Iterative Feedback Tuning for robust controller design and optimization

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***Abstract*— This paper introduces a new approach for robust controller design using Iterative Feedback Tuning (IFT) method. Based on some robustness principles a new criterion is proposed reflecting both performance and robustness specifications. Then, some analysis of the new criterion is made and we show how to apply the standard IFT procedure using the new criterion. The approach is transparently illustrated on one model-based controller design example.**

I. INTRODUCTION

Iterative Feedback Tuning (IFT) is an iterative procedure for controller optimization [2]. It is useful mainly for tuning of low-order controllers. It can tune a badly adjusted controller to upgrade its performance, where the desired performance has to be formulated as a criterion function of some specific form. Although the main application of IFT is controller tuning (optimization starting from an initial controller), it is successfully used for model-free and model-based controller design as well (for some application examples see [1], or [3]). The IFT procedure was primarily developed for tuning of single-input single-output (SISO) one-degree-of-freedom (1DOF) or two-degree-of-freedom (2DOF) discrete-time controllers. Later, the procedure was modified to allow optimization of other controller structures [1].

The IFT procedure does not require a plant model; only closed-loop system experiments are necessary to estimate a gradient of a criterion function which is to be minimized. In the classical IFT method the criterion is typically constructed with two terms: one reflecting a desired tracking property, the second one restricting control action energy. This kind of criterion is obviously insufficient if other specifications, such as robust stability, are imposed. The robustness specification belongs to the most commonly required specifications and it would be useful to handle it within the IFT framework.

One attempt to deal with the robustness was made in [6], where the H_∞-norm approximation of a sensitivity matrix (matrix with the four sensitivities) is introduced to the criterion. In this paper we introduce a robust stability measure to the criterion in a simpler and more flexible way. We use the elements of the sensitivity matrix, i.e. the sensitivity functions and we add the square of their weighted H_2-norm separately to the criterion using closed-loop signals. We focus only on a single-input single-output (SISO) discrete-time linear 1DOF controller structure in this paper, but the same approach is easily extendable to a 2DOF structure.

The paper is organized as follows: Section 2 introduces the considered closed-loop system and the classical IFT procedure with some details on possible configurations. In section 3 we develop the new robustness criterion and the corresponding modified IFT procedure. We show that an unbiased estimate of the required gradient can be computed from closed-loop signals. Section 4 illustrates the functionality of the developed approach on one controller design example. Finally section 5 gives some conclusions.

II. BASICS

A. Closed-loop system

A one-degree-of-freedom linear discrete-time controller optimization is considered. A corresponding closed-loop (CL) diagram is shown in Figure 1. The block G is either a discrete-time plant model $G(z^{-1})$ or a real plant G with a sample and hold device and the block $C(z^{-1})$ represents a linear discrete-time controller to be optimized. The operator z^{-1} is considered to be either a frequency operator in a Z-transform, or a time delay operator. The external signals $r(t)$, $d(t)$, $v(t)$ are an output reference, an external control action input, and a perturbation signal, respectively. The output signal $y(t)$ and the control action $u(t)$ are internal closed-loop signals.

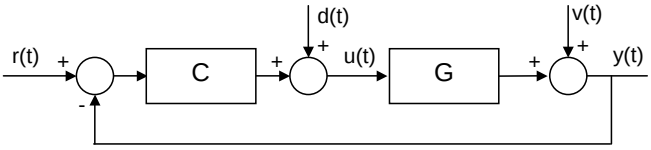

Fig. 1. Considered closed-loop system.

The reference input $r(t)$ and the control input $d(t)$ are exogenous signals added to the closed loop either as a reference for the output, or as a closed-loop excitation providing us with some plant information. The perturbation $v(t)$ is an unwanted and unknown perturbing signal.

Considering the plant model $G(z^{-1})$ in the closed loop, the system in Figure 1 can be described by four closed-loop transfer functions (generalized sensitivities): $S(z^{-1})$,

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$C(z^{-1})S(z^{-1})$, $G(z^{-1})S(z^{-1})$, and $G(z^{-1})C(z^{-1})S(z^{-1})$, with the sensitivity $S(z^{-1})$ defined as follows:

$$S(z^{-1}) = (1 + G(z^{-1})C(z^{-1}))^{-1} \quad (1)$$

(To simplify equations, we will often use the following simplified notations for the sensitivities: S, CS, GS, GCS and for the signals: r, d, v.)

The sensitivities represent the behavior of closed-loop internal signals ($u(t)$ and $y(t)$) with respect to an external signal. They are typically used to describe the following input-output signal relations:

$$\begin{aligned} y(t) &= Sv(t) \\ u(t) &= -CSv(t) \\ y(t) &= GSd(t) \\ y(t) &= GCSr(t) \end{aligned} \quad (2)$$

Recall that by analyzing the sensitivities (2) we can measure the system robust stability with respect to different kinds of uncertainties, and noise and perturbation signals (see for example [7]). Conventional H_∞ design techniques typically use the so called sensitivity matrix $T(z^{-1})$:

$$T = \begin{pmatrix} GCS & GS \\ CS & S \end{pmatrix} \quad (3)$$

which contains the four sensitivities. To design a robust controller, these techniques minimize the H_∞-norm of the weighted matrix $W_1(z^{-1})T(z^{-1})W_2(z^{-1})$:

$$\min_{C(z^{-1})} \|W_1(z^{-1})T(z^{-1})W_2(z^{-1})\|_\infty \quad (4)$$

The weighting filters W_i allow the designer to scale and to weight appropriately the sensitivities in order to obtain the best compromise in the sensitivity minimization. Weighting selection is one of the main designer tasks in the H_∞ controller design techniques.

Using the sensitivities as they are defined in (2), the two internal closed-loop signals $u(t)$ and $y(t)$ (see Figure 1) have the following expressions:

$$\begin{aligned} y(t) &= GCSr(t) + GSd(t) + Sv(t) \\ u(t) &= CSr(t) + Sd(t) - CSv(t) \end{aligned} \quad (5)$$

B. Classical Iterative Feedback Tuning

The classical IFT scheme for 1DOF controller optimization considers the closed-loop system as it is shown in Figure 1 but without the control excitation input: $d(t) = 0$. IFT generally minimizes the following criterion function:

$$\begin{aligned} J_P(\rho) = E\left[\left(L_y(z^{-1})(y(t) - y_d(t))\right)^2\right] \\ +\lambda E\left[\left(L_u(z^{-1})u(t)\right)^2\right] \end{aligned} \quad (6)$$

(the index P denotes "performance"), where ρ is a vector of controller coefficients to be optimized, e.g. $C = C(\rho)$ and $E[\cdot]$ denotes expectation with respect to all random signals. The first term in the criterion reflects some tracking performance specifications ($y_d(t)$ is a desired closed-loop time response, $y(t)$ is the output (5)). The second term adds a possible control action energy restriction. The transfer functions $L_y(z^{-1})$ and $L_u(z^{-1})$ are some stable weighting filters adjusting the criterion and λ is a scaling factor added for the same purpose.

Replacing the signals $u(t)$ and $y(t)$ with the sensitivity expressions (5) and with $d(t) = 0$, we obtain the following criterion:

$$\begin{aligned} J_P(\rho) = E[L_u(GCSr + Sv - y_d)^2] \\ +\lambda E[L_u(CSr - CSv)^2] \end{aligned} \quad (7)$$

where C and S depend on the optimization vector ρ.

Notice that the criterion (7) contains three of the four sensitivities (2). Only two of these sensitivities (GCS and CS) are driven by the external signal $r(t)$ that is at the designer's disposal, while the third one (S) is driven by the uncontrollable perturbation signal $v(t)$. This implies that we can see the optimization of the criterion J_P from two different points of view:

1) By minimizing the criterion J_P we improve the tracking performance (term $E[(y - yd)^2]$) and limit the control action (term $E[u^2]$).
2) By minimizing the criterion J_P we minimize a weighted and perturbed H_2 norm of the sensitivities GCS and CS. The sensitivity S is neglected since it is weighted with the unknown random perturbation $v(t)$.

Controller optimization using IFT minimizes the criterion function (6) step by step by iteratively tuning the controller. The following style of procedure sets out one such step:

1) *Step 1: 1st real-time experiment.* An external excitation signal ($r_1(t)$ or $d_1(t)$) is applied to the closed-loop system. In this experiment we obtain the internal closed-loop signals $y_1(t)$ and $u_1(t)$ containing some weighted sensitivity functions as shown by (5). We can establish three general configurations for the 1st experiment:
   - Configuration 1: The excitation is applied at $r(t)$: $r(t) = r_1(t)$, $d(t) = 0$.
   - Configuration 2: The excitation is applied at $d(t)$: $d(t) = d_1(t)$, $r(t) = 0$.
   - Configuration 3: The excitation is applied at both $r(t)$ and $d(t)$: $r(t) = r_1(t)$, $d(t) = d_1(t)$.
2) *Step 2: 2nd real-time experiment.* Another excitation signal ($r_2(t)$ or $d_2(t)$) is applied to the closed loop, in order to obtain an estimate of the signal gradients: $est\frac{\partial y_1}{\partial \rho}$ and $est\frac{\partial u_1}{\partial \rho}$. These derivatives are necessary to compute the gradient of the criterion. The following design choices are available for the 2nd experiment:
   - The excitation in the 2nd experiment may be applied either at $r(t) = r_2(t)$ or at $d(t) = d_2(t)$).
   - The excitation signal applied in the 2nd experiment is not arbitrary, but highly though not completely constrained, as described in detail below. The theory of IFT shows that the computation of the desired gradients requires a special excitation

signal containing a feedback from a signal generated in the 1st experiment, possibly filtered with a selected stable filter $K(z^{-1})$.

The obtained derivative estimate is perturbed (i.e. deviates from the exact value) by a noise term. The form of the noise term is determined by the 1st experiment configuration, and the 2nd experiment choice, as we shall see later.

3) *Step 3: Criterion gradient estimation.* Using the estimated signal derivatives $est\frac{\partial y_1}{\partial \rho}$ and $est\frac{\partial u_1}{\partial \rho}$ obtained from the 2nd experiment, the criterion gradient $est\frac{\partial J}{\partial \rho}$ is estimated.
4) *Step 4: New coefficients calculation.* A new controller parameter vector ρ is computed from the estimated criterion gradient. This new controller parameter vector defines a new controller $C(\rho)$.

Notice that in the classical IFT procedure, the excitations in steps 1 and 2 are always applied at $r(t)$.

III. IFT FOR ROBUST CONTROLLER DESIGN

To introduce a robust stability measure to IFT, we can incorporate the four sensitivities (2) to the optimized criterion function. To do so we add a new term $J_R(\rho)$ (R denotes robustness) to the classical criterion $J_P(\rho)$; we will call J_P a performance index and J_R a robustness index. The new IFT criterion becomes:

$$J(\rho) = J_P(\rho) + J_R(\rho) \tag{8}$$

where $J_P(\rho)$ has the standard form (6) and $J_R(\rho)$ contains some norms of weighted sensitivity terms $L_S S$, $L_{CS} CS$, $L_{GS} GS$, $L_{GCS} GCS$. The transfer functions L_S, L_{CS}, L_{GS}, L_{GCS} are stable and proper weighting filters to be chosen by the designer. The exact structure of $J_R(\rho)$ is given below, since it differs according to the configuration chosen for the IFT tuning.

A. Robustness index

Consider the IFT scheme where configuration no. 1 is chosen for the 1st experiment; i.e. the excitation is applied to $r(t)$: $r(t) = r_1(t)$, $d(t) = 0$. The internal closed-loop signals have the following form:

$$\begin{aligned} y(t) &= GCSr(t) + Sv(t) \\ u(t) &= CSr(t) - CSv(t) \end{aligned} \tag{9}$$

We propose the following robustness index (the criterion for robustness):

$$\begin{aligned} J_R(\rho) &= J_{GCS}(\rho) + J_{CS}(\rho) + J_{GS}(\rho) + J_S(\rho) \\ &= \lambda_{GCS} E\left[(L_{GCS} y)^2\right] + \lambda_{CS} E\left[(L_{CS} u)^2\right] \\ &+ \lambda_{GS} E\left[(L_{GS} C^{-1} y)^2\right] + \lambda_S E\left[(L_S C^{-1} u)^2\right] \end{aligned} \tag{10}$$

where the λ_i are some weighting (scaling) coefficients to adjust the relative impact of each particular robustness term. The terms L_i are some stable and proper weighting filters to impose some specific sensitivity attenuations. The reason for the subscripts on the four summands in (10) is suggested by equations (11) below.

Replacing the signals $y(t)$ and $u(t)$ by (9) in the criterion (10), we have:

$$\begin{aligned} J_{GCS}(\rho) &= \lambda_{GCS} E\left[(L_{GCS}(GCSr + Sv))^2\right] \\ J_{CS}(\rho) &= \lambda_{CS} E\left[(L_{CS}(CSr - CSv))^2\right] \\ J_{GS}(\rho) &= \lambda_{GS} E\left[(L_{GS}(GSr + C^{-1}Sv))^2\right] \\ J_S(\rho) &= \lambda_S E\left[(L_S(Sr - Sv))^2\right] \end{aligned} \tag{11}$$

We can observe that the sub-criteria J_{GCS}, J_{CS}, J_{GS}, J_S of the robustness index J_R always contain two sensitivities. One sensitivity is different in each term and it is driven (or weighted) by the excitation $r(t)$, the other is driven by the perturbation $v(t)$. In order to impose robustness we shall rely only on the sensitivities driven by the excitation $r(t)$ because we have control over the energy of that signal, while the other term is considered as a perturbation.

Each sub-criterion in (11) contains the H_2-norm of a corresponding weighted sensitivity perturbed with the filtered perturbation $v(t)$. To measure robust stability we would, of course, prefer an H_∞-norm instead. To improve the quality of the robustness measure for J_R we can choose an appropriate filter L_i. For example:

- A band-pass filter selecting the problematic frequency band (where a sensitivity modulus is too high) can help to lower the sensitivity in the appropriate frequency band. A similar idea using band-pass filters is advanced in [6] to approximate the H_∞-norm.
- A filter where the frequency response is the inverse of the spectral factor of the external excitation signal allows one to remove the sensitivity weighting caused by the excitation.

Notice that the criterion can be adjusted at each iteration to capture the robust stability property as it evolves with successive controllers.

The proposed robustness index (10) allows one to shape each sensitivity independently in a transparent manner and it gives us a sensitivity shaping design tool. On the other hand, we have to take into account that the criterion minimizes an H_2-norm instead of the H_∞-norm and this may result in different weighting filters with respect to standard sensitivity shaping techniques. One key advantage of this IFT approach to robust controller design is that the filters used are not limited in complexity since they do not influence the final controller order. This is an important advantage with respect to H_∞ design methods, where a subsequent controller reduction step is often used to reduce the complexity of the controller.

The criterion (10) has the following gradient:

$$\frac{\partial J_R}{\partial \rho} = \frac{\partial J_{GCS}}{\partial \rho} + \frac{\partial J_{CS}}{\partial \rho} + \frac{\partial J_{GS}}{\partial \rho} + \frac{\partial J_S}{\partial \rho} \tag{12}$$

where the sub-criteria gradients are as follows:

$$\begin{aligned}\frac{\partial J_S}{\partial \rho} &= 2\lambda_S E\left[L_S C^{-1} u L_S\left(\frac{\partial C^{-1}}{\partial \rho}u + C^{-1}\frac{\partial u}{\partial \rho}\right)\right]\\ \frac{\partial J_{CS}}{\partial \rho} &= 2\lambda_{CS} E\left[L_{CS} u L_{CS}\frac{\partial u}{\partial \rho}\right]\\ \frac{\partial J_{GS}}{\partial \rho} &= 2\lambda_{GS} E\left[L_{GS} C^{-1} y L_{GS}\left(\frac{\partial C^{-1}}{\partial \rho}y + C^{-1}\frac{\partial y}{\partial \rho}\right)\right]\\ \frac{\partial J_{GCS}}{\partial \rho} &= 2\lambda_{GCS} E\left[L_{GCS} y L_{GCS}\frac{\partial y}{\partial \rho}\right] \qquad (13)\end{aligned}$$

The exact internal closed-loop signal derivatives are as follows:

$$\begin{aligned}\frac{\partial y}{\partial \rho} &= GS\frac{\partial C}{\partial \rho}(Sr - Sv) = GS\frac{\partial C}{\partial \rho}(r - y)\\ \frac{\partial u}{\partial \rho} &= S\frac{\partial C}{\partial \rho}(Sr - Sv) = S\frac{\partial C}{\partial \rho}(r - y) \qquad (14)\end{aligned}$$

The term $(Sr - Sv)$ in (14) represents the required form of the excitation for the 2nd experiment; as is indicated in (14) it can be generated as $(r(t) - y(t))$ using the reference signal $r(t)$ and the output signal $y(t)$. The expressions (14) shows that, to obtain the derivatives $\frac{\partial y}{\partial \rho}$ and $\frac{\partial u}{\partial \rho}$ the excitation signal $(r(t) - y(t))$ has to be filtered by the closed-loop sensitivities GS and S.

B. Robustness index gradient estimation

To estimate the gradient (12) properly, we need to present a more explicit version of steps 2 and 3 of the IFT procedure. We consider the configuration 1 for the 1st step of the IFT procedure: the excitation is applied to $r(t) = r_1(t)$ with $d(t) = 0$. The obtained internal signals are then:

$$\begin{aligned}y_1(t) &= GCSr_1(t) + Sv_1(t)\\ u_1(t) &= CSr_1(t) - CSv_1(t) \qquad (15)\end{aligned}$$

In the 2nd experiment, the excitation can be applied either to $r(t)$ or to $d(t)$. As we show later, these two possibilities are equivalent. The excitation signal is considered to be in the following form:

$$r_2(t) = d_2(t) = K(Sr(t) - Sv(t)) = K(r_1(t) - y_1(t)) \quad (16)$$

$K(z^{-1})$ is any chosen stable proper filter with $K^{-1}(z^{-1})$ stable and proper. We show below that with this 2nd experiment we are able to estimate the internal signal derivatives $est\frac{\partial y}{\partial \rho}$ and $est\frac{\partial u}{\partial \rho}$. The estimates will serve to estimate the gradient of both the standard performance criterion J_P and the robustness criterion J_R. With the excitation $r_2(t)$ from (16) added to the closed loop and with $d_2 = 0$, the obtained 2nd experiment signals are:

$$\begin{aligned}y_2(t) &= GCSr_2(t) + Sv_2(t)\\ u_2(t) &= CSr_2(t) - CSv_2(t) \qquad (17)\end{aligned}$$

The derivative estimates are evaluated as follows:

$$est\frac{\partial y}{\partial \rho} = K^{-1}C^{-1}\frac{\partial C}{\partial \rho}y_2 = \frac{\partial y_1}{\partial \rho} + K^{-1}C^{-1}S\frac{\partial C}{\partial \rho}v_2 \quad (18)$$

$$est\frac{\partial u}{\partial \rho} = K^{-1}C^{-1}\frac{\partial C}{\partial \rho}u_2 = \frac{\partial u_1}{\partial \rho} - K^{-1}S\frac{\partial C}{\partial \rho}v_2 \quad (19)$$

where the terms with $v_2(t)$ represent the estimation error. Notice that if necessary C^{-1} can be unstable. A stable approximation of the inverses C^{-1} is then used instead using all-pass stable filters; for details see [2].

The estimation of the gradient (12) is computed using the estimated derivatives (18), (19), which are unbiased. By computing the estimation errors of the unbiased sub-criterion gradients $est\frac{\partial J_{GCS}}{\partial \rho}$, $est\frac{\partial J_{GS}}{\partial \rho}$, $est\frac{\partial J_{CS}}{\partial \rho}$, and $est\frac{\partial J_S}{\partial \rho}$, one observes that the estimate of the gradient $est\frac{\partial J_R}{\partial \rho}$ obtained this way is also unbiased. The errors on the estimated gradients have the following form:

$$\frac{\partial J_S}{\partial \rho} - est\frac{\partial J_S}{\partial \rho} = 2\lambda_S \sum^{*}\left[L_S(Sr_1 - Sv_1)L_S C^{-1}K^{-1}S\frac{\partial C}{\partial \rho}v_2\right] \qquad (20)$$

$$\frac{\partial J_{CS}}{\partial \rho} - est\frac{\partial J_{CS}}{\partial \rho} = 2\lambda_{CS} \sum^{*}\left[L_{CS}(CSr_1 - CSv_1)L_{CS}K^{-1}S\frac{\partial C}{\partial \rho}v_2\right] \qquad (21)$$

$$\frac{\partial J_{GS}}{\partial \rho} - est\frac{\partial J_{GS}}{\partial \rho} = 2\lambda_{GS} \sum^{*}\left[L_{GS}(GSr_1 + C^{-1}Sv_1)L_{GS}C^{-2}K^{-1}S\frac{\partial C}{\partial \rho}v_2\right] \quad (22)$$

$$\frac{\partial J_{GCS}}{\partial \rho} - est\frac{\partial J_{GCS}}{\partial \rho} = 2\lambda_{GCS} \sum^{*}\left[L_{GCS}(GCSr_1 + Sv_1)L_{GCS}C^{-1}K^{-1}S\frac{\partial C}{\partial \rho}v_2\right] \quad (23)$$

with the sum defined as follows:

$$\sum^{*}[f] = \frac{1}{N}\sum_{t=1}^{N}[f(t)] \mid t = 1, 2, \ldots \qquad (24)$$

It is easy to see that all these estimation errors have zero mean, because the disturbances $v_2(t)$ and $v_1(t)$ in the two experiments are uncorrelated, each having zero mean.

Remark 1: If the 2nd experiment excitation is applied to $d(t) = d_2(t)$ instead of $r(t)$, the estimation errors on the estimates (18), (19) and the gradient estimation errors (20) - (23) are the same except that they are multiplied with the controller C. The reason is that to estimate $\frac{\partial y}{\partial \rho}$ and $\frac{\partial u}{\partial \rho}$ we do not need to filter the internal signals $y_2(t)$ and $u_2(t)$ with C^{-1} as we do in (18), (19). This remark is valid for all three IFT configurations as they are defined in the IFT procedure step 1 (section II-B). The analysis of the two different possibilities of excitation point in the 2nd experiment leads us to the conclusion that with only one 2nd experiment possibility (either $r_2 = 0$ or $d_2 = 0$) we can produce the other one by simply adding the controller C or its inverse C^{-1} (or its stable approximation) in the excitation filter K.

Remark 2: Consider the complete controller C_c consisting of a fixed part H and a part C to be optimized as follows:

$$C_c(z^{-1}) = H(z^{-1})C(z^{-1}) \qquad (25)$$

where H is a stable and proper transfer function. The errors as they are mentioned in Remark 2 are again the same, only multiplied with the fixed part H. The derivative estimates (18), (19) are slightly different, since the signals $y_2(t)$ and $u_2(t)$ have to be filtered with the fixed part H.

C. IFT configuration 2 and 3

For the 1st experiment, and with configuration no. 2 (excitation with $d_1(t)$, and $r(t) = 0$) the internal closed-loop signals are as follows:

$$\begin{aligned} y_1(t) &= GSd_1(t) + Sv_1(t) \\ u_1(t) &= Sd_1(t) - CSv_1(t) \end{aligned} \tag{26}$$

and the proposed robustness index has the following form:

$$\begin{aligned} J_R(\rho) &= J_{GCS}(\rho) + J_{CS}(\rho) + J_{GS}(\rho) + J_S(\rho) \\ &= \lambda_{GCS} E\left[(L_{GCS}Cy)^2\right] + \lambda_{CS} E\left[(L_{CS}Cu)^2\right] \\ &+ \lambda_{GS} E\left[(L_{GS}y)^2\right] + \lambda_S E\left[(L_S u)^2\right] \end{aligned} \tag{27}$$

The choice of the robustness index again reflects the need to introduce into the criterion all four sensitivities weighted with the exogenous excitation. Again, the sub-criteria gradient estimates $est\frac{\partial J_{GCS}}{\partial \rho}$, $est\frac{\partial J_{GS}}{\partial \rho}$, $est\frac{\partial J_{CS}}{\partial \rho}$, and $est\frac{\partial J_S}{\partial \rho}$ contain some errors. The errors are unbiased and they have a similar form as in configuration no. 1.

Configuration no. 3 has two excitations: $r_1(t)$ and $d_1(t)$ are applied simultaneously. The closed-loop internal signals are given by (5). A robustness index can then be defined as follows:

$$\begin{aligned} J_R(\rho) &= J_1(\rho) + J_2(\rho) \\ &= \lambda_1 E\left[(L_1 y)^2\right] + \lambda_2 E\left[(L_2 u)^2\right] \\ &= \lambda_1 E\left[\left(L_1(GCSr + GSd + Sv)\right)^2\right] \\ &\quad + \lambda_2 E\left[\left(L_2(CSr + Sd - CSv)\right)^2\right] \end{aligned} \tag{28}$$

Again the robustness index contains all four sensitivities weighted by an exogenous signal. However, notice that in this configuration the four sensitivities cannot all be weighted independently. Unbiased gradient estimates $est\frac{\partial J_1}{\partial \rho}$, $est\frac{\partial J_2}{\partial \rho}$ can be generated as in the previous configurations.

IV. ROBUST CONTROLLER OPTIMIZATION EXAMPLE

To demonstrate the effect and the use of the new robustness criterion terms we will optimize a discrete-time PID controller for the following discrete-time model:

$$G(z^{-1}) = \frac{0.1813z^{-1} + 0.2122z^{-2}}{1 - 0.6065z^{-1}} \tag{29}$$

The model (29) is taken from [4] (p.137). It is a discretized continuous-time plant model (with gain 1, time constant 10s and delay 3s). The sampling time is 5s. The required tracking performance specifications for the PID are: rise time about 20s and maximum overshoot under 10%. Additionally, we will require the gain margin to be as large as possible. The margin corresponds to the inverse of the maximum of the output sensitivity: $|S(z^{-1})|_{max}$. It gives us one of the main robustness measures, since it represents the minimum distance of the open loop frequency response (Nyquist plot) to the critical point $(-1, 0i)$. The initial controller coefficients are set to $\rho = [0.1, 0, 0]^T$, which gives us a low-gain proportional controller.

Controller 1: Tracking performance controller optimization. In the first stage we use the following optimization criterion:

$$J_P(\rho) = E\left[\left(y(t) - y_d(t)\right)^2\right] \tag{30}$$

where the desired response $y_d(t)$ reflecting the tracking performance specification is given by the following tracking reference model $T_d(z^{-1})$:

$$y_d(t) = T_d(z^{-1})r(t) = \frac{0.2257z^{-1} + 0.1533z^{-2}}{1 - 0.939z^{-1} + 0.318z^{-2}} r(t) \tag{31}$$

The excitation $r_1(t)$ is a unit step of 80 samples.

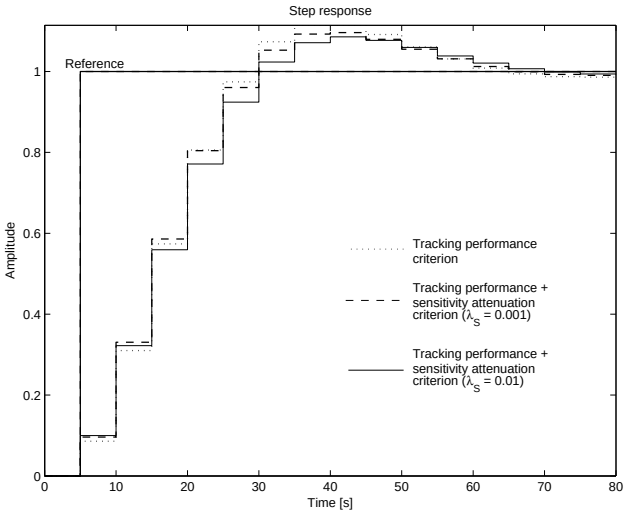

Fig. 2. Closed-loop step response for the three controllers.

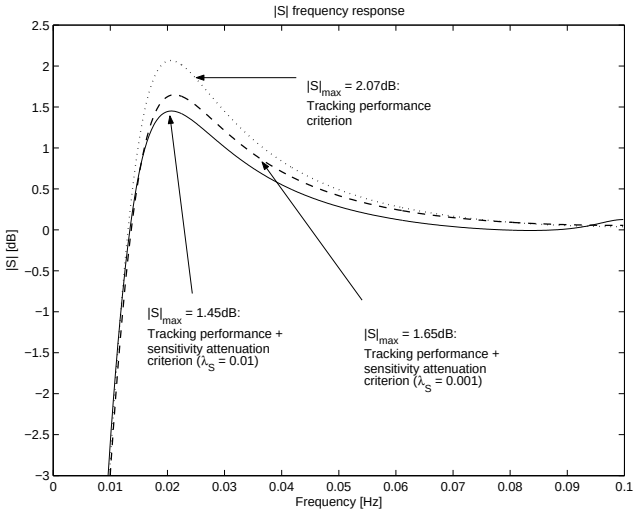

Fig. 3. Output sensitivity frequency response modulus $|S(z^{-1})|$.

Running the IFT optimization on the criterion (30) gives us the following optimal PID controller:

$$C_{PID1}(z^{-1}) = \frac{0.51771 + 0.14448z^{-1}}{(1 - z^{-1})(1 - 0.19749z^{-1})} \quad (32)$$

The corresponding closed-loop step response is shown in Figure 2 (dotted line). The value of the criterion with this controller is $J(\rho) = 5.3 \cdot 10^{-6}$. We observe from Figure 2 that the controller fulfills the performance requirement. The corresponding output sensitivity response is in Figure 3 (dotted line): the maximum is about 2.07dB, which represents a gain margin of about 0.78.

Controller 2: Tracking performance and robust stability controller optimization. To improve the gain margin while preserving reasonable tracking performance we propose the following criterion:

$$J(\rho) = E\left[(y(t) - y_d(t))^2\right] + \lambda_S E\left[\left(L_S C^{-1} u(t)\right)\right)^2\right] \quad (33)$$

The first term reflects the tracking requirement and is the same as in (30), while the second term represents minimization of the output sensitivity maximum. The scaling factor is set to $\lambda_S = 0.001$ (this makes the two terms approximately equivalent, see the final criterion values), the weighting filter $L_S(z^{-1})$ is an inverted 2nd order notch filter [5] with the frequency of band-stop $f_b = 0.02$Hz, band attenuation $M_a = -10$dB, and the denominator damping $\zeta_d = 0.5$. The frequency response of the weighting filter L_S is in Figure 4.

The IFT procedure with this modified criterion gives us the following second optimal PID controller:

$$C_{PID2}(z^{-1}) = \frac{0.58743 + 0.02857z^{-1}}{(1 - z^{-1})(1 + 0.13235z^{-1})} \quad (34)$$

The resulting closed-loop step response is shown in Figure 2 (dashed line), the criterion reached: $J(\rho) = 5.7 \cdot 10^{-6} + 8.2 \cdot 10^{-5}$ (first term represents tracking, the second one robustness). The second controller again fulfills the tracking performance requirement. Although its tracking behavior is almost identical to the first controller, the gain margin is higher as can be seen from the corresponding output sensitivity response in Figure 3 (dashed line). The sensitivity maximum is now 1.65dB, which represents a gain margin of about 0.83. Hence using a supplementary robustness index in the IFT criterion we have increased one of the main robust stability measures without any tracking performance degradation.

Controller 3: This controller is designed using the same criterion as the 2nd controller but with the scaling factor λ_S set to 0.01 in order to accentuate the robustness requirement. The obtained controller has the following form:

$$C_{PID3}(z^{-1}) = \frac{0.61199 + 0.29753z^{-1}}{(1 - z^{-1})(1 + 0.79105z^{-1})} \quad (35)$$

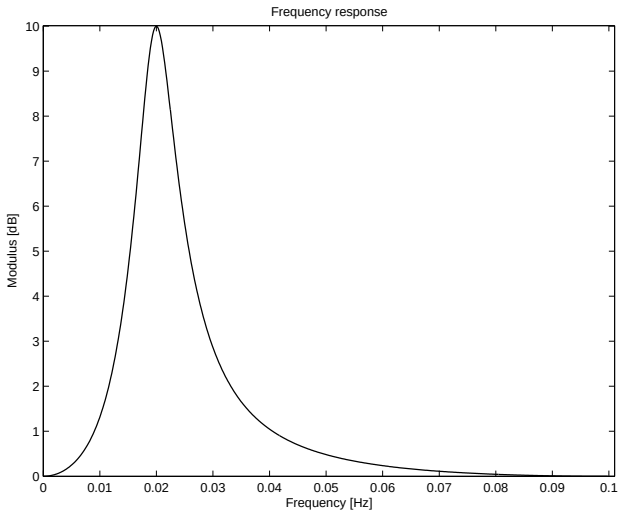

Fig. 4. Frequency response modulus for the weighting filter $L_S(z^{-1})$.

The closed-loop step response for the 3rd controller is shown in Figure 2 (solid line), the resulting criterion value is $J(\rho) = 6.8 \cdot 10^{-6} + 8.1 \cdot 10^{-4}$. The controller guarantees the required tracking performance and the sensitivity maximum is now 1.45dB, which gives a gain margin of 0.85. The sensitivity response in shown in Figure 3 (solid line).

V. CONCLUSIONS

In this paper we have presented a simple and flexible approach to adding some robustness measures to the classical IFT optimization criterion. The approach allows one to optimize or design a controller using the model-free IFT methodology, while considering not only the tracking performance and the energy preservation but also the robust stability. The robust stability is treated by a sensitivity shaping approach, which gives the designer a simple and flexible tool. The presented model-based controller design example illustrates transparently the proposed approach.

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