Connecting identification and robust control : a new challenge *

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Abstract - Much of the recent work on robust control is based on assumptions about the uncertainties around are at odds with the type of information that is classically available about an identified model. Conversely, unti theory had not focused upon delivering accurate uncertainty bounds around the estimated model. In addition, in not much effort has been devoted to the effect of the controller on the quality of the identified model. In this between identification and control design will be illustrated, and a number of recent results will be presented tha synergy between these two fields.

1 Introduction

The last ten years have seen the emergence of robust control theory as a major research activity. During the same period, research in system identification has dwindled, and it might be tempting to believe that most of the theoretical questions in identification theory have been resolved for some time. The surprising fact is that much of robust control theory is based on prior descriptions of model uncertainty or model errors which classical identification theory has been incapable of delivering. Conversely, until recently identification theorists have not spent much effort in trying to produce the accurate uncertainty bounds around their estimated models that their robust control design colleagues were taking for granted. It is as if, until a few years ago, the control design community and the identification community had not been talking much to each other. The gap between the surrealistic premises on which much of robust control design theory is built and the failure of identification theory to deliver accurate uncertainty bounds in the face of unmodelled dynamics has brought to light major deficiencies in both theories, and a sudden awareness from around 1988 of the need to understand

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better the interactions between both theories.

Surely, a natural place to search for an understanding of the interactions between identification and robust control design is in the adaptive control community. Indeed, adaptive control combines the design of an on-line identifier with that of a control law. After the establishment of the first proofs of global stability and convergence of adaptive controllers in the case of exact knowledge of the model structure in 1978 (see e.g. Goodwin, Ramadge, Caines (1980)), the adaptive control community did indeed turn its attention rapidly to the questions of robust adaptive control, namely the establishment of stability and performance results in cases where the nominal model has a lower complexity than the true system : see e.g. Anderson et al. (1986). However, in their pursuit of an adaptive controller possessing some degree of robustness to unmodelled dynamics, the adaptive control community split into two camps. There were those who focused upon the specification of more and more sophisticated control law design procedures in the hope that a super control law would be able to overcome the vagaries of any identification method. And there were those who, by cajoling and forcing the parameter estimates to conform to certain boundedness conditions *whatever the control law*, were aiming at developing a super identifier able to leap over any reasonable controller in a single bound. An essential feature of adaptive control, however, is that the identification is performed in closed loop and that the controller therefore impacts on the estimated model and on its quality (i.e. its error with respect to the true system). It is therefore to be expected that the separate designs of the identifier and of the controller without regard for the effect of the control law on the identified model, or of the identified model on the robustness of the control law, may not lead to a maximization of the global robustness of the identifier/controller schema. While the theory of the identifiability of closed loop systems dates back to the late seventies and early eighties, the effect of the controller on the convergence points of the model parameter estimates in an adaptive scheme was first analyzed in Lin, Kumar, Seidman (1985), Polderman (1987) and Gunnarson (1988). The first instance in which a robust design procedure is proposed that takes jointly into account the effect of the control design criterion on the identified model and the effect of the identification criterion on the controller robustness is probably to be found in Bitmead, Gevers, Wertz (1990). The central idea of this synergistic design prodecure will be presented later.

In light of the above motivation, this paper will focus on several points of interaction between identification and control design. The first part of the paper will be tutorial; the second part will present some recent results on the estimation of uncertainty bounds around nominal models, and on the interaction between control design and identification design in closed loop identification.

In the course of our tutorial presentation, our arguments will unfold along the following thread.

- First we shall review some fundamentals of classical (i.e. Ljungian) identification

theory (Ljung, 1987, Söderström, Stoica, 1990). In the case where the model structure is of lower complexity than the true system (called restricted complexity modelling), the error on the estimated transfer function can be split into an error due to unmodelled dynamics (often called bias error) and an error due to noise and the finite amount of data (often called variance error). An implicit expression is available for the characterization of the bias error, which allows for qualitative design guidelines.

- We then show the effect of the input signal spectrum on the estimated model, and hence on the model error. We illustrate how a feedback controller that stabilizes an identified restricted complexity model may destabilize an otherwise stable system when the model has been estimated with a poorly chosen input.

- In adaptive control, the input spectrum is influenced by the reference signal and the feedback controller. The potentially destabilizing effect of a poorly chosen input signal mentioned above clearly points to a potential regulator induced mechanism for instability in an adaptive control loop.

- This instability of the closed loop finds its roots in the application of a certainty equivalence control law without due satisfaction of a robustness criterion. In other words, the control law is computed from the estimated model, but is applied to the true plant, while the plant/model error is larger than is authorized by an appropriate robustness criterion. To understand this mechanism, some basic concepts of stability robustness will be reviewed.

- There are various forms of stability robustness criteria, but *in fine* they all constrain some frequency domain expression of the plant/model error to be smaller than some frequency function of the designed (i.e. nominal) closed loop system, which is itself a function of the feedback controller. In order to design this controller it is therefore important to obtain as precise a description as is possible of the error on the estimated model. This leads to the important new subject of the estimation of bounds on model uncertainty (or model error).

- We shall first argue that, contrary to some popularly held beliefs, the estimation of such bounds is trivial in the case where the process is noise-free. In fact, in such case one can estimate not just the error bound but the error itself by first resorting to high order modelling followed by model reduction techniques. We therefore turn to the noisy case.

- Classical (Ljungian) identification theory offers only a qualitative description of the distribution of bias over frequency, while the variance error description, although of a more quantitative nature, is based on a simplifying assumption that is not always satisfied. For the class of linearly parametrized models, we shall

present a new procedure, due to Goodwin, Gevers, Ninness (1990), that allows one to compute explicit upper bounds on the plant/model error.

- In the case of adaptive control, the adaptive controller, through its effect on the shaping of the input signal spectrum, has a direct effect on the plant model estimate, which itself influences the controller at the next step, as explained above. We shall end this paper by explaining how an understanding of the role of the regulator on the identified model can be exploited to design the identification method (and in particular the identifier filters) and the control law in a synergistic way so that the combined robustness is larger than that which would result from the separate designs of these two components of an adaptive controller. Such idea was first developed in Bitmead, Gevers, Wertz (1990).

2 A brief review of classical identification

In this section we introduce our model assumptions and we briefly recall some classical results concerning the bias and variance error of an identified model. For the sake of simplicity, this tutorial presentation will be entirely written for single input single output systems. We shall assume throughout that the true system is linear and is described by

$$y_t = G_T(q)u_t + v_t, \tag{1}$$

where q is the shift operator, $G_T(q)$ is the unknown "true" transfer function of the plant and v_t is additive noise.

We shall use a parametrized model set of the form

$$y_t = G(q,\theta)u_t + H(q)e_t. \tag{2}$$

Here $G(q,\theta)$ is a proper rational transfer function parametrized by some vector $\theta \in \mathbb{R}^d$, e_t is a white noise sequence, while $H(q)$ is some fixed noise model chosen by the user. We note that the noise model is often also identified. Here we shall not do so : our purpose is to keep the problem sufficiently simple while getting some important ideas across. We shall be interested in the identification of $G(q,\theta)$ from input-output data in the practically relevant situation of undermodelling (or restricted complexity modelling).

Undermodelling assumption

The model structure $G(q,\theta)$ is such that there exists no value of the parameter vector θ such that

$$G(z,\theta) = G_T(z) \qquad \forall z \in \mathbb{C}.$$

This is of course a typical situation in practice, where the system is more complex than the model set used for identification and control design. For any given value of θ, the true transfer function can therefore be written as the sum of a nominal model, $G(q,\theta)$, and an error, denoted $G_\Delta(q)$:

$$G_T(q) = G(q,\theta) + G_\Delta(q). \tag{3}$$

The error $G_\Delta(q)$ is not parametrized, but being the difference between the true transfer

function and the nominal model parametrized by θ, it depends on the specific value of θ given to the nominal model. To stress this implicit dependence of $G_\Delta(q)$ on θ, we shall sometimes denote it $G_{\Delta,\theta}(q)$, defined as

$$G_{\Delta,\theta}(q) \triangleq G_T(q) - G(q,\theta). \tag{4}$$

2.1 The prediction error method

From the model set (**??**) it is easy to write the one-step ahead prediction for y_t:

$$\hat{y}_{t|t-1}(\theta) = H^{-1}(q)G(q,\theta)u_t + [1 - H^{-1}(q)]y_t. \tag{5}$$

The one-step ahead prediction error is

$$\begin{aligned} \epsilon_t(\theta) &\triangleq y_t - \hat{y}_{t|t-1}(\theta) \\ &= H^{-1}(q)[(G_T(q) - G(q,\theta))u_t + v_t] \end{aligned} \tag{6}$$

In prediction error identification, the estimation of the parameter vector θ on the basis of N input-output data is obtained by minimizing some function of the prediction errors $\{\epsilon_t(\theta), t = 1, \ldots, N\}$, typically a sum of the squares of these errors. However, for reasons that will become transparent later, it is often desirable to minimize a frequency weighted sum or, equivalently, to filter the errors by some stable filter with transfer function $D(q)$. We denote by $\epsilon_t^f(\theta)$ the filtered errors :

$$\epsilon_t^f(\theta) \triangleq D(q)\epsilon_t(\theta). \tag{7}$$

Least-squares prediction error identification amounts to estimating θ that minimizes

$$V_N(\theta) \triangleq \frac{1}{N}\sum_{t=1}^{N}[\epsilon_t^f(\theta)]^2. \tag{8}$$

The parameter estimate is then defined as

$$\hat{\theta}_N = \arg\min_{\theta \in D_\theta} V_N(\theta) \tag{9}$$

where D_θ is a predefined set of admissible values. Assuming that $\hat{\theta}_N$ has been obtained by some minimization algorithm, it then defines an estimated input-output model $G(q, \hat{\theta}_N)$.

2.2 Bias and variance error

With $\hat{\theta}_N$ having been obtained by minimization of (**??**), we now analyze the error between $G_T(e^{j\omega})$ and the estimated model $G(e^{j\omega}, \hat{\theta}_N)$ obtained from N data. First we recall (see Ljung (1987)) that under reasonable conditions on the data, $\hat{\theta}_N$ converges as $N \to \infty$ to

$$\theta^* = \arg\min_{\theta \in D_\theta} \bar{V}(\theta), \tag{10}$$

where

$$\bar{V}(\theta) = \lim_{N\to\infty} EV_N(\theta). \tag{11}$$

By Parseval's identity :

$$\theta^* = \arg\min_{\theta \in D_\theta} \int_{-\pi}^{\pi} \{|G_T(e^{j\omega}) - G(e^{j\omega},\theta)|^2\Phi_u(\omega) + \Phi_v(\omega)\} \times \frac{|D(e^{j\omega})|^2}{|H(e^{j\omega})|^2} d\omega \tag{12}$$

The relationship (**??**) is very important, because it gives an implicit characterization of the model (or models) $G(e^{j\omega}, \theta^*)$ to which $G(e^{j\omega}, \hat{\theta}_N)$ will converge if the number of data tends to infinity. The expression raises observations that are so important that they deserve specific comments.

Comment 2.1.

a) Notice that in this case of a fixed noise model $H(e^{j\omega})$, the convergence point (or points) of the parameter estimate depends only on the combined frequency weighting $\frac{\Phi_u(\omega)|D(e^{j\omega})|^2}{|H(e^{j\omega})|^2}$. It is independent of the actual noise, but depends on the assumed noise model $H(e^{j\omega})$. Observe also that the identifier filter $D(e^{j\omega})$ allows a complete manipulation of the frequency fit between $G_T(e^{j\omega})$ and $G(e^{j\omega},\theta)$. In particular it can counteract whatever effect the assumed noise model might have.

b) Whereas expression (**??**) is very useful in terms of understanding the role of the filters and spectra on the plant/model error fit, and whereas it allows for a certain manipulation of this fit, it does not offer much help in quantifying the actual asymptotic error between $G_T(e^{j\omega})$ and $G(e^{j\omega},\theta^*)$. A quantification of this error will force us to venture into post-Ljungian[1] identification theory.

With θ^* defined by (**??**), the error between $G_T(e^{j\omega})$ and $G(e^{j\omega},\theta^*)$ will be called *Bias Error* :

$$B(e^{j\omega}) \triangleq G_T(e^{j\omega}) - G(e^{j\omega},\theta^*). \qquad (13)$$

We now consider the total error $G_{\Delta,\hat{\theta}_N}(e^{j\omega})$ between $G_T(e^{j\omega})$ and $G(e^{j\omega},\hat{\theta}_N)$:

$$\begin{aligned} G_{\Delta,\hat{\theta}_N}(e^{j\omega}) &= G_T(e^{j\omega}) - G(e^{j\omega},\theta^*) + G(e^{j\omega}\theta^*) \\ &\quad -G(e^{j\omega},\hat{\theta}_N) \\ &\triangleq B(e^{j\omega}) + N(e^{j\omega},\hat{\theta}_N) \qquad (14) \end{aligned}$$

We have decomposed this total error into the sum of a bias error, $B(e^{j\omega})$, and a noise-induced error, denoted $N(e^{j\omega},\hat{\theta}_N)$. This second term is often called the variance error, with some abuse of language.

Comment 2.2.

The bias error is independent of the number of data (since it is by definition an asymptotic error) and of the noise. The noise error depends very clearly on the number of data (it actually tends to zero, by definition of θ^*, when $N \to \infty$), and it is induced by the noise. If there were no noise, there would be no noise error $N(e^{j\omega},\hat{\theta}_N)$.

It is almost as difficult to quantify the noise term $N(e^{j\omega},\hat{\theta}_N)$ as it is quantify the bias error. In Ljung (1987), the following approximation was obtained for systems of sufficiently high order :

$$\mathrm{cov}N(e^{j\omega},\hat{\theta}_N) \sim \frac{n}{N}\ \frac{\Phi_v(\omega)}{\Phi_u(\omega)} \qquad (15)$$

where $\sim$ indicates proportionality, n the order of the model $G(e^{j\omega},\theta)$, N the number of data, $\Phi_v(\omega)$ the noise spectrum and $\Phi_u(\omega)$ the input spectrum. This expression was obtained under the assumptions that the model order n is large and that $G(e^{j\omega},\theta)$ can be described as a piecewise constant function of frequency, with the constants being independent over different frequency intervals. This assumption is not always realistic. A more accurate, but more complicated expression, was obtained in Goodwin, Gevers, Mayne (1991).

[1] This term refers of course to the theory developed after publication of Ljung (1987); it should in no way cause concern as to Lennart Ljung's good health.

3 Control/identification interactions

The bias and variance formulae of Section 2 show, at least qualitatively, how the input spectrum affects the identified model and hence the model error. In this section we shall illustrate these effects through an example, at the same time highlighting some specific modes of interaction between an identified model and a controller in a closed loop situation, and the possibly noxious consequences of such interactions. In particular, we shall illustrate the following points :

- In the case of undermodelling, different inputs used for identification may lead to very different estimated model transfer functions $G(e^{j\omega}, \hat{\theta}_N)$. This is of course well-known.
- A control design based on a nominal model estimated with a poorly chosen input signal may destabilize the actual plant, even if the plant itself was stable to start with.
- When the identification is performed in closed loop with an external reference input, the frequency distribution of the actual input is determined by the spectrum of the reference input, the actual plant and the controller. If the "identification" - "control design" is performed in iterative steps, as is the case in adaptive control, the succession of controllers may, via their effect on the input spectrum, produce models that in turn yield a destabilizing controller.

Throughout this paper we shall consider the following to be the true sustem :

$$G_T(q) = \frac{0.0364q^{-1}(1 + 1.2q^{-1})}{1 - 1.6q^{-1} + 0.68q^{-2}}. \qquad (16)$$

This second-order "plant" has two stable poles at $z = 0.8 \pm j0.2$, and a non-minimum phase zero at $z = -1.2$. It is a low pass system with a steady-state gain $G_T(1) = 1$. We shall consider a class of first-order models of the form

$$G(q, \theta) = \frac{bq^{-1}}{1 + aq^{-1}}, \qquad (17)$$

where $\theta = (a \quad b)^T$.

3.1 Influence of the input signal on the estimated model

We consider noise-free identification to illustrate clearly the effect of the input on the estimated model. Provided the input signal is sufficiently rich (i.e. contains at least one cosine function for this first order model), the error between $G_T(e^{j\omega})$ and $G(e^{j\omega}, \theta)$ will then be entirely a bias error.

We first consider an input that contains a single frequency : $u_t = \cos(\omega t)$. For three different values of the frequency, $\omega_1 = 0.1$, $\omega_2 = 0.2, \omega_3 = 1$ rad/sec, and using 1000 data points with a sampling frequency of $\omega_e = 2\pi$ rad/sec, an ARX model is identified yielding three different parameter vectors $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$: see Table 3.1.

Frequency (radians/sec)	$\hat{a}$	$\hat{b}$
0.1	-0.8037	0.2197
0.2	-0.8667	0.2304
1	2.8550	-0.3300

Table 3.1.
Estimated parameters with single frequency input

Figure 1 shows the Nyquist plots of the true frequency function $G_T(e^{j\omega})$ (full line), as well as those of the models $G(e^{j\omega}, \hat{\theta}_1)$ (dots), $G(e^{j\omega}, \hat{\theta}_2)$ (plus) and $G(e^{j\omega}, \hat{\theta}_3)$ (dash). The figures clearly show the influence of the input: in this noiseless case and with a single frequency input, the fit between $G_T(e^{j\omega})$ and $G(e^{j\omega}, \hat{\theta})$ is exact at the frequency of the applied input.

Figure 1: Nyquist plot of true system and 3 estimated models

Next we apply an input that contains a mixture of two frequencies applied earlier : $u_t = \cos(0.1t) + 0.2\cos(t)$. Now an exact fit of the model to the plant at each of these frequencies is impossible. An identification under the same conditions as above yields $\hat{\theta}_4 = (-0.8343\ 0.1852)$, and the corresponding model $G(e^{j\omega}, \hat{\theta}_4)$, which strikes a compromise between a fit at $\omega_1 = 0.1$ and at $\omega_2 = 1$ rad/sec.

3.2 Effect of the estimated model on closed loop stability

To keep things simple we shall assume here that, on the basis of the identified model, we design a very simple pole-placement controller. For our first order model this is achieved by proportional output feedback. If we want the nominal closed loop system to have a pole at some $z = d$ inside the unit circle, we apply

$$\begin{aligned} u_t &= k(\hat{\theta})(r_t - y_t) \\ &= -\frac{(d+\hat{a})}{\hat{b}}(r_t - y_t), \end{aligned} \tag{18}$$

where r_t is an external reference signal. With this controller the actual closed loop system becomes :

$$y_t = \frac{0.0364k(\hat{\theta})q^{-1}(1+1.2q^{-1})}{1-(1.6-0.0364k(\hat{\theta}))q^{-1}+(0.68+0.0436k(\hat{\theta}))q^{-2}} \tag{19}$$

$$\triangleq F(q, k(\hat{\theta}))r_t \tag{20}$$

Table 3.2. shows the closed loop poles for some of the models identified above with a deadbeat control strategy, i.e. d=0.

$\hat{\theta}$	Poles of $F(q, k(\hat{\theta}))$
$\hat{\theta}_1$	$0.7334 \pm j0.5492$
$\hat{\theta}_3$	$0.6425 \pm j0.8027$
$\hat{\theta}_4$	$0.7180 \pm j0.6008$

Table 3.2.
Closed loop poles obtained from models identified with three different inputs

This table shows that a controller based on an estimated nominal model may destabilize an open loop stable plant: indeed $F(q, k(\hat{\theta}_3))$ is unstable. This phenomenon may be due to a poorly chosen control design criterion, or to the fact that the model has been fitted to the true system in a poorly chosen frequency range, or to a combination of these two factors. It is precisely the interaction between these two elements (plant/model error, control design) that is the theme of this paper.

3.3 Mechanism for instability in adaptive control

We consider now a situation where the identification and control design are performed in a succession of off-line steps using the same system and model structure as above. With a given input u_t a first order model is identified first in open loop. Using this model and a pole placement criterion, a controller is computed and inserted in a feedback loop around the system. With a reference signal r_t identical to the initial input signal u_t, a new first order model is now identified and, from it, a new controller is computed, which replaces the first one in the feedback loop, etc... For the reference signal we choose the mixture of two frequencies used in Section 3.1 :

$$r_t = \cos(0.1t) + 0.2\cos(t). \qquad (21)$$

The first open loop identification yields $\hat{\theta}_4$: see above. With the same pole placement criterion as above, this yields $k(\hat{\theta}_4) = 4.5058$. Inserting the feedback controller (**??**) with this value of $k(\hat{\theta}_4)$ into the loop and reidentifying in closed loop with the same r_t now yields $\hat{\theta}_5 = (-0.9616\ 0.0374)$. This in turn would produce $k(\hat{\theta}_5) = 25.7320$, which yields a new unstable closed loop system : the poles of $F(q, k(\hat{\theta}_5))$ are $0.3317 \pm j1.3007$.

To understand this instability mechanism, we note that in closed loop the input signal u_t is determined by the reference signal r_t, the actual plan G_T and the controller k. Indeed

$$u_t = [1 + kG_T(q)]^{-1} k r_t. \qquad (22)$$

With r_t as in (**??**), u_t will be a combination of the same two frequencies, but the relative weights of these two frequencies will have been altered by the transfer function $[1 + kG_T(e^{j\omega})]^{-1}$. With $k = k(\hat{\theta}_4)$ in (**??**) we get

$$u_t = 0.8192\cos(0.1t + \varphi_1) + 1.5397\cos(t + \varphi_2) \qquad (23)$$

We note that the ratio of high-versus low-frequency weighting has been increased by 9.4 between r_t and u_t, a significant shift towards higher frequencies, induced by the controller.

This iterative control design/identification scheme, though not truly adaptive since the identification is performed off-line, is representative of the instability mechanism that may occur in an adaptive control loop, where

closed loop stability may be lost due to a frequency shift of the model fit, induced by the control law.

4 A quickie on robust stability

A controller $K(z)$, designed to achieve closed loop stability and some performance specification for an assumed nominal plant model $\hat{G}(z)$, might not achieve closed loop stability and acceptable performance when applied to the true plant $G_T(z)$ if the error between $G_T(z)$ and $\hat{G}(z)$ is "too large" in some sense. The example of the previous section has illustrated this.

The idea of robust stability design is as follows. Assuming that some bound on the error between $G_T(z)$ and $\hat{G}(z)$ is given *a priori*, can we design a controller $K(z)$ that will of course produce closed loop stability and acceptable performance with $\hat{G}(z)$, but also with all possible $G(z)$ that lie within the *a priori* assumed error bound around $\hat{G}(z)$. Conversely, one could ask : is there a control design that will allow for a large plant/model error in the frequency bound where a large error is expected? These questions clearly indicate the interplay between control design and model uncertainty. We shall now formalize this interplay by enunciating some very basic results of stability robustness. As explained before, we shall limit our analysis to the SISO case. A much more thorough analysis can be found in, e.g., Lunze (1989), Morari and Zafiriou (1989) or Maciejowski (1989).

We consider the true plant $G_T(z)$ in a unity feedback loop with a controller $K(z)$: see Figure 2. 8025Fig.4.1.800

Figure 2.
Unity feedback system

In practice the controller has been computed on the basis of a plant model $\hat{G}(z)$; e.g. $\hat{G}(z) = G(z, \hat{\theta})$ in an identification context. There are various ways of characterizing the error between $G_T(z)$ and $\hat{G}(z)$: additive, multiplicative, feedback descriptions. Here we shall consider a multiplicative description of the error :

$$G_T(z) = \hat{G}(z)(1 + L(z)) \tag{24}$$

Simple block-diagram manipulations show that, by replacing G_T by (**??**), the unity feedback system of Figure 2 can be replaced by the feedback configuration of Figure 3 with the following definitions of L and M :

$$L(z) = \frac{G_T(z) - \hat{G}(z)}{\hat{G}(z)}, \; M(z) = \frac{K(z)\hat{G}(z)}{1 + K(z)\hat{G}(z)} \tag{25}$$

4820FIG.4.2.800

Figure 3.
Perturbation feedback system.

We note that the transfer function $M(z)$ is a computable quantity : it depends on the nominal model $\hat{G}(z)$ and on the controller to be designed. The error $L(z)$, on the other hand, is most often unknown. We shall now make the following assumption about $G_T(z)$, $\hat{G}(z)$ and $K(z)$.

Assumption A

The plant $G_T(z)$ and the model $\hat{G}(z)$ have the same number of unstable poles;

The designed closed loop (with $K(z)$ and $\hat{G}(z)$) is stable.

We note that part 1 of Assumption A may be hard to validate, while part 2 is trivial. Under Assumption A, the closed loop will remain stable provided the number of encirclements of -1 in the Nyquist diagram of $K(e^{j\omega})G(e^{j\omega})$ remains unchanged when $\hat{G}(e^{j\omega})$ is replaced by $G_T(e^{j\omega})$. This will be so if

$$1 + K(e^{j\omega})G_T(e^{j\omega}) \neq 0 \quad \forall\omega. \tag{26}$$

Note that

$$1 + KG_T = (1 + LM)(1 + K\hat{G}) \tag{27}$$

Since $1 + K(e^{j\omega})\hat{G}(e^{j\omega}) \neq 0$ by Assumption A, (**??**) is implied by :

$$|L(e^{j\omega})M(e^{j\omega})| < 1 \quad \forall\omega \tag{28}$$

This is in turn implied by

$$|\frac{G_T(e^{j\omega}) - \hat{G}(e^{j\omega})}{\hat{G}(e^{j\omega})}| \quad < \quad |\frac{1 + K(e^{j\omega})\hat{G}(e^{j\omega})}{K(e^{j\omega})\hat{G}(e^{j\omega})}| \ \forall\omega. \tag{29}$$

The left hand side is entirely dependent on the model error : it is actually the relative plant/model error. The right hand side is computable and entirely determined by the nominal model and the controller.

An alternative expression can be obtained at the expense of an additional assumption.

Assumption B

The plant $G_T(z)$ and the model $\hat{G}(z)$ have the same poles on the unit circle.

We note that $1 + LM$ can also be written

$$1 + LM = (\frac{\hat{G} - G}{G} \times \frac{1}{1 + K\hat{G}} + 1)\frac{G}{\hat{G}}. \tag{30}$$

By assumption B, $\frac{G}{\hat{G}}$ is nonzero on the unit circle. Therefore $1 + L(e^{j\omega})M(e^{j\omega}) \neq 0$ is implied by

$$|\frac{G_T(e^{j\omega}) - \hat{G}(e^{j\omega})}{G_T(e^{j\omega})}| \quad < \quad |1+K(e^{j\omega})\hat{G}(e^{j\omega})| \ \forall\omega. \tag{31}$$

Expressions (**??**) and (**??**) provide two alternative bounds on the relative error between plant and model. Note that in (**??**) the absolute error is divided by $\hat{G}$, while in (**??**) it is divided by G_T. We shall call $\frac{G_T - \hat{G}}{\hat{G}}$ the relative plant/model error and $\frac{G_T - \hat{G}}{G_T}$ the relative model/plant error. Various other forms of robust stability criteria can be formulated. The important point to be made here is that they all require that some function of the model error (or relative model error) be bounded by some function of the *designed* feedback controller at each frequency. This clearly shows the identification/control design interplay.

The robust control community has focused on developing design methods for $K(z)$ that would satisfy the stability robustness criterion (**??**), which is often expressed as $\|LM\|_\infty < 1$. This is usually accomplished on the basis that the model error is god-given (i.e. a priori known), a rather unrealistic assumption. This shows the need for a quantification of uncertainty bounds in the case of identified models. We now turn to this question.

5 Uncertainty bounds on estimated models

We first recall the expression of the error between the true system and a model estimated from N data :

$$G_{\Delta,\hat{\theta}_N}(e^{j\omega}) = B(e^{j\omega}) + N(e^{j\omega},\hat{\theta}_N) \qquad (32)$$

where B and N are the bias error and the noise error, respectively :

$$B(e^{j\omega}) \triangleq G_T(e^{j\omega}) - G(e^{j\omega},\theta^*) \qquad (33)$$
$$N(e^{j\omega}) \triangleq G(e^{j\omega},\theta^*) - G(e^{j\omega},\hat{\theta}_N). \qquad (34)$$

We shall not, in this section, discuss the questions of how to shape the identification experiment (e.g. the filter $D(q)$) in such a way as to make the error $G_{\Delta,\hat{\theta}_N}$ small, or rather to manipulate this error into satisfying the stability robustness criterion (**??**). This is a design issue that will be addressed in the next section. Rather, we shall focus here on ways of obtaining reasonable bounds on $|G_{\Delta,\hat{\theta}_N}(e^{j\omega})|$, given that the availability of such error bounds have been shown to be an essential but missing feature of robust control design.

Before we embark on an attempt to produce computable error bounds, and at the risk of being somewhat controversial, we should like to make clear a few points about model error estimation that appear not to be fully understood.

- Firstly, it should be clear that the computation of uncertainty bounds around an estimated model makes sense only in the case of noisy data. When the data are noiseless, the only error is of course the bias error. Various suggestions have been made about ways to estimate this bias error in this case. However, in the noiseless case, one can estimate a very high order model with high accuracy, the only limit being in the number of data and their richness. It is therefore fair to say that the true system can be approximated as accurately as desired by fitting very high order models. If, for practical or other reasons, a low order model is desired for control design, this low order model can be obtained by model reduction or by identification. In such case, the exact model error is known, not just an upper bound. Hence the computation of uncertainty bounds in the noiseless case is not a critical issue.

- Secondly, if a choice of model structure has been made for the restricted complexity model, say a rational transfer function model $G(q,\theta)$ of order n, then the task is to obtain an upper bound on the unmodelled dynamics $G_\Delta(q)$ without actually estimating $G_\Delta(q)$. We believe that methods that parametrize $G_\Delta(q)$ as a new, say $m-th$ order, model parametrized by a new parameter vector, say ξ, amount to "model the unmodelled dynamics", i.e. they amount to estimating a new nominal model of order $n+m$. This is cheating in some sense.

- Thirdly, we want to raise the issue of *hard bounds versus soft bounds*. Clearly the robust stability criteria (**??**), (**??**) require that some form of the error be-

tween plant and model transfer function be strictly bounded above by some other frequency function. These are hard bounds. Such criteria have spurred some members of the robust control community to create or re-create an identification theory that would yield hard bounds on transfer function estimates. Such theory is based on the premise that the noises acting on the system are themselves "hard bounded noises", and leads to set membership descriptions for the parameters or H_∞ transfer function estimation. While certainly not denigrating the interesting theoretical effort that is underway (see e.g. Helmicki, Nett, Jacobson (1991)), we believe that the great "hard-versus-soft bound debate" will ultimately prove to be a non-issue. Clearly there are situations where it is known a priori that some noise is always smaller than some finite bound, but in most cases disturbances or measurement errors may occasionnally be larger than they are on average. To impose a priori bounds on noise sources that will never be exceeded will result in very large error bounds on plant models, and hence exceedingly conservative control designs. Traditional noise descriptions and parameter estimators will, through the central limit theorem, lead to Gaussian parameter estimates and confidence ellipsoids. A reasonable approach would then be to replace the hard bounds on model error in the robustness criteria by confidence interval bounds corresponding to a sufficiently high probability. Whereas this approach does not offer a 100% guarantee of robust stability, it will certainly lead to less conservative control designs.

On the basis of the second remark above it is important to realize the distinction between *estimating the unmodelled dynamics* and *estimating a bound on the unmodelled dynamics*. We believe that the former is just like changing the order of the nominal model, but we shall attempt to do the latter.

Various methods have been proposed for quantifying the uncertainty around an estimated model. One approach is to design several identification experiments with input sequences having different spectral distributions. This approach is typically suggested by the H_∞ identification community. We shall instead consider the situation where a single input-output data sequence is available, containing N data points.

One not so crazy idea would be to filter the available data with different data filters $D_i(q)$, thereby emphasizing different input signal frequency bands. However, one should bear in mind that one does not create new information by this filtering operation. Each one of the identified models will have been identified with a reduced amount of information, since some information has been filtered out. These models will therefore exhibit different bias erors, but they will all have a larger noise error.

It therefore seems unavoidable that, in order to compute uncertainty bounds around estimated models, some additional prior information needs to be injected. Various

strategies have been proposed, all of which imply some prior quantified assumption on the unmodelled dynamics $G_\Delta(e^{j\omega})$ or on the noise v_t. Examples of such assumed prior knowledge are a prior bound on the noise, a prior bound and/or Lipschitz constant on the variation of $G_\Delta(e^{j\omega})$ over frequency, or a known stochastic prior distribution on $G_\Delta(e^{j\omega})$: see e.g. Kosut, Lau, Boyd (1990), or Goodwin, Salgado (1989).

Here we present some recent results of Goodwin, Gevers, Ninness (1990), where a smoothness prior assumption is made on $G_\Delta(e^{j\omega})$ but *without actually imposing the parameters of the smoothness constraint.* Instead, the prior model on the variation of $G_\Delta(e^{j\omega})$ is itself parametrized, and these parameters are estimated from the data.

5.1 A stochastic model for the uncertainty

In Goodwin, Gevers, Ninness (1990), $G_T(q)$ is assumed to be a realization of a random variable whose mean is the nominal model $G(q,\theta)$ for some θ, and whose residual, $G_\Delta(q)$, has zero mean :

$$G_T(q) = G(q,\theta) + G_\Delta(q). \qquad (35)$$

A parametrized prior probability density function (p.d.f.) is then assumed for $G_\Delta(q)$ and it is shown that the parameters of this p.d.f. can be estimated by maximum likelihood. Hence, confidence intervals can be estimated for the unmodelled dynamics. For pedagogical reasons, and to keep things simple, we shall not describe the procedure here in its full generality, but we shall assume a particular parametrized model for the p.d.f. of $G_\Delta(q)$ which has been shown to produce robust results in a large number of simulations.

Assumption on the unmodelled dynamics

$G_\Delta(e^{j\omega})$ is a zero mean Gaussion process with covariance function :

$$E\{G_\Delta(e^{j\omega_1})G_\Delta(e^{j\omega_2})\} = \frac{\alpha e^{j\omega}}{e^{j\omega}-\lambda}; \omega \triangleq \omega_1 - \omega_2. \qquad (36)$$

We note that this stochastic assumption is just one of several ways of imposing smoothness on the variation of $G_\Delta(e^{j\omega})$ with ω. From (**??**) we get :

$$E\{|G_\Delta(e^{j\omega_1}) - G_\Delta(e^{j\omega_2})|^2\} \leq \frac{\alpha\lambda(1+\lambda)}{(1-\lambda)^3}\omega^2 \qquad (37)$$

We could of course also have constrained the square variation of $G_\Delta(e^{j\omega})$ to be bounded by a quantity like the right hand side of (**??**), thereby replacing a soft bound by a hard one.

Without loss of generality, $G_\Delta(q)$ can be written as

$$G_\Delta(q) = \sum_1^\infty \eta_k q^{-k}. \qquad (38)$$

The model assumption (**??**) corresponds to the time-domain assumption that the coefficients η_k are independent but non identically distributed random variables drawn from the following zero-mean Gaussian p.d.f. :

$$f\eta_k(\xi) = \frac{1}{\sqrt{2\pi\alpha\lambda^k}} \exp\{-\frac{1}{2}\,\frac{\xi^2}{\alpha\lambda^k}\}, \alpha > 0, \;\; 0 < \lambda < 1. \qquad (39)$$

Assumption on the nominal model

The nominal model is a finite linear combination of known rational basis functions :

$$G(q,\theta) = T(q)\theta \quad ,\theta \in \mathbb{R}^d, \tag{40}$$

with

$$T(q) = [T_1(q), \ldots, T_d(q)]. \tag{41}$$

Although our present theoretical developments are limited to these linear-in-the-parameter models, we note that this class does contain Finite Impulse Response (FIR) models, Laguerre rational function models, or first order Taylor series expansions of rational transfer functions around an assumed prior $\bar{\theta}$.

Assumption on the noise v_t

*The additive noise v_t in (**??**) is assumed to be a zero mean independent identically distributed Gaussian process with unknown variance σ^2; it is independent of $G_\Delta(e^{j\omega})$.*

The adoption of a stochastic prior model for $G_\Delta(e^{j\omega})$ and v_t will lead to a quantification of the uncertainty around the estimated model $G(e^{j\omega},\hat{\theta})$ expressed in terms of the variance of $G_T(e^{j\omega}) - G(e^{j\omega},\hat{\theta})$. This variance will be a function of α, λ and σ^2. A main new contribution of Goodwin et al. (1990) is to show how these parameters can be estimated. Replacing α, λ, γ by their estimates will then produce estimates of $E\{|G_T(e^{j\omega}) - G(e^{j\omega},\hat{\theta})|^2\}$, which are the desired uncertainty bounds.

We shall not go here into the details of the estimation of θ (by Least Squares) and of α, λ and σ^2 (by Maximum Likelihood, after a projection to account for the estimation of $\hat{\theta}$). We refer to Goodwin et al. (1990). We shall instead focus on the expression of the mean square error of $G_T(e^{j\omega}) - G(e^{j\omega},\hat{\theta})$ and on its interpretation.

5.2 Computation of uncertainty bounds around $G(e^{j\omega},\hat{\theta}_N)$.

We assume that $G_\Delta(q)$ of (**??**) can be approximated sufficiently closely by a long FIR model :

$$G_\Delta(q) = \sum_1^L \eta_k q^{-k} = S(q)\eta \tag{42}$$

with

$$S(q) \triangleq [q^{-1}, \ldots, q^{-L}], \quad \eta \triangleq [\eta_1, \ldots, \eta_L]^T. \tag{43}$$

If $\hat{\theta}_N$ is the Least Squares estimate of θ obtained from N data, then it can be shown that

$$\theta - \hat{\theta}_N = R\eta + MV, \tag{44}$$

where $R \in \mathbb{R}^{d\times L}$ and $M \in \mathbb{R}^{d\times N}$ are known functions of the input signals and $V \triangleq (v_1, \ldots, v_N)^T$ is the unknown vector of noise signals. The model error is then, using (**??**), (**??**) and (**??**) :

$$\begin{aligned} G_{\Delta,\hat{\theta}}(e^{j\omega}) &\triangleq G_T(e^{j\omega}) - G(e^{j\omega},\hat{\theta}_N) \\ &= T(e^{j\omega})(\theta - \hat{\theta}_N) + S(e^{j\omega})\eta \\ &= [T(e^{j\omega})R + S(e^{j\omega})]\eta + T(e^{j\omega})MV \end{aligned} \tag{45}$$

In this expression, T and S are known functions of ω, R and M are known functions of the input u_t, while η and V are independent random vectors whose second order moments

are known functions of the unknown parameters α, λ and σ^2. We can therefore compute the total Mean Square Modelling Error (MSME) as a function of α, λ and σ^2 :

$$\begin{aligned} MSME(\omega) &\triangleq E\{|G_T(e^{j\omega}) - G(e^{j\omega}, \hat{\theta}_N)|^2\} \\ &= \alpha[T(e^{j\omega})R + S(e^{j\omega})]\Lambda(\lambda)[T(e^{j\omega})R + S(e^{j\omega})]^* \\ &\quad + \sigma^2 T(e^{j\omega})MM^T T^*(e^{j\omega}), \end{aligned} \tag{46}$$

where

$$\Lambda(\lambda) \triangleq \mathrm{diag}(\lambda, \lambda^2, \ldots, \lambda^L), \tag{47}$$

and where $*$ denotes conjugate transpose.

Expression (**??**) gives the total Mean Square Modelling Error between the true system and the nominal $G(e^{j\omega}, \hat{\theta}_N)$ where $\hat{\theta}_N$ is estimated by Least Squares. The first term accounts for the actual bias error : $\alpha S \Lambda S^*$ is the a priori assumed bias error corresponding to $G_{\Delta,\theta}$, while the term TR is a correction term that accounts for the difference between the a priori parameter vector θ and the a posteriori estimate $\hat{\theta}_N$. The input exerts its influence on the bias error through R and the model structure through $T(e^{j\omega})$. The second term is the error due to the noise v_t. It is roughly proportional to the ratio of the noise variance to the input energy (see Goodwin et al. (1990)); in particular, MM^T is inversely proportional to N, the number of data. Here again, the model structure exerts its influence through $T(e^{j\omega})$.

It is important to note that,even though η appears in the expression (**??**) of the model error, neither η nor $G_\Delta(q)$ are estimated or used in the expression of the MSME. Only the parameters α and λ of the statistics of the unmodelled dynamics appear in (**??**).

By replacing α, λ and σ^2 by their Maximum Likelihood estimates, (**??**) provides a computable estimate of the total $MSME$ on $G(e^{j\omega}, \hat{\theta})$ for each frequency. This then provides a tool for the estimation of the optimal order with finite data: see Goodwin et al. (1990). More importantly, as regards model uncertainty, expression (**??**) allows one to compute bounds on the amplitude of the error in the model estimate. Alternatively, from (**??**) one can compute the 2×2 covariance matrix of

$$\tilde{g}(e^{j\omega}) \triangleq 1ReG_{\Delta,\hat{\theta}}(e^{j\omega}) \tag{48}$$

$$ImG_{\Delta,\hat{\theta}}(e^{j\omega}). \tag{49}$$

One can then compute a confidence ellipse at each ω around $G(e^{j\omega}, \hat{\theta})$. A typical such plot is shown in Figure 4.

Figure 4 : Nyquist plot of true and estimated model with error bounds

6 Merging identification and control design

We have seen in the previous section how to compute uncertainty bounds around identified models for linearly parametrized models. Even though the method needs to be extended to wider classes of models, at least we have a preliminary grasp on the problem. With this method for model uncertainty estimation under our wings, we now turn to the problem of organizing the identification and control design in a way that the combined robustness is larger than that which would result from the separate designs of the identifier and the controller. The search for such a synergistic design is still in its infancy, and no complete algorithmic procedure is available today. Therefore we shall concentrate on a presentation of ideas and suggestions.

We recall that, under the assumption that the plant and the model have the same number of unstable poles, robust stability will be guaranteed if the true plant $G_T(z)$, the plant model $G(z,\theta)$ and the feedback controller $K(z)$ are such that $|L(e^{j\omega})M(e^{j\omega})| < 1 \quad \forall\omega$. We have seen in Section 4 that this inequality is satisfied if either (**??**) or (**??**) holds. We recall these two sufficient conditions here for reader comfort :

$$|\frac{G_T(e^{j\omega}) - \hat{G}(e^{j\omega})}{\hat{G}(e^{j\omega})}| \quad < \quad |\frac{1 + K(e^{j\omega})\hat{G}(e^{j\omega})}{K(e^{j\omega})\hat{G}(e^{j\omega})}| \ \forall\omega, \tag{50}$$

or :

$$|\frac{G_T(e^{j\omega}) - \hat{G}(e^{j\omega})}{G_T(e^{j\omega})}| \quad < \quad |1+K(e^{j\omega})\hat{G}(e^{j\omega})| \ \forall\omega. \tag{51}$$

An important observation is that in control design based on open loop identification, the left hand side (LHS) of (**??**) and (**??**) is entirely determined by the identification part of the design, while the right hand side (RHS) is a function of the controller and the nominal (i.e. identified) model. However, in closed loop identification (and hence in adaptive control), the controller influences both sides of these inequalities through its effect on the input signal spectrum. These observations are the key to understanding the identifier/controller interplay, which is the focus of our present motherly attention.

Ideally, and assuming that a model set $G(z,\theta)$ and a class of admissible controllers $K(z)$ have been chosen a priori, one would like to globally minimize $|L(e^{j\omega})M(e^{j\omega})|_\infty$ w.r.t. θ and all admissible controllers $K(z)$. Such frontal attack on the problem does not appear to be feasible given the present state of H_∞ technology. An alternative might be to minimize the H_∞ norm of either the

absolute or the relative plant/model error. However, presently available H_∞ identification techniques rely on data in the form of estimates of $G_T(e^{j\omega})$ at a finite number of frequency points, together with hard error bounds around these estimates (see e.g. Parker, Bitmead (1987), and Helmicki, Jacobson, Nett (1991)). More importantly, a straight H_∞ minimization of the LHS of (**??**) or (**??**) would yield a worst case identifier which would not take account of the combined effects of controller and identifier on both sides of (**??**) and (**??**).

Even though a direct H_∞ minimization of $|LM|$ does not appear to be presently feasible, it should serve as a useful guide for the design of more classical H_2 identification and control design methods based on the minimization of a Least Squares criterion involving input-output signals only. Indeed, by carefully cajoling the identifier filter $D(z)$, one may encourage the relative plant/model error $\frac{G_T-\hat{G}}{\hat{G}}$ to be small at those frequencies where the designed closed loop transfer function $\frac{K\hat{G}}{1+K\hat{G}}$ is large. Conversely, by selecting the control design law properly, one may encourage the RHS of (**??**) or (**??**) to be large where the left hand side is large. In this way, one can expect to achieve a better performance/stability robustness compromise than by straight H_∞ (i.e. worst case) design.

6.1 Identification in closed loop

We recall that, with Least Squares identification, the identified model is asymptotically determined by (**??**). Assume now that the identification is performed in closed loop with $u_t = K(z)(r_t - y_t)$, as shown in Figure 2. Then (**??**) is replaced by

$$\begin{aligned}\theta^* &= \arg\min_\theta \int_{-\pi}^{\pi} \{|(G_T - G)\frac{K}{1+G_TK}|^2 \\ &\quad (\Phi_r + \Phi_v) + \Phi_v\}\frac{|D|^2}{|H|^2}d\omega. \qquad (52)\\ &= \arg\min_\theta \int_{-\pi}^{\pi} \{|LM|^2|\frac{1+GK}{1+G_TK}|^2(\Phi_r + \Phi_v) + \Phi_v\} \\ &\quad \times\frac{|D|^2}{|H|^2}d\omega. \qquad (53)\end{aligned}$$

This expression shows how $D(z)$ can be chosen to keep $|LM|^2$ small. Assuming that $\Phi_r \gg \Phi_v$ in the frequency band of the reference signal, and that the fit between estimated model and true plant is such that $|\frac{1+GK}{1+G_TK}| \simeq 1$ in that same frequency band, then $D(z)$ should be chosen so as to flatten the weighting on $|LM|^2$ over the designed closed loop bandwidth. It should fall off thereafter, so as to eliminate the negative effects of high-frequency noise. In particular $D(z)$ should cancel the effect of the noise model, i.e. contain $H(z)$ as a factor.

Assume that $\Phi_r\frac{|D|^2}{|H|^2}$ is a flat spectrum over the passband of the closed loop system, and that $\Phi_v \ll \Phi_r$ in that passband. We notice from (**??**) that an identification performed in closed loop has a natural tendency to keep $|LM|$ small within this passband, a robustness enhancement feature. To obtain the same effect with an open loop identification would require a shaping of the filtered input spectrum so that

$$\frac{\Phi_u|D|^2}{|H|^2} \simeq |\frac{K}{1+KG}|^2 \qquad (54)$$

This confirms a general rule in experiment design that the identification experimental conditions should resemble as much as possible the conditions in which the model will be used. One can proceed further in the analysis by choosing a specific controller design scheme.

6.2 Least Squares identification with LQG/LTR controller

The first instance in which the controller/identifier design was jointly analyzed is probably in Bitmead, Gevers, Wertz (1990). There a Linear Quadratic Gaussian (LQG) controller with Loop Transfer Recovery (LTR), in combination with a Least Squares (LS) identifier, is analyzed. Here we briefly illustrate how the resulting controller, called CRAP, is an example of a synergistic controller/identifier design.

It is shown in Bitmead et al. (1990) that with the CRAP controller design choices, the characterization (**??**) of the model identified in the closed (i.e. adaptive) loop is replaced by:

$$\theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \{|\frac{(G_T - G)}{G_T}|^2 (\Phi_r + \Phi_v) + \Phi_v\} \frac{|D|^2}{|H|^2} d\omega. \quad (55)$$

In the CRAP controller, a LQG/LTR controller structure is chosen a priori, and it so happens (but would you believe it's just a happy coincidence?) that it makes the weighting $\frac{K}{1+G_T K}$ in (**??**) look like $\frac{1}{G_T}$. This means that the identifier filter allows a direct manipulation of the relative model/plant error. Assume now as before that $\Phi_v \ll \Phi_r$ in the passband of the closed loop system. A comparison between (**??**) and (**??**) then suggests the following identifier filter :

$$|D| \simeq |\frac{H}{1 + K\hat{G}}|. \quad (56)$$

This is the design choice suggested and analyzed in Bitmead et al. (1990).

6.3 An H_2 iterative identification and control design

In Bitmead and Zang (1991) the synergistic design is pushed a step further by manipulating both the control design and the identifier design through appropriate frequency weighting filters. A succession of off-line control designs with fixed plant model and identifications designs with fixed controller are performed. This iterative design therefore becomes quasi-adaptive. Assume that the global objective function is to minimize

$$J^* = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} [(y_t - r_t)^2 + \lambda u_t^2], \quad (57)$$

where r_t is the reference signal. J^* depends only on the controller $K(z)$, but since the true plant $G_T(z)$ from which $K(z)$ should be computed is unknown, the idea is to approach the minimization of J^* by iteratively minimizing a succession of local H_2 identification/control criteria.

Consider now that, at some stage of these iterations, a model $G(z, \hat{\theta})$ and a controller

$K(z)$ are considered. Then the *designed* closed loop control signal and output predictions, assuming zero noise, are:

$$\hat{u}_t^c = \frac{K(z)}{1+G(z,\hat{\theta})K(z)} r_t,\ \hat{y}_t^c = \frac{G(z,\hat{\theta})K(z)}{1+G(z,\hat{\theta})K(z)} r_t. \quad (58)$$

Correspondingly, the *designed* closed loop model performance is

$$\hat{J} \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} \{(\hat{y}_t^c - r_t)^2 + \lambda(\hat{u}_t^c)^2\} \quad (59)$$

If $K(z)$ is used in the feedback system, then actual y_t and u_t signals are generated in the loop. By replacing these in (**??**), the *achieved* performance J^* is obtained, with which $\hat{J}$ must be compared.

Assuming that, at some iteration, a plant model $G(z,\hat{\theta})$ is given, then Bitmead and Zang minimize the following local control objective:

$$J_c = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} \{[F(z)(\hat{y}_t^c(\hat{\theta}) - r_t)]^2 + \lambda[F(z)\hat{u}_t^c]^2\} \quad (60)$$

where $F(z)$ is a frequency weighting filter. One can then choose the filter $F(z)$ so that the designed J_c resembles the actual J^* as much as possible. This leads to the following design choice, assuming that Φ_v is small in the support of Φ_r:

$$F(e^{i\omega}) = 1 + \frac{\Phi_{y-\hat{y}}^{1/2}(\omega)}{\Phi_r^{1/2}(\omega)} \quad (61)$$

Note that $\Phi_{y-\hat{y}}$, which can be estimated from the data, gives information on the modelling error. This controller design scheme penalizes the control signal and the tracking error more heavily in frequency bands where the model fit is poor relative to the reference signal energy.

Now consider the identification design, i.e. assume that $K(z)$ is fixed and $G(z,\theta)$ is to be optimized over θ. First note that in classical Least Squares identification, one would minimize some frequency weighted function of $y_t - \hat{y}_t$, where $\hat{y}_t = \frac{G(z,\hat{\theta})K(z)}{1+G_T(z)K(z)} r_t \neq \hat{y}_t^c$. It is important to reflect on this difference between the *actual* closed loop prediction $\hat{y}_t$ and the *designed* closed loop prediction $\hat{y}_t^c$. Bitmead and Zang advocate estimating θ by minimizing the following local identification criterion:

$$J_{id} = \frac{1}{N} \sum_{t=0}^{N} \{(y_t - \hat{y}_t^c)^2 + \lambda(u_t - \hat{u}_t^c)^2\}. \quad (62)$$

They show that minimizing J_{id} over θ for a fixed controller $K(z)$ forces the designed closed loop model performance $\hat{J}$ of (**??**) to approach the actual performance J^*. We stress again that this scenario is one in which the identification is performed under a criterion that, as much as possible, approaches the global optimization criterion.

The interesting point is that minimizing J_{id} is equivalent with minimizing the square of the errors $y_t - \hat{y}_t$ filtered by

$$|D|^2 \simeq |\frac{H}{1+K\hat{G}}|^2 (1+\lambda|K|^2). \quad (63)$$

A remarkable observation is that, for $\lambda = 0$, this prescription coincides with that obtained via different arguments in (**??**) for the same

case of singular optimal (or LQG/LTR) control.

We have thus given two design examples in which the controller structure and the identifier filter are matched to encourage the satisfaction of the robustness criterion $|LM|_\infty < 1$, or one of its variants, by a judicious combination of controller design and identification design.

7 Conclusions

The emergence of a new theory of robust control with its demands for quantified error bounds on estimated models has produced a revival of the development of identification theory. The last two years have witnessed a concerted effort by identification aficionados to produce an identification theory that is more in tune with the requirements of robust control design. In the process, the connections between identification and control are beginning to be understood better.

In this paper we have first illustrated these interactions between control design and identification design. We have then focused upon recent results in two directions. The first aims at providing an explicit quantification of the uncertainty around estimated transfer functions. The second aims at designing the controller and the identifier in a way that they are mutually supportive in terms of an overall stability robustness criterion. In both cases, and particularly the latter, much further work remains to be done. There are exciting years ahead for those who want to contribute to this challenging new area of system identification.

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