INPUT DESIGN: FROM OPEN-LOOP TO CONTROL-ORIENTED DESIGN

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Abstract: In this paper we briefly review the evolution of the main tools and results for optimal experiment design for system identification. The initial work dates back to the seventies and focused on the accuracy of the parameters of the input-output transfer function estimate. In the eighties, new formulas for the variance of transfer function estimates based on high-order model approximations led to the first goal-oriented experiment design results. The recent trend is to address control-oriented optimal design questions using the more accurate parameter covariance formulas for finite order models.

Keywords: identification, optimal experiment design, optimization, input signals, control accuracy

The early work on experiment design

Optimal input design for system identification was an active area of research in the 1970's, with different quality measures of the identified model being used for this optimal design (Mehra, 1974; Zarrop, 1979; Goodwin and Payne, 1977). Up until very recently, the optimal input design literature has focused almost exclusively on the minimization of some measure of the variance error of the estimated quantity. The objective functions that were minimized in the 1970's were various measures of the covariance matrix P_θ, where θ is the parameter vector of the model structure used for the open-loop model, which is being estimated.

Let the "true system" be given by:

$$\mathcal{S} : y(t) = \overbrace{G(z,\theta_0)}^{G_0(z)}\, u(t) + \overbrace{H(z,\theta_0)e(t)}^{v(t)} \qquad (1)$$

for some unknown parameter vector $\theta_0 \in \mathbf{R}^k$, where $e(t)$ is white noise of variance σ_e^2, while $G(z,\theta_0)$ and $H(z,\theta_0)$ are stable discrete-time transfer functions, with $H(z,\theta_0)$ monic and minimum-phase.As stated above, in most of the optimal input design literature, it is assumed that the system is identified with a model structure $\mathcal{M} = \{G(z,\theta),\ H(z,\theta)\}$, $\theta \in \mathbf{R}^k$, that is able to represent the true system.

When Prediction Error identification is used with a full order model structure, the estimated parameter vector $\hat{\theta}_N$ is known to converge, under mild assumptions, to a Gaussian distribution:

$$(\hat{\theta}_N - \theta_0) \stackrel{N\to\infty}{\longrightarrow} N(0, P_\theta), \qquad (2)$$

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where the asymptotic parameter covariance matrix P_θ can be estimated from the data. Important examples of optimal design criteria developed in the 1970's are A-optimal design which minimizes $tr(P_\theta)$, D-optimal design which minimizes $det(P_\theta)$, E-optimal design which minimizes $\lambda_{max}(P_\theta)$, and L-optimal design which minimizes $tr(WP_\theta)$, where W is a nonnegative weighting matrix.

Although the design can be performed directly with respect to the time-domain input signal sequence $u(1), \ldots, u(N)$, this leads to a complex nonlinear optimal control problem involving a large number of unknowns. Examples of optimal design directly with respect to the input sequence can be found in e.g. (Cooley and Lee, 2001). The problem is significantly simplified if one restricts attention to quasi-stationary signals (Ljung, 1999), which admit a power spectral density $\Phi_u(\omega)$. Indeed, in open-loop identification, the inverse of the covariance matrix P_θ can be expressed as the following expression of the input signal spectrum:

$$P_\theta^{-1} = \left(\frac{N}{\sigma_e^2} \frac{1}{2\pi} \int_{-\pi}^{\pi} F_u(e^{j\omega}, \theta_0) F_u(e^{j\omega}, \theta_0)^* \Phi_u(\omega) d\omega \right) + \left(N \frac{1}{2\pi} \int_{-\pi}^{\pi} F_e(e^{j\omega}, \theta_0) F_e(e^{j\omega}, \theta_0)^* d\omega \right) \quad (3)$$

This matrix is called the information matrix (or Fisher information matrix): $M_\theta \triangleq P_\theta^{-1}$. Here, $F_u(z, \theta_0) = \frac{\Lambda_G(z,\theta_0)}{H(z,\theta_0)}$, $F_e(z, \theta_0) = \frac{\Lambda_H(z,\theta_0)}{H(z,\theta_0)}$, $\Lambda_G(z, \theta) = \frac{\partial G(z,\theta)}{\partial \theta}$ and $\Lambda_H(z, \theta) = \frac{\partial H(z,\theta)}{\partial \theta}$. The formula shows that the data length N and the input spectrum $\Phi_u(\omega)$ appear linearly in the expression of the information matrix M_θ, and that, for a given data length N, the input spectrum is the only design quantity that can shape the parameter covariance matrix. It is also clear from (3) that, provided the first matrix has full rank, the covariance matrix P_θ can be made arbitrarily small by raising the power of the input signal. Thus, the most common approach to optimal input design is to minimize some reasonable function of P_θ (e.g. $det(P_\theta)$) with respect to $\Phi_u(\omega)$ under a constraint on $\Phi_u(\omega)$ (e.g. $\int \Phi_u d\omega < \alpha$ for a fixed α). An optimal input signal is then defined as any realization of length N of a quasistationary signal $u(t)$ having $\Phi_u^{opt}(\omega)$ as spectral density.

An important contribution of the optimal experiment design work of the seventies was to parametrize Φ_u in such a way that the information matrix (3) can be expressed as an affine combination of a finite number of parameters of that spectrum. For example, Zarrop used Tchebycheff system theory (see e.g. (Karlin and Studden, 1966)) to parametrize the input spectrum in terms of its so-called "trigonometric moments" with respect to the system (Zarrop, 1979); these moments are real numbers. The information matrix $M_\theta \triangleq P_\theta^{-1}$ can then be expressed as a finite linear combination of these moments, with respect to which the optimization can then be performed.

Another important result of the optimal experiment design work of the seventies, first established in (Mehra, 1974), was to establish that the solutionof this optimal input design problem could always be obtained in the form of a discrete power spectrum, i.e. the optimal input can always be generated as a finite linear combination of sinusoids (multisine). The number of sinusoids required depends on the particular model structure and on the constraints. For example, it was shown in (Goodwin and Payne, 1977) that, if a Box-Jenkins model structure is used with $G(z, \theta)$ containing $2n$ parameters, then an optimal input for the criterion $det(P_\theta)$ under a constraint on the input power can be achieved with no more than $2n$ sinuoids. By using the theory of Tchebycheff systems, Zarrop showed that the number of sinusoids required can actually be reduced to n (Zarrop, 1979).

Even though some of the experiment design work of the 1970's considered closed-loop experiments (Ng *et al.*, 1977*a*; Ng *et al.*, 1977*b*), the objective functions considered at that time were limited to functions of the covariance of the open-loop model parameters.

Experiment design based on L_2 control performance criteria

In the mid-eighties, Ljung and collaborators produced bias and variance formulas (Ljung, 1985; Wahlberg and Ljung, 1986) directly for the transfer function estimates, rather than for the parameter estimates which only serve as auxiliary variables in the representation of these transfer functions. The asymptotic variance formulas were derived under the assumption that the model order n tends to infinity in some appropriate way when the data length N tends to infinity. Thus, for the variance of the input-output transfer function estimate $G(z, \hat{\theta}_N)$, the following approximations were obtained in (Ljung, 1985) under an assumption of high model order, for the open-loop (O.L.) and closed-loop (C.L.) experimental conditions, respectively:

$$Var(G(e^{j\omega}, \hat{\theta}_N)) \approx \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u(\omega)} \quad \text{in O.L.} \quad (4)$$

$$Var(G(e^{j\omega}, \hat{\theta}_N)) \approx \frac{n}{N} \frac{\Phi_v(\omega)}{\Phi_u^r(\omega)} \quad \text{in C.L.} \quad (5)$$

where n is the model order, N is the number of data, $\Phi_u(\omega)$ is the input spectrum, $\Phi_v(\omega)$ is the output disturbance spectrum, and $\Phi_u^r(\omega) \triangleq$

$|\frac{C_{id}(e^{j\omega})}{1+C_{id}(e^{j\omega})G_0(e^{j\omega})}|^2\Phi_r(\omega)$ is the part of the input spectrum that is caused by the external reference excitation r (here C_{id} is the feedback controller present during the data collection experiment). These formulas explicitly contain the effect of the experimental conditions (e.g. number of data, input spectrum, noise spectrum, feedback configuration, feedback controller C_{id}, etc) on the error measure. They paved the way for the formulation of goal-oriented experiment design problems, including control-oriented problems (Gevers and Ljung, 1986; Hjalmarsson *et al.*, 1996; Forssell and Ljung, 2000).

The optimal design criterion used in these contributions was the squared error between the output of the optimal loop (i.e. the loop that would be obtained if the optimal controller, dependent on the unknown true system, were applied to the system), and the output of the achieved loop (i.e. the loop in which the controller obtained from the estimated model is applied to the true system). The results were all based on the transfer function variance formulas (4)-(5), derived under the assumption that the model order tends to infinity, and it was observed in recent years that the use of these formulas for finite order models can sometimes lead to erroneous conclusions. This observation triggered a revival of interest in optimal design formulations based on *variance expressions for finite order models*.

Experiment design for robust control

Robust stability and robust performance criteria are typically expressed as constraints on frequency weighted expressions of the variance of the transfer function error, rather than as L_2 performance criteria. For example, a robust stability constraint is typically formulated as

$$Var\ G(e^{j\omega},\hat{\theta}_N) \leq b(e^{j\omega})\ \forall\omega \quad (6)$$

where $b(e^{j\omega})$ is a frequency weighting function that takes account of closed-loop properties (e.g. robust stability condition). In order to formulate optimal input design problems in terms of control-oriented quality measures on $G(e^{j\omega},\hat{\theta}_N)$ such as in (6), using the finite model order formula (3) rather than the asymptotic (in model order) variance formulas, several approaches can be taken.

One commonly used approach to go from parameter covariance to transfer function covariance is to use the following first order Taylor series approximation:

$$Var\ G(e^{j\omega},\hat{\theta}_N) \approx \frac{\partial G^*(e^{j\omega},\theta_0)}{\partial\theta} P_\theta \frac{\partial G(e^{j\omega},\theta_0)}{\partial\theta} \quad (7)$$

This approach, initiated in the L_2 framework in (Lindqvist, 2001), was subsequently adopted in (Jansson and Hjalmarsson, 2004*b*)-(Jansson and Hjalmarsson, 2004*a*), where it is shown that several useful H_∞ design criteria can be reformulated as weighted trace optimal input design problems subject to LMI constraints. A sensible open-loop optimal input design problem can then be formulated as follows:

$$\min_{\Phi_u(\omega)} \max_{\omega} tr[W(e^{j\omega})P_\theta] \quad \text{subject to} \quad (8)$$
$$\int_{-\pi}^{\pi} \Phi_u(\omega)d\omega \leq \alpha, \quad \text{and } \Phi_u(\omega) \geq 0\ \forall\omega,$$

where α is some positive constant and $W(e^{j\omega})$ is a function of the model structure and reflects the robustness objectives. This is still a difficult, infinite dimensional optimization problem. However, by the use of Schur complement, the problem can be reformulated as a convex optimization problem under Linear Matrix Inequality (LMI) constraints. The numerical solution of such problems became possible in the nineties with the advent of interior point optimization methods (Nesterov and Nemirovskii, 1994; Boyd *et al.*, 1994). The problem becomes finite dimensional if the input spectrum $\Phi_u(\omega)$ can be finitely parametrized. As we have stated above, this can always be achieved, either exactly by a finite dimensional expansion based on the trigonometric moments using Tchebycheff system theory, or by restricting the class of admissible to subclasses which, by construction, admit a finite parametrization. One such possible approximation, used in (Lindqvist and Hjalmarsson, 2001) is to use input signals generated by passing white noise through FIR filters; other finite-dimensional approximations of the input signal spectrum have been used and analyzed in the thesis (Jansson, 2004) which contains a wealth of useful results on optimal experiment design; see also (Jansson and Hjalmarsson, 2004*c*).

All the results quoted above use the first order Taylor series approximation (7). An alternative to the use of this approximation formula is to use the formulas that have recently been obtained for the variance of finite order transfer function estimates (Xie and Ljung, 2001; Ninness and Hjalmarsson, 2004). For example, for an Output Error (OE) model structure, the open-loop variance formula (4) is replaced by

$$Var(G(e^{j\omega},\hat{\theta}_N)) \approx \kappa_n(\omega)\frac{\Phi_v(\omega)}{\Phi_u(\omega)} \quad (9)$$

where $\kappa_n(\omega)$ depends on the poles of the true system and on $\Phi_u(\omega)$. The use of these new transfer function variance formulas for input design has been advocated in (Hjalmarsson and Jansson, 2003), but one additional difficulty, as the authors point out, is that the function $\kappa_n(\omega)$ depends on the unknown system.

A rather different approach to optimal input design for robust control, which directly uses the covariance matrix P_θ without the need for an approximation is based on the use of the ellipsoidal uncertainty set U_θ:

$$U_\theta = \{\theta | (\theta - \hat{\theta}_N)^T P_\theta^{-1} (\theta - \hat{\theta}_N) < \chi^2\}. \quad (10)$$

It follows from the property (2) that the true parameter vector $\theta_0 \in \mathbb{R}^d$ belongs to U_θ with probability $\alpha(d, \chi^2) = Pr(\chi^2(d) \leq \chi^2)$, where $\chi^2(d)$ denotes the χ^2 distribution with d degrees of freedom. The results in (Bombois *et al.*, 2001; Gevers *et al.*, 2003), which connect robust stability and robust performance measures directly to the ellipsoidal uncertainty region U_θ, now allow one to formulate experiment design problems for robust control in terms of the minimization of some appropriate function of U_θ (or of P_θ) without the need for the intermediate step of transfer function variance estimation, which typically requires both a Taylor series approximation and/or a conservative step of overbounding of the uncertainty set.

The first open-loop optimal input design problem for robust control based on the direct use of the uncertainty ellipsoid U_θ was formulated in (Hildebrand and Gevers, 2003). The robust stability measure minimized in that paper, with respect to the input spectrum $\Phi_u(\omega)$, was the worst-case ν-gap $\delta_{WC}(G(z, \hat{\theta}_N), \mathcal{D})$ between the identified model $G(z, \hat{\theta}_N)$ and all models in the Prediction Error uncertainty set $\mathcal{D} \triangleq \{G(z, \theta) | \theta \in U_\theta\}$:

$$\delta_{WC}(G(z, \hat{\theta}_N), \mathcal{D}) = \sup_{\theta \in U_\theta} \delta_\nu(G(z, \hat{\theta}_N), G(z, \theta)) \quad (11)$$

where the ν-gap is defined in (Vinnicombe, 1993). One of the merits of the worst-case ν-gap is that it is directly related to the size of the set of its stabilizing controllers: the smaller the worst-case ν-gap of the uncertainty set $\mathcal{D}$, the larger is the set of controllers that stabilize all models in $\mathcal{D}$. The optimal input design problem solved in (Hildebrand and Gevers, 2003) was

$$\min_{\Phi_u} \delta_{WC}(G(z, \hat{\theta}_N), \mathcal{D}) \text{ subject to} \quad (12)$$
$$\int_{-\pi}^{\pi} \Phi_u(\omega) d\omega \leq \alpha, \quad \text{and } \Phi_u(\omega) \geq 0 \ \forall \omega.$$

The solution uses Tchebycheff system theory: the input spectrum is parametrized in terms of its n moments with respect to the system. The optimal solution can always be obtained as a multisine.

Optimal experiment design in closed loop

All the results discussed so far are for open-loop identification, whereas identification for control is typically performed in closed-loop, often in an iterative way. As it happens, the parameter covariance formula (3) can easily be extended to closed-loop identification as follows (Bombois *et al.*, 2005; Jansson and Hjalmarsson, 2005):

$$P_\theta^{-1} = N \overbrace{\left(\frac{1}{\sigma_e^2} \frac{1}{2\pi} \int_{-\pi}^{\pi} F_r(e^{j\omega}, \theta_0) F_r(e^{j\omega}, \theta_0)^* \Phi_r(\omega) d\omega \right)}^{\mathcal{P}_r^{-1}(\Phi_r(\omega), \theta_0, \sigma_e^2)}$$
$$+ \ N \ \overbrace{\left(\frac{1}{2\pi} \int_{-\pi}^{\pi} F_e(e^{j\omega}, \theta_0) F_e(e^{j\omega}, \theta_0)^* d\omega \right)}^{\mathcal{P}_v^{-1}(\theta_0)} \quad (13)$$

Here, $F_r(z, \theta_0) = C_{id} S_{id} \frac{\Lambda_G(z,\theta_0)}{H(z,\theta_0)}$, $F_e(z, \theta_0) = \frac{\Lambda_H(z,\theta_0)}{H(z,\theta_0)} - C_{id} S_{id} \Lambda_G(z, \theta_0)$, $\Lambda_G(z, \theta) = \frac{\partial G(z,\theta)}{\partial \theta}$ and $\Lambda_H(z, \theta) = \frac{\partial H(z,\theta)}{\partial \theta}$. Note that P_θ^{-1} is made up of a part depending on $\Phi_r(\omega)$ and a part which does not depend on $\Phi_r(\omega)$. Both parts are linear in N and both parts depend on the operating controller C_{id}. For a given controller C_{id} and a fixed data length, we observe that the covariance matrix is again linear in the reference spectrum $\Phi_r(\omega)$, which is now the design object. Instead of using a fixed controller, and optimizing over the external reference spectrum $\Phi_r(\omega)$, closed-loop optimal design problems can also be formulated with respect to both the reference spectrum $\Phi_r(\omega)$ and the operating controller C_{id}. It turns out to be easier to use the input spectrum $\Phi_u(\omega)$ and the cross-spectrum $\Phi_{ue}(\omega)$ as design variables; note that there is a one-to-one relationship between the pair $\{\Phi_r(\omega), C_{id}(e^{j\omega})\}$ and the pair $\{\Phi_u(\omega), \Phi_{ue}(\omega)\}$. Such approach has been proposed in (Jansson and Hjalmarsson, 2005).

Why do more work than is needed ?

The traditional approach to optimal input design, as exemplified by the problem formulations (8) or (12), has been to optimize some measure of the resulting uncertainty, subject to a constraint on the input signal power. However, in an identification for robust control setting, one should not spend more effort on the identification than is needed to ensure that the controller designed with the identified model achieves a prescribed level of performance with all systems in the uncertainty region. This robustness constraint can always be translated into a condition similar to (6). This idea has led to the recent concept of "least costly identification for control", which was first proposed in (Bombois *et al.*, 2004*b*). Instead of minimizing some measure of the uncertainty set, the objective is to deliver an uncertainty set that is just within the bounds required by the robust control specifications, and to do so at the smallest possible cost. In (Bombois *et al.*, 2004*a*) open-loop identification is considered and the cost is then defined as the total input signal power. The idea of least costly (or minimum energy) identification experiment for control has been further developed in an open-loop framework in (Jansson and Hjalmarsson, 2004*b*).

From a practical point of view, the cost of identification is an issue of major importance. This has been thoroughly discussed in (Rivera *et al.*, 2003) where the concept of "plant-friendly" identification is presented. It is often estimated that 75% of the cost associated to an advanced control project goes into model development. Even though the definition of the cost used in the recent work on "least costly identification for control" does by no means cover all the practical costs of modelling, the disruption caused to normal operation and the time required to arrive at a satisfactory model are considered to be very significant elements of this total modelling cost. These two costs are incorporated in the "least costly" criterion of (Bombois *et al.*, 2005) in a closed-loop framework.

Is optimal design really worth the effort?

One might wonder whether it pays to perform optimal input design computations, given that the optimal solution necessarily depends on the unknown system, which means that a preliminary model estimate must be obtained first before an approximately optimal input signal can be computed. This is sometimes referred to as *adaptive (or iterative) optimal input design*. In (Barenthin *et al.*, 2005) the possible benefits of optimal input design for control have been quantified for two benchmark problems. It is shown that significant savings can sometimes be obtained by the application of a two-step identification procedure, where the second step uses an optimally designed input signal computed from a preliminary model estimate.

Many experiment design issues remain to be addressed, let alone solved. A fundamental issue is the fact that the optimal experiment depends on the unknown system. Thus, the practical implementation of optimal input design results requires that a preliminary model be estimated quickly on the basis of non-optimal inputs, after which an estimate of the optimal input can be computed. This raises the very important issue of the robustness of the optimal design to model errors, and of the convergence of such adaptive implementations. Some preliminary observations and recommendations on this robustness question issue have been made in (Jansson and Hjalmarsson, 2004*b*).

Other important issues to be addressed are to formulate the input design problem directly in terms of the properties (robust stability and performance) of the controller that is designed from the identified model. Some preliminary results in this direction can be found in (Barenthin and Hjalmarsson, 2005). Finally, the new phase of research results that have been briefly described here are all based on variance results for finite order models, under the assumption that the true system is in the model set. Results for the case of undermodelling are only just beginning to emerge (Bombois and Gilson, 2006). As pointed out in (Hjalmarsson, 2005), a proper choice of input is even more important when a restricted complexity model is used with a particular objective (e.g. control) in mind: it is then always better that the input excite only those parts of the system dynamics that need to be modelled.

Conclusions and additions

This will be for the final version.

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