AN ITERATIVE FEEDBACK TUNING PROCEDURE FOR LOOP TRANSFER RECOVERY

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Abstract: Iterative Feedback Tuning (IFT) is a data-based method for the tuning of restricted-complexity controllers with a standard $\mathscr{H}_2$ criterion which in general gives no a priori robustness guarantees. In this paper we elaborate on Loop Transfer Recovery (LTR) LQG synthesis techniques designed to achieve robustness of the feedback loop. We propose an IFT procedure that achieves approximate LTR and its associated robustness. The proposed procedure is illustrated with a numerical simulation example.

Keywords: Iterative Feedback Tuning, Robust Control, Loop Transfer Recovery

1. INTRODUCTION

Iterative Feedback Tuning (IFT) is a method for tuning a parameterised controller in a feedback loop when a mathematical description of the plant is not available and the controller must be tuned on the basis of input-output measurements. In IFT the tuning of the controller parameters is performed through an iterative procedure where a sequence of parameter updates is calculated. Each update is calculated on the basis of data obtained from a specific closed-loop experiment where the previously calculated parameter vector is implemented in the controller closing the loop and a particular reference input is injected in the system. The sequence of updates converges to a locally optimal parameter vector according to an $\mathscr{H}_2$ control criterion which is the weighted combination of a tracking and a disturbance rejection term. The essence of the method is an algorithm to construct an estimate of the gradient of this control design criterion as a function of the parameter of the controller on the basis of the input-output data collected during the closed-loop experiment. The tuning of the controller eventually consists in a gradient based descent of the control design criterion on the space of controller parameters.

The IFT method was introduced in (Hjalmarsson *et al.*, 1994); see also (Hjalmarsson *et al.*, 1998). Following the original formulation several variants and improvements of the procedure have been proposed and a number of successful applications have also been reported (Hjalmarsson, 2002). One of the main features of the method is that the term of the design criterion associated to disturbance rejection automatically takes into account the disturbances acting on the plant during the tuning procedure. The asymptotic convergence rate of the IFT method for pure disturbance rejection has been analysed in (Hildebrand *et al.*, 2005*a*) and (Hildebrand *et al.*, 2005*b*).

At the current state of the art, an issue that certainly requires more investigation is the degree of robust stability of a feedback loop tuned through the IFT method. Since IFT minimises an $\mathscr{H}_2$ criterion, it does not carry with it any a priori robustness guarantees. On the other hand, increasing the degree of robust stability can often be the main motivation for a practitioner to re-tune the controller in an existing feedback loop.

An approach based on the IFT methodology which deals with the issue of robust stability has been proposed in (Veres and Hjalmarsson, 2002). This approach considers $\mathscr{H}_\infty$ robust stability concepts. In particular, it is based on the fact that one can obtain a rough estimate of the $\mathscr{H}_\infty$ stability margin, i.e. the maximum singular value of the closed-loop frequency response (Zhou *et al.*, 1996), through the following procedure: (i) feed the system with a white noise input and collect the corresponding output; (ii) estimate the values of the frequency dependent stability margin on a frequency grid in $[0\ \pi]$ as the sampled variances of filtered versions of the collected output obtained through a battery of narrow-band filters centred on the elements of the grid; (iii) take the maximum of the values of the estimated frequency dependent stability margin. The tuning procedure of (Veres and Hjalmarsson, 2002) then consists of estimating the stability margin at each iteration and of taking a descent step on an $\mathscr{H}_2$ criterion with a very narrow band weighting term centred on the frequency corresponding to the maximum estimated singular value. As can be observed, the procedure is computationally heavy and rather ad-hoc. The main reason for this is that it is based on an $\mathscr{H}_\infty$ robust stability criterion, which does not lend itself naturally to an IFT implementation. Indeed, the key feature of IFT, and the reason for its success, is the fact that an $\mathscr{H}_2$ criterion is miminized via the on-line estimation of its gradient with respect to the vector of controller parameters. While the computation of the gradient of an $\mathscr{H}_2$ criterion poses no problem, for an $\mathscr{H}_\infty$ criterion this represents a major computational burden.

A suboptimal but simpler approach to the introduction of robustness into a controller computed by the IFT methodology has recently been proposed in (Prochàzka *et al.*, 2005). There the control performance criterion is modified by the addition of one or more $\mathscr{H}_2$ terms that specifically introduce weighted versions of one or all of the sensitivity functions in the criterion. Each sensitivity function can be weighed independently with a view of obtaining specific robustness features; the choice of these weighting filters is typically performed in a trial and error fashion. Thus, the procedure of (Prochàzka *et al.*, 2005) is inspired by the sensitivity shaping objective of $\mathscr{H}_\infty$ methods, but it pursues this objective using $\mathscr{H}_2$ criteria which lend themselves to an IFT implementation, i.e. to a computation of the gradient of the cost function using IFT-like experiments on the real system.

In this paper, we propose to approach the issue of robust stability within the $\mathscr{H}_2$ framework, which is the natural framework of IFT, but using a robust control approach that does not attempt to approximate or mimic the $\mathscr{H}_\infty$ design. Our approach is inspired instead by the research on LQG control design which considered the covariance and weighting matrices in the formulation of the LQG control problem as design variables, rather than models of the real world, which can be manipulated in order to meet some design requirements. In particular, here we refer to the results of (Maciejowski, 1985) on Loop Transfer Recovery (LTR) for discrete-time LQG design. In (Maciejowski, 1985) it has been shown that for some specific choices of these design variables and under some specific assumptions, the designed control system meets the excellent robustness properties of continuous-time LQ state feedback, i.e. infinite gain margin and at least 60° of phase margin (Maciejowski, 1989). The terminology LTR refers to the fact that this robustness is obtained as a result of the matching of the open-loop return ratio of the designed feedback loop with the open-loop return ratio of the Kalman filter associated to the selected noise model.

The contribution of this paper is an IFT procedure to obtain approximate loop recovery and its associated robustness in a parameterised feedback loop. In parallel with the fact that in LTR LQG synthesis fictitious covariances and weighting terms are used to obtain loop recovery, our procedure is based on the injection of synthetic disturbances in the loop and the formulation of a specific disturbance rejection problem.

The paper is organised as follows. In the next section we recall the results of (Maciejowski, 1985) on LTR. In Section 3 we introduce an IFT procedure for approximate LTR. A numerical example that illustrates the procedure is discussed in Section 4. Section 5 contains conclusions and future objectives.

2. LOOP TRANSFER RECOVERY (LTR) FOR DISCRETE-TIME SYSTEMS

Consider the problem of designing an output feedback controller for a plant P described by the state-space equations

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $x \in \mathbb{R}^n$, $y, u \in \mathbb{R}^m$ ($n \geq m$), and w and v are process and measurement disturbances modelled as white noise signals with covariances W and V respectively. Here we focus on LQG methods where the output feedback controller is obtained as the concatenation of a Kalman filter, for state estimation, and an LQ-optimal state feedback law. The Kalman filter takes the form

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + Bu_k + K_f^p\,(\hat{y}_{k|k-1} - y_k) \tag{3}$$

$$\hat{y}_{k|k-1} = C\hat{x}_{k|k-1} \tag{4}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} - K_f^f\,(\hat{y}_{k|k-1} - y_k) \tag{5}$$

where $\hat{x}_{k|k}$ and $\hat{x}_{k|k-1}$ are the state estimates at time instant k based on data up to time instants k and $k-1$ respectively. The feedback matrix K_f^p and the feedforward matrix K_f^f are obtained from the solution of the appropriate Riccati equation (Åstrom and Wittenmark, 1997). In the sequel we consider the case in which it is actually possible to use $\hat{x}_{k|k}$ to calculate the control action, i.e. the computational time is negligible with respect to the interval between observations of plant variables. The output feedback control is then obtained as

$$u_k = -K_c \hat{x}_{k|k} . \tag{6}$$

where state feedback matrix K_c is also defined by the appropriate Riccati equation after the selection of a pair of cost weights in the LQ control design criterion. We denote by C_f the output feedback controller defined by equations (3–6).

In general the control system described by equations (1–6) does not have good robust stability properties. The lack of guaranteed robust stability properties is a drawback of output feedback LQG design. This drawback arises in the discrete time case considered in this paper as well as in the continuous time case (Doyle, 1978). In order to overcome this limitation, techniques to select the weighting terms in the design procedure have been developed which aim to the recovery of the excellent robustness properties of continuous time LQ full-state feedback. The continuous time case is considered for example in (Doyle and Stein, 1981). Here, we recall some results for the discrete time case.

In (Maciejowski, 1985) the particular case in which the output feedback controller C_f is designed according to the design criterion

$$J = \sum_{k=1}^{\infty} y'_k y_k \tag{7}$$

is considered. Let us recall that for this particular criterion the optimal state-feedback matrix K_c has an explicit expression (Shaked, 1985). It is then shown that, under some suitable assumptions, the loop transfer function of the resulting control system (1-6) recovers the open loop transfer function of the state feedback loop in the corresponding Kalman filter (3-5). Moreover, conditions under which the Kalman filter state feedback loop presents robustness properties comparable to a continuous time LQ state feedback loop are derived.

The results of (Maciejowski, 1985) are summarized below. Here $\Phi(z)$ denotes the open loop transfer function of the Kalman filter. The transfer function $\Phi(z)$ is given by

$$\Phi(z) = C(zI - A)^{-1} K_f^p . \tag{8}$$

and is the transfer function from $(\hat{y}_{k|k-1} - y_k)$ to $\hat{y}_{k|k-1}$ obtained by cutting the internal feedback loop in (3-5). The recovery properties are stated as follows.

Proposition 1 (Maciejowski, 1985)
Let the following assumptions hold: (i) $\det(CB) \neq 0$; (ii) $P(z) = C(zI - A)^{-1}B$ is minimum-phase. Let $C_f(z)$ be the LQG output feedback controller designed according to (7). Then

$$P(z)C_f(z) = \Phi(z) . \tag{9}$$

The following proposition deals with the robustness of the loop determined by $\Phi(z)$.

Proposition 2 (Maciejowski, 1985)
Suppose that, for some $z_0 = e^{j\omega_0}$

$$\bar{\sigma}[\Phi(z_0)] < \varepsilon \tag{10}$$

$$\bar{\sigma}[C(z_0 I - A)^{-1} W^{1/2}] < \varepsilon \tag{11}$$

$$\underline{\sigma}(V) = \bar{\sigma}(V) \tag{12}$$

$$\varepsilon \leq 2\bar{\sigma}(W) \tag{13}$$

where $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ denote, respectively, the maximum and the minimum singular value of a matrix. Then $\varepsilon \ll 1$ implies that $\Phi(z)$ has the stability margins of the continuous-time LQ state feedback loop.

The following comments are in order:

- As pointed out in (Maciejowski, 1985), assumptions (10) and (11) will usually hold with ε small over some high-frequency interval provided that the sampling interval is small enough or, equivalently, provided that the bandwidth of $\Phi(z)$ is low enough.
- The robustness of the continuous-time LQ state feedback loop is characterised by infinite gain margin and at least 60° of phase margin. In the SISO case, on which we shall focus in the next section, the designed loop satisfies the condition

  $$|1 + \Phi(e^{j\omega})| \geq 1 \tag{14}$$

  which means that the Nyquist locus stays outside the circle with centre -1, and radius 1 and implies the stability margins mentioned above.
- Good robustness properties of the designed loop are guaranteed as long as the assumptions of Proposition 2 are met. Some freedom is then left to the designer who can modify W and V, in order to obtain useful feedback action (sensitivity reduction) as well as stability robustness. For example, an increase of W with respect to V results, roughly speaking, in an increase of the bandwidth of the designed loop. The reader is referred to (Maciejowski, 1989) for a discussion of more sophisticated procedures, which may include augmentation of the plant dynamics, to allow the designer to affect also the shape of the designed open-loop return ratio.

3. IFT PROCEDURE FOR APPROXIMATE LOOP RECOVERY

In this section we propose an IFT procedure to achieve approximate LTR in a parameterised feedback loop. The procedure is inspired by the results of Propositions 1 and 2. Here, though, we restrict ourselves to the SISO case.

We make the usual assumption that the transfer function of the plant is not known and that an output feedback controller $C(\rho)$ belonging to a set of parameterised controllers with parameter $\rho \in \mathbb{R}^r$, is connected to the plant. The objective is to tune the controller $C(\rho)$, in order to obtain approximate loop recovery and its associated robustness. In order to do so we adopt the restriction of the LQG criterion (7), over the set of available controller parameters, as a design criterion.

The basic idea, which allows us to obtain (7) as a design criterion, is to consider the experimental set-up depicted in Figure 1. Here $\tilde{w}$ and $\tilde{v}$ are synthetically generated white-noise signals with variances σ_w^2 and σ_v^2 respectively. The desired design criterion is then

$$J(\rho) = \mathbf{Var}[\tilde{y}(\rho)]\,. \tag{15}$$

The above criterion is the restriction of (7) over the set of available controllers for a particular choice of W and V. It can be expected that, providing that the system fulfils the assumptions of Proposition 1 and 2, the controller tuned so as to minimise (15) will achieve approximate loop recovery.

The following comments are in order:

- Since we inject the synthetic process noise $\tilde{w}$ at the input of the plant, the proposed experimental setup corresponds to the choice $W = \sigma_w^2 BB'$ and $V = \sigma_v^2$ in the formulation of the LQG control problem. Here we assume that other disturbances acting on the system are not taken into account in the design criterion. In practice we realise this condition by making the IFT procedure insensitive to the effect of other possible disturbances.
- In the SISO case the transfer function of the plant must have exactly unit delay in order to fulfil assumption (i) in Proposition 1. (This occurs generically when sampling a continuous-time plant.) In addition, we recall that here we consider the case where the controller has no delay.
- The assumptions of Proposition 2 will be fulfilled if the bandwidth of the system is kept low enough (relative to the sampling frequency) as discussed at the end of the previous section. The ratio σ_w^2/σ_v^2 is an approximate tuning knob for the bandwidth. Condition (12) is automatically satisfied since V is in fact a scalar.

The data-based IFT procedure to minimise (15) is given below. Apart from the introduction of the synthetic disturbances the procedure does not take other major departures from the standard IFT procedure of (Hjalmarsson *et al.*, 1998). Here Step 2 is introduced to make the tuning independent from other disturbances acting on the loop whose effect is in fact canceled in the sampled estimate (16) below.

IFT PROCEDURE FOR LOOP RECOVERY

Assume that controller $C(\rho_n)$ is attached to the plant.

1. Generate synthetic white-noise signals $\{\tilde{w}_k\}_{k=1:N}$ and $\{\tilde{v}_k\}_{k=1:N}$ and collect output data $\{\tilde{y}_k(\rho_n)\}_{k=1:N}$ by performing the experiment illustrated in Figure 1.

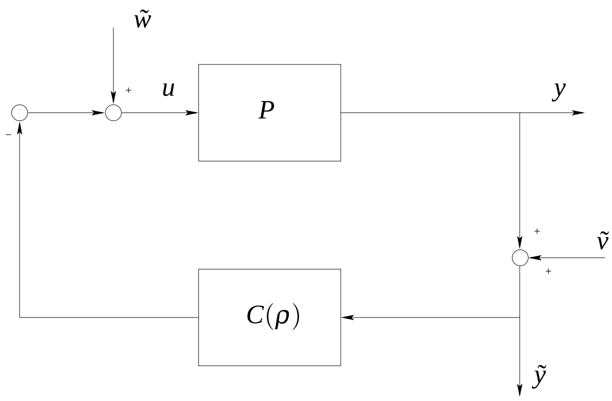

Fig. 1. Experimental setup

2. Repeat the experiment with the same inputs and collect another set of output data $\{\tilde{y}'_k(\rho_n)\}_{k=1:N}$.

3. Collect output data $\{\tilde{y}^2_k(\rho_n)\}_{k=1:N}$ data by performing a third experiment with input $\tilde{w}^2_k = \tilde{y}_k(\rho_n)$.

4. Form the estimate of the gradient of $J(\rho)$ at ρ_n as

$$est_N\left[\frac{\partial J}{\partial \rho}(\rho_n)\right] = \frac{1}{N}\sum_{k=1}^{N} \tilde{y}'_k(\rho_n) est\left[\frac{\partial \tilde{y}_k}{\partial \rho}(\rho_n)\right] \tag{16}$$

$$est\left[\frac{\partial \tilde{y}_k}{\partial \rho}(\rho_n)\right] = \frac{\partial C}{\partial \rho}(q,\rho_n)\,\tilde{y}^2_k(\rho_n) \tag{17}$$

5. Calculate the new parameter vector ρ_{n+1} according to

$$\rho_{n+1} = \rho_n - \gamma_n R_n^{-1}\, est_N\left[\frac{\partial J}{\partial \rho}(\rho_n)\right] \tag{18}$$

where γ_n is a positive step size and R_n is a symmetric positive definite matrix.

6. Update the controller parameter with ρ_{n+1} and loop to step 1.

Under some technical assumptions the sequence ρ_n converges to a local minimiser of (15) (Hildebrand *et al.*, 2005*a*; Hjalmarsson *et al.*, 1998). The reader is referred to (Hildebrand *et al.*, 2005*a*; Hjalmarsson *et al.*, 1998) also for the choice of γ_n and R_n in the algorithm. In the case that the external disturbances are negligible, with respect to the power of inputs $\tilde{w}_k$ and $\tilde{v}_k$, the experiment in Step 2 can be avoided and one can set $\tilde{y}'_k(\rho_n) = \tilde{y}_k(\rho_n)$ in (16).

4. NUMERICAL EXAMPLE

In this section we present a numerical example which illustrates the use of the IFT procedure proposed in the previous section. The design case illustrated here is a fair representative of many different examples which confirmed the effectiveness of the proposed procedure.

The transfer function of the plant is

$$P(z) = \frac{0.259(z-0.3)^3}{(z-0.8)(z-0.5)(z^2-z+0.89)}\,. \tag{19}$$

The Bode plots of $P(z)$ are displayed in Figure 2. The parameterised feedback controller is given by

$$C(z;\rho) = \rho_0 + \rho_1 z^{-1} + \rho_2 z^{-2}\,. \tag{20}$$

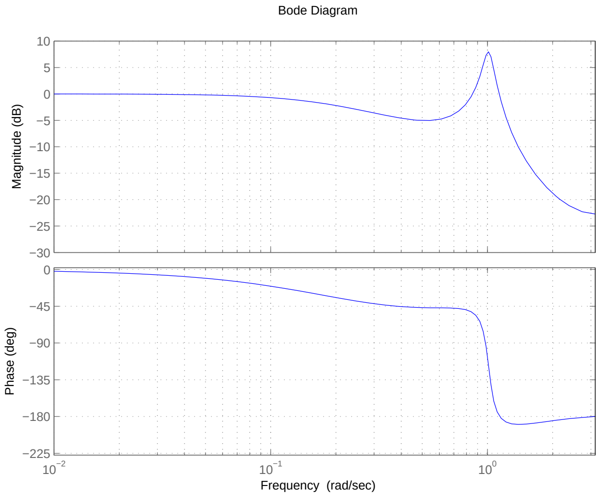

Fig. 2. Bode plots of the plant transfer function

Since the plant is minimum-phase and has unit delay, and the controller has no delay, the conditions of Proposition 1 — which imply recovery of good robustness properties — are fulfilled. Therefore, due to the result of Proposition 2, we can expect good robustness properties at least for a design with a low target bandwidth.

We have tuned the parameter of the feedback controller according to the data-based procedure of Section 3 for different values of the ratio σ_w^2/σ_v^2. In each case σ_v^2 was fixed at $\sigma_v^2 = 1$ while σ_w^2 was given by $\sigma_w^2 = 0.5$, $\sigma_w^2 = 2$ and $\sigma_w^2 = 4$ respectively. For each case, a large number of IFT iterations have been performed, with a small step size, in order to ensure convergence of the algorithm. The figures documented below are plotted with the IFT controllers $C(z;\rho)$ obtained at convergence of the algorithm.

Figure 3 displays the Nyquist diagrams of $P(z)C(z;\rho)$ and of $P(z)C_f(z)$ where $C_f(z)$ is the LQG controller defined by $W = \sigma_w^2 BB'$, $V = \sigma_v^2$ and (7). The Bode plots of the corresponding sensitivites are displayed in Figure 4.

Notice how by increasing the ratio σ_w^2/σ_v^2 we obtained an increase in the bandwidth of the designed closed-loop system — namely an increase in the gain of $P(z)C(z;\rho)$, and a corresponding reduction of the corresponding sensitivity over an increasing range of frequencies. We also obtained good robustness properties in terms of stability margins for all three cases. As was expected, the robustness of the design, which is implied by fulfilment of the assumptions of Proposition 2, degrades slightly as the bandwidth is increased. This can be seen from the slight increase in the peak value of the sensitivity.

5. CONCLUSIONS

We have proposed an IFT procedure that achieves Loop Transfer Recovery and the robustness which is associated with it. The objective of current research is the development of an iterative procedure for con-

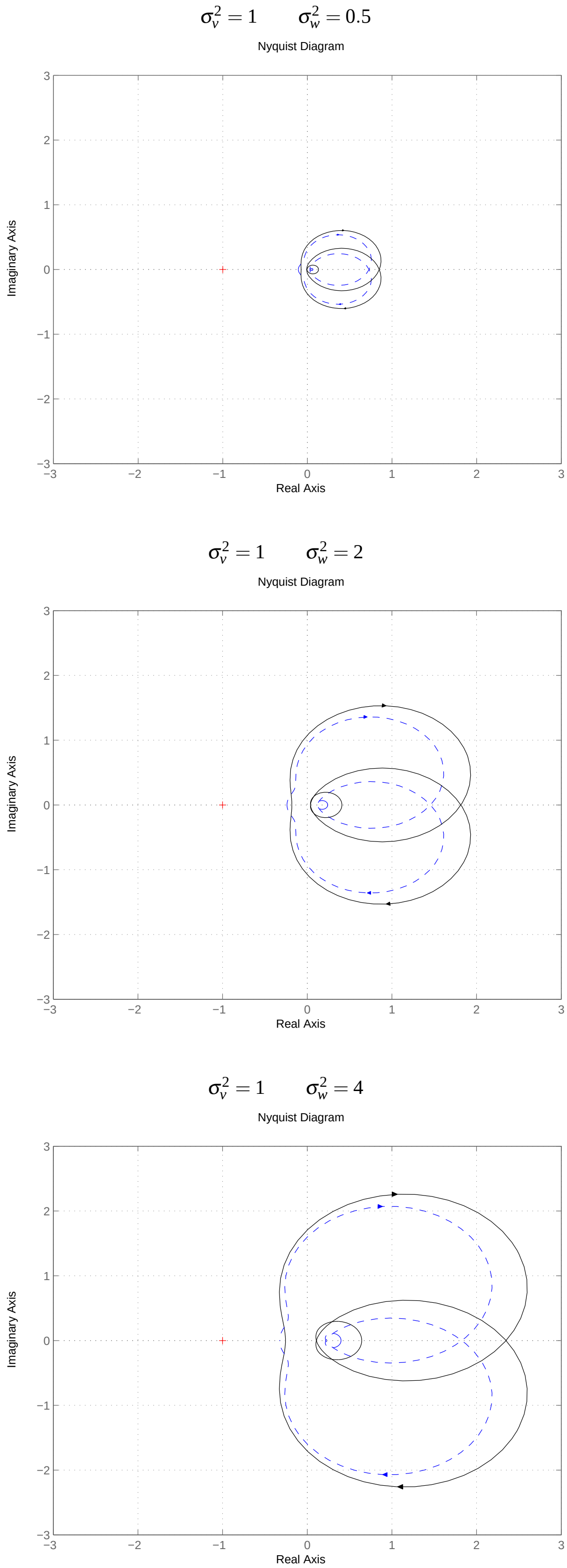

Fig. 3. Nyquist diagrams of $P(z)C(z;\rho)$ (dash) and of $P(z)C_f(z)$ (solid)

troller tuning where the designer can progressively enlarge the bandwidth of a initial controller while maintaining robustness of the loop. A simulation example which illustrates such a procedure has been given in this paper. We will investigate the use of more sophisticated techniques to shape the designed loop which possibly make use of approximate closed-loop models of the plant.

REFERENCES

Åstrom, K.J. and B. Wittenmark (1997). *Computer-Controlled Systems*. Prentice Hall.

Doyle, J.C. (1978). Guaranteed margins for LQG regulators. *IEEE Transactions on Automatic Control* **23**, 756–757.

Doyle, J.C. and G. Stein (1981). Multivariable feedback design: Concepts for a classical/modern synthesis. *IEEE Transactions on Automatic Control* **26**, 4–16.

Hildebrand, R., A. Lecchini, G. Solari and M. Gevers (2005*a*). Asymptotic accuracy of iterative feedback tuning. *IEEE Transactions on Automatic Control* **50**, 182– 1185.

Hildebrand, R., A. Lecchini, G. Solari and M. Gevers (2005*b*). Optimal prefiltering in iterative feedback tuning. *IEEE Transactions on Automatic Control* **50**, 1196–1200.

Hjalmarsson, H. (2002). Iterative feedback tuning - an overview. *International Journal of Adaptive Control and Signal Processing* **16**, 373–395.

Hjalmarsson, H., M. Gevers, S. Gunnarson and O. Lequin (1998). Iterative Feedback Tuning: theory and applications. *IEEE Control Systems Magazine* **18**, 26–41.

Hjalmarsson, H., S. Gunnarson and M. Gevers (1994). A convergent iterative restricted complexity control design scheme. *Proc. 33rd IEEE Conference on Decision and Control* pp. 1735–1740.

Maciejowski, J.M. (1985). Asymptotic recovery for discrete-time systems. *IEEE Transactions on Automatic Control* **30**, 602–605.

Maciejowski, J.M. (1989). *Multivariable Feedback Design*. Addison-Wesley.

Prochàzka, H., M. Gevers, B.D.O. Anderson and C. Ferrera (2005). Iterative feedback tuning for robust controller design and optimization. *Proc. Joint IEEE CDC-ECC Conference*.

Shaked, U. (1985). Explicit solution to the singular discrete-time stationary linear filtering problem. *IEEE Transactions on Automatic Control* **30**, 34–47.

Veres, S. and H. Hjalmarsson (2002). Tuning for robustness and performance using iterative feedback tuning. *Proc. 41st IEEE Conference on Decision and Control* pp. 4682–4686.

Zhou, K., J.C. Doyle and K. Glover (1996). *Robust and Optimal Control*. Prentice Hall.

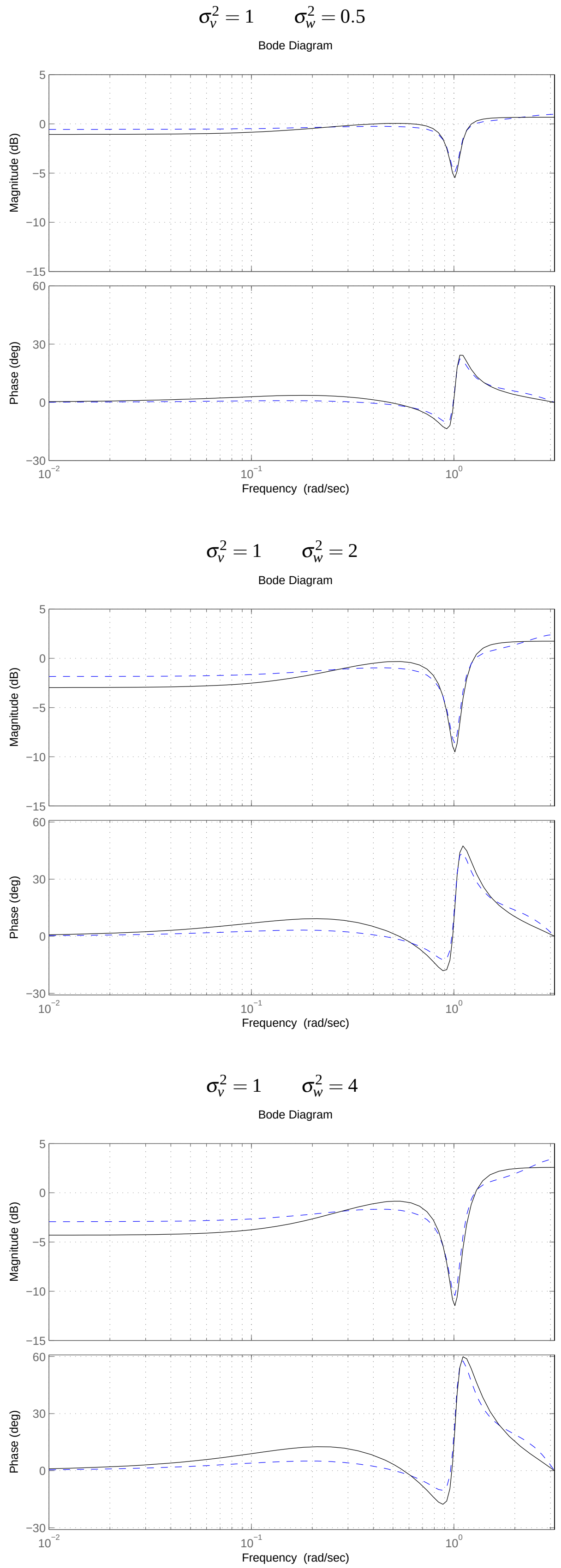

Fig. 4. Bode plots of $[1+P(z)C(z;\rho)]^{-1}$ (dash) and of $[1+P(z)C_f(z)]^{-1}$ (solid)