$ is obtained from

$$\hat{P} = \frac{\hat{T}_{12}}{1 - K\hat{T}_{12}}, \quad (6)$$

and the nominal closed-loop transfer matrix is

$$T\left(\hat{P}, K\right) = \begin{bmatrix} \hat{T}_{12}K & \hat{T}_{12} \\ (1-\hat{T}_{12}K)K & 1-\hat{T}_{12}K \end{bmatrix}. \quad (7)$$

Observe that

- the presence of *nonminimum phase zeros in* K does not affect the stability of $T(\hat{P}, K)$;
- *if K has unstable poles*, however, the three reconstructed entries of $T(\hat{P}, K)$ will be unstable. In addition, notice that if $K(e^{j\omega_0}) = \infty$ at some frequency ω_0 (e.g. if K contains an integrator), then $T_{12}(e^{j\omega_0}) = 0$, $\phi_y^{r_2}(\omega_0) = 0$, and the output SNR will be 0 at that frequency and very low around it, yielding once

[1] All transfer functions and matrices must be understood as rational functions of the inverse delay operator z.

again a bad estimate at that frequency. Such a unit-circle pole will be called a *blocking pole* with respect to $r_2(t)$.

Case 3: estimate T_{21}. A stable estimate $\hat{T}_{21}$ of T_{21} in (1) is obtained from measurements of $r_1(t)$ and $u(t)$, and then $\hat{P} = \frac{1}{\hat{T}_{21}} - \frac{1}{K}$. This is the dual situation of Case 2. The stability of $T(\hat{P}, K)$ is not affected by the presence of unstable poles in K while nonminimum phase zeros in K make the three reconstructed entries of $T(\hat{P}, K)$ unstable. In addition, zeros located on the unit circle have a blocking effect on $r_1(t)$.

Case 4: estimate T_{22}. A stable estimate $\hat{T}_{22}$ of T_{22} in (1) is obtained from measurements of $r_2(t)$ and $u(t)$, and then $\hat{P} = \frac{1}{K}(\frac{1}{\hat{T}_{22}} - 1)$. This is the dual situation of Case 1, and $T(\hat{P}, K)$ will be unstable if K is unstable or nonminimum phase.

Conclusion: With the indirect approach, one must

- identify $\hat{P}$ from $\hat{T}_{12}$ using $\{y(t), r_2(t)\}$ data if K *has nonminimum phase zeros;*
- identify $\hat{P}$ from $\hat{T}_{21}$ using $\{u(t), r_1(t)\}$ data if K *has unstable poles.*

All other strategies lead to unstable closed loops for the estimated $T(\hat{P}, K)$. If K is both unstable and nonminimum phase, the nominal closed loop will be unstable whichever entry of $T(P_0, K)$ is identified.

3.2 *The joint input-output approach*

Here the choice is between estimating $\hat{P}$ as

$$\hat{P} = \hat{T}_{11}\hat{T}_{21}^{-1} \tag{8}$$

using stable estimates of T_{11} and T_{21} obtained from $\{y(t), u(t), r_1(t)\}$ data or

$$\hat{P} = \hat{T}_{12}\hat{T}_{22}^{-1} \tag{9}$$

using stable estimates of T_{12} and T_{22} obtained from $\{y(t), u(t), r_2(t)\}$ data.

Case 1: estimate T_{11} and T_{21}. If one replaces P_0 in (1) by $\hat{P}$ computed from (8), one gets:

$$T\left(\hat{P}, K\right) = \begin{bmatrix} \dfrac{\hat{T}_{11}}{\frac{\hat{T}_{21}}{K} + \hat{T}_{11}} & \dfrac{\frac{\hat{T}_{11}}{K}}{\frac{\hat{T}_{21}}{K} + \hat{T}_{11}} \\ \dfrac{\hat{T}_{21}}{\frac{\hat{T}_{21}}{K} + \hat{T}_{11}} & \dfrac{\frac{\hat{T}_{21}}{K}}{\frac{\hat{T}_{21}}{K} + \hat{T}_{11}} \end{bmatrix} \tag{10}$$

$$\neq \begin{bmatrix} \hat{T}_{11} & \dfrac{\hat{T}_{11}}{K} \\ \hat{T}_{21} & \dfrac{\hat{T}_{21}}{K} \end{bmatrix} \tag{11}$$

because $\hat{T}_{11} \neq \frac{\hat{P}K}{1+\hat{P}K}$ and $\hat{T}_{21} \neq \frac{K}{1+\hat{P}K}$ with $\hat{P}$ computed via (8). In view of this rather unpromising observation, it is obvious that, for the identification procedure to be half-way acceptable, there will need to hold

$$\frac{\hat{T}_{21}}{K} + \hat{T}_{11} \approx 1 \tag{12}$$

if $\hat{T}_{11}$ and $\hat{T}_{21}$ are fair estimates of T_{11} and T_{21}.

If K has a strictly nonminimum phase zero, then (12) cannot hold because the left-hand side is unstable. In addition, the $(1, 2)$ and $(2, 2)$ entries of $T(\hat{P}, K)$ in (10) will be unstable. Thus, Case 1 will then deliver an unstable nominal closed-loop system. Unstable poles of K do not pose a problem.

Now, let us assume that K has a zero on the unit circle in $z_0 = e^{j\omega_0}$, say. Then, $T_{11}(z_0) = 0$, $T_{21}(z_0) = 0$, and this zero will make the SNR in $y(t)$ and $u(t)$ equal to 0 at ω_0 and very low around it. As a result, $\hat{T}_{11}$ and $\hat{T}_{21}$ will typically be very bad estimates of T_{11} and T_{21} around ω_0; in particular, $\hat{T}_{11}(z_0) \neq 0$, $\hat{T}_{21}(z_0) \neq 0$, and, instead of (12), it is then required that

$$\left|\frac{\hat{T}_{21}(z_0)}{K(z_0)} + \hat{T}_{11}(z_0)\right| = \infty \tag{13}$$

for $T(\hat{P}, K)$ given by (10) to be close to the true $T(P_0, K)$ around $z = z_0$. This will indeed be the case since $K(z_0) = 0$. Furthermore, (12) should hold at frequencies where the blocking zero in $z = z_0$ does not affect the quality of the estimates.

As a conclusion, *the approach described here will deliver a stable matrix $T(\hat{P}, K)$ only if K has no strictly nonminimum phase zeros.* In this case, condition (12) must hold, except at possible unit-circle zeros of K where (13) will hold. Condition (12) serves as a way of validating the quality of the estimates $\hat{T}_{11}$ and $\hat{T}_{21}$.

Case 2: estimate T_{12} and T_{22}. A model $\hat{P}$ of the plant P_0 is reconstructed from estimates $\hat{T}_{12}$ and $\hat{T}_{22}$ according to (9), and the nominal generalised closed-loop transfer matrix is

$$T\left(\hat{P}, K\right) = \begin{bmatrix} \dfrac{\hat{T}_{12}K}{\hat{T}_{22} + \hat{T}_{12}K} & \dfrac{\hat{T}_{12}}{\hat{T}_{22} + \hat{T}_{12}K} \\ \dfrac{\hat{T}_{22}K}{\hat{T}_{22} + \hat{T}_{12}K} & \dfrac{\hat{T}_{22}}{\hat{T}_{22} + \hat{T}_{12}K} \end{bmatrix}. \tag{14}$$

The same reasoning as in Case 1 leads to the conclusion that $T(\hat{P}, K)$ *will be stable only if K has no strictly unstable poles.* Then, $\hat{T}_{22} + \hat{T}_{12}K \approx 1$ must hold except at possible unit-circle poles of K, where $|\hat{T}_{22} + \hat{T}_{12}K| = \infty$. Nonminimum phase zeros of K do not pose any problem in Case 2.

Conclusion: With the joint input-output approach, one must

- identify $\hat{P}$ from $\hat{P} = \hat{T}_{11}\hat{T}_{21}^{-1}$, if K has *strictly unstable poles and no strictly nonminimum phase zeros*, using $\{y(t), u(t), r_1(t)\}$ data. $T(\hat{P}, K)$ will be stable if $\frac{\hat{T}_{21}}{K} + \hat{T}_{11} \approx 1$ except at possible unit-circle zeros of K;
- identify $\hat{P}$ from $\hat{P} = \hat{T}_{12}\hat{T}_{22}^{-1}$, if K has *strictly nonminimum phase zeros and no strictly unstable poles*, using $\{y(t), u(t), r_2(t)\}$ data. $T(\hat{P}, K)$ will be stable if $\hat{T}_{22} + \hat{T}_{12}K \approx 1$ except at possible unit-circle poles of K.

If K has both poles and zeros outside the unit circle, the joint input-output approach cannot be used to obtain a model $\hat{P}$ stabilised by K, since $T(\hat{P}, K)$ will be unstable whatever the reference signal being used. Possible unit-circle poles and zeros of K do not pose problems. Note that the conditions given here are only necessary, and that it is not possible to guarantee the stability of $T(\hat{P}, K)$ *a priori* for the joint input-output approach, for instance by choosing stable structures for $\hat{T}_{11}$ and $\hat{T}_{21}$, respectively $\hat{T}_{12}$ and $\hat{T}_{22}$.

3.3 *The direct approach*

In this approach, the model $\hat{P}$ of the system is directly identified using measurements of $u(t)$ and $y(t)$. The nominal closed-loop transfer matrix obtained by this approach is

$$T\left(\hat{P}, K\right) = \begin{bmatrix} \frac{\hat{P}K}{1+\hat{P}K} & \frac{\hat{P}}{1+\hat{P}K} \\ \frac{K}{1+\hat{P}K} & \frac{1}{1+\hat{P}K} \end{bmatrix}. \quad (15)$$

Its stability does not hinge on the cancellation of unstable poles or zeros in K by the transfer function that is identified ($\hat{P}$ here), contrary to what happens in the indirect and joint input-output approaches, where this constraint can make these methods *a priori* unusable in some cases. Hence, the direct approach can be used with any external excitation, $r_1(t)$ or $r_2(t)$, even if K has unstable poles and/or nonminimum phase zeros. However, there is no guarantee that the obtained nominal closed-loop model will be stable, and the stability of $T(\hat{P}, K)$ will have to be checked *a posteriori.*

Notice that if $K(e^{j\omega_0}) = 0$ at some frequency ω_0 and if $r_1(t)$ is used ($r_2(t) = 0$), there will hold $\phi_y^{r_1}(\omega_0) = 0$. Hence, the output SNR will be 0 at ω_0, yielding a model $\hat{P}$ that can be very different from P_0 around ω_0, which could result in a nominal closed loop that can be unstable or close to instability. Therefore, it is better to use $r_2(t)$ to excite the system if K has a zero on the unit circle. Conversely, any unit-circle pole of K will have a blocking effect on $r_2(t)$, and it is then better to use $r_1(t)$. Notice that the direct method makes it possible to use both reference signals simultaneously.

3.4 *Guidelines for an optimal experiment design*

The observations made above lead to the following experiment design guidelines, summarised in Table 1.

In order to guarantee the stability of the nominal closed-loop model, and if the indirect or joint input-output approach is used, one must

- excite the system with $r_2(t)$ if K has nonminimum phase zeros;
- excite the system with $r_1(t)$ if K has unstable poles.

The **indirect approach** then guarantees the stability of $T(\hat{P}, K)$ *a priori*, provided the correct cross-diagonal entry of $T(P_0, K)$ is identified and its estimate is stable (which can be guaranteed by the choice of an appropriate model structure). However, this method cannot be used if the controller has both unstable poles and nonminimum phase zeros (even if these poles and/or zeros are on the unit circle).

With the **joint input-output approach**, the stability of the nominal closed-loop system can only be checked *a posteriori* (after computing $T(\hat{P}, K)$). This method cannot be used if the controller has both poles and zeros outside the unit circle (but poles and zeros on the unit circle are allowed).

With the **direct approach**, either of the excitation signals can be used (they can also be used simultaneously). The direct approach is the only method that can be used if the controller has both zeros and poles outside the unit circle. However, the closed-loop stability of the resulting model can also be checked only *a posteriori.*

None of the three classical closed-loop prediction-error identification methods guarantees nominal closed-loop stability if the controller is both unstable and nonminimum phase. This problem can be overcome by means of the Hansen scheme (Hansen, 1989), which is based on the dual Youla parameterisation of all systems that are stabilised by a given controller K. However, we have shown in the full version of this paper (Codrons et al., 1999) that the quality of the estimate does not only rely on the presence of blocking poles or zeros in K, but also on the frequency response of its coprime factors, and that the obtained model may present a small (although guaranteed nonzero) stability margin if some precautions are not taken.

3.5 *Consequences for robust control design*

This instability problem can be a serious drawback when the purpose is to use the model $\hat{P}$ for control design. We have shown in the full version of this

Table 1. Stability of the nominal closed-loop model w.r.t. the identification method, the excitation signal and the singularities of K: stability is guaranteed (+); stability has to be checked *a posteriori* (0); instability is guaranteed (–). When K has several listed singularities, the most unfavourable one outclasses the others.

Singularities of K	Indirect				Joint i/o		Direct ($u \to y$)		
	$r_1 \to y$	$r_2 \to y$	$r_1 \to u$	$r_2 \to u$	$r_1 \to (y, u)$	$r_2 \to (y, u)$	r_1	r_2	$r_1 + r_2$
Strictly unstable poles	–	+	–	–	0	–	0	0	0
Unit-circle poles	–	+	–	–	0	0	0	0	0
Strictly nonmin. phase zeros	–	–	+	–	–	0	0	0	0
Unit-circle zeros	–	–	+	–	0	0	0	0	0

paper (Codrons et al., 1999) that if the nominal model is unstable in closed loop with the controller used during identification, i.e. if $b_{\hat{P},K} = 0$, it may be impossible to use this model to design a new controller for the true system P_0. Indeed, the performance such new controller will achieve with P_0 will generally be very different from the nominal performance it will achieve with $\hat{P}$. It may even be impossible, in practice, to find a controller that stabilises both $\hat{P}$ and P_0.

4. NUMERICAL ILLUSTRATION

4.1 *Problem setting*

We consider as 'true system' the following ARX model:

$$P_0(z) = \frac{0.1028z + 0.1812}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941};$$

$$H_0(z) = \frac{z^4}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941}.$$

It describes a flexible transmission system that was proposed by Landau et al. (1995) as a benchmark for testing various control design methods. The experiments are carried out with a controller that was obtained via an iterative feedback tuning scheme by Hjalmarsson et al. (1995):

$$K(z) = \frac{0.5517z^4 - 1.765z^3 + 2.113z^2 - 1.296z + 0.4457}{z^3(z-1)}.$$

It achieves $b_{P_0,K} = 0.2761$. $K(z)$ has an unstable pole in $z = 1$, i.e. on the unit circle; it will have a blocking effect on $r_2(t)$. $K(z)$ also has a pair of strictly nonminimum phase complex zeros in $z = 1.2622 \pm 0.2011j$, which will pose problems if the indirect or joint input-output approaches are used with $r_1(t)$.

We now test the closed-loop identification methods addressed in Section 3, when the system is excited with $r_2(t)$. The signals $r_2(t)$ and $e(t)$ used during the simulations are taken as mutually independent Gaussian sequences with zero mean and variances 1 and 0.05, respectively [2].

[2] We also made experiments with $r_1(t)$. With this source of excitation, only the direct method delivered a model that was stabilised by K, as expected.

4.2 *The indirect approach*

The system was simulated with $r_2(t)$ and $e(t)$. Using 1000 measurements of $r_2(t)$ and $y(t)$, and an output error model OE[3,8,3] with exact structure for T_{12}, an estimate $\hat{T}_{12}$ was obtained, from which a model $\hat{P}$ of the plant was derived using (6). This model has degree 9. Fig. 2 shows the Bode diagrams of P_0 and $\hat{P}$. As expected, the estimate is very bad at low frequency, i.e. around the unit-circle pole of K in $z = 1$. Furthermore, the three reconstructed entries of the nominal closed-loop transfer matrix given by (7) all have a pole in $z = 1$, meaning that the nominal closed-loop transfer matrix is unstable and $b_{\hat{P},K} = 0$. This is because the zero in $z = 1$ of T_{12} is imperfectly estimated as a zero in $z = 0.9590$ in $\hat{T}_{12}$, which does not cancel the integrator in K.

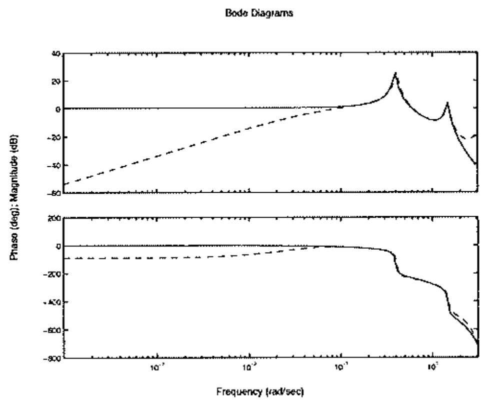

Fig. 2. Indirect approach: P_0 (—), $\hat{P}$ (– –)

4.3 *The joint input-output approach*

The system was simulated with $r_2(t)$ and $e(t)$. $\hat{T}_{12}$ was obtained as in the indirect approach, while an output error model with exact structure (OE[6,8,0]) was used to estimate $\hat{T}_{22}$ from 1000 samples of $r_2(t)$ and $u(t)$. A model $\hat{P}$ for the plant was then reconstructed according to (9). It has degree 13. Fig. 3 shows the Bode diagrams of P_0 and $\hat{P}$, which are close to one another. The nominal generalised closed-loop transfer matrix is given by (14). It is stable and the nominal stability margin is close to the actual one: $b_{\hat{P},K} = 0.2519 \approx$

$b_{P_0,K} = 0.2761$. It can also be shown that $\hat{P}$ is a good model for control design (see Codrons et al., 1999). Fig. 4 shows that $\hat{T}_{22} + \hat{T}_{12}K \approx 1$ except near $z = 1$, i.e. $\omega = 0$, where it goes to infinity. Recall that this is a necessary condition to ensure the stability of the nominal closed-loop transfer matrix (14) when the controller contains an integrator.

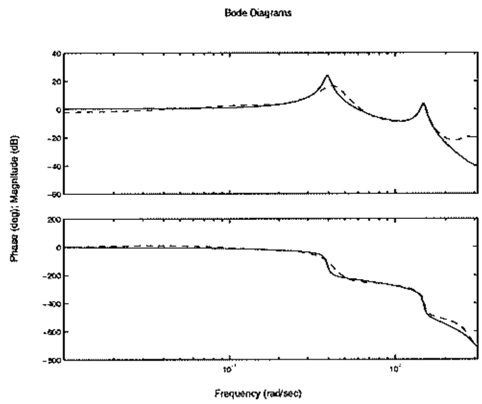

Fig. 3. Joint i/o approach: P_0 (—), $\hat{P}$ (−−)

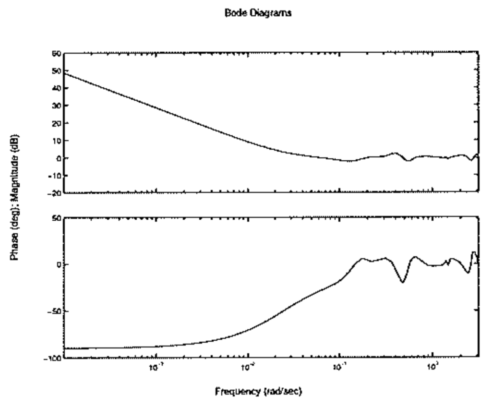

Fig. 4. Joint i/o approach: $\hat{T}_{22} + \hat{T}_{12}K$

4.4 *The direct approach*

The simulation was carried out with $r_2(t)$ and $e(t)$, and 1000 samples of $u(t)$ and $y(t)$ were used to identify a model $\hat{P}$ with the correct structure (ARX[4,2,3]). Because of the blocking effect of the unit-circle pole of K on $r_2(t)$, we could expect a bad estimate of P_0 at low frequency. However, the obtained model is very close to the true system (see Fig. 5) and achieves a nominal stability margin $b_{\hat{P},K} = 0.2725$. We can explain the – perhaps surprisingly – good quality of $\hat{P}$ in the low frequency range by the fact that not only T_{12} but also T_{22} and thus $T_{22}H_0$ have a zero in $z = 1$, i.e. in $\omega = 0$ (see Fig. 5). Hence, the output SNR does not tend to 0 but remains constant as the frequency tends to zero. This would not be true if, for instance, step disturbances were acting on the output of the system. In this case, indeed, H_0 would have a pole in $z = 1$ which would cancel the corresponding zero in T_{22}.

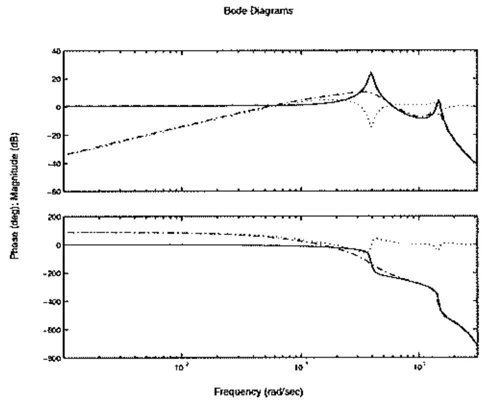

Fig. 5. Direct approach: P_0 (—), $\hat{P}$ (−−), T_{12} (−·) and $T_{22}H_0$ (···)

5. CONCLUSIONS

When an unstable or nonminimum phase controller is used during the closed-loop identification process, the generalised stability margin of the closed-loop model can be zero although the plant is stabilised by the controller. All methods do not have the same properties regarding this stability issue. To avoid the emergence of this problem, experiment design guidelines must be followed. These are summarised in Table 1.

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