Closed-loop Identification of Multivariable Systems: With or Without Excitation of All References? ⋆

L. Mišković [b,⋆⋆], A. Karimi [b], D. Bonvin [b], M. Gevers [a,⋆⋆⋆]

[a] *Center for Systems Engineering and Applied Mechanics (CESAME), Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium*

[b] *Laboratoire d'Automatique, Ecole Polytechnique Fédérale de Lausanne (EPFL), CH–1015 Lausanne, Switzerland.*

Abstract

The accuracy of plant parameters estimated in closed-loop operation is investigated for a class of multivariable systems and for the situation where only some of the reference inputs are excited. This issue is important for at least two reasons: (i) there are control applications where it is preferable not to excite all references in order to avoid performance degradation, and (ii) it is not clear whether the results in the context of open-loop identification, where the existence of common parameters between the various transfer functions is a condition for improved accuracy with additional excitation, also hold in the closed-loop case. The paper examines the effect of the non-excitation of some reference inputs on the variance of the estimated parameters. The proposed expressions are valid for all conventional model structures used in prediction error identification. Although exciting all reference inputs is not necessary for identifiability, this work shows that, regardless of the parametrization, the excitation of all references never worsens and, in most cases, improves the accuracy of the parameter estimates. The analytical results developed in this work are illustrated by two simulation examples.

Key words: Closed-loop identification, covariance matrices, multivariable systems, system excitation.

1 Introduction

A common approach in closed-loop identification of multivariable systems is to excite all external reference signals simultaneously and then use the acquired data to identify the parameters of the selected model structure. However, in practice, it is not rare to encounter the situation where it is not convenient to excite all references due to process limitations or for economic reasons. For example, in an industrial process where product quality is one of the reference signals, exciting this reference would result in manufacturing a product of non-uniform quality, which is not acceptable in most cases. Instead, it is preferable to perform the identification by exciting the other reference inputs. Another incentive for not exciting all reference inputs is of practical nature: When performing identification on a real plant, a control engineer has to specify, for each of the reference inputs, the experimental conditions such as the type of acceptable input signals, the acceptable level of excitation, the experiment time, etc. It is clear that exciting only one or a few reference inputs makes this task much simpler and thus more appealing to engineers.

In this context, the identifiability of multivariable linear systems has been recently re-examined in [1]. It has been shown that, in contrast to commonly held beliefs, it is not necessary to excite all reference signals for the identification of a multivariable system operating in closed loop with a linear time-invariant controller. In fact, provided that the controller is of sufficient complexity, it is possible to identify a multivariable system even without any external excitation. In such case, it is the excitation due to noise that provides the information for the estimation of the parameters. On the other hand, relying on information from the noise source only might mean that one has to acquire an unreasonable long data sequence in order to satisfy the prescribed level

⋆ An early version of this paper was presented at the 14th IFAC Symposium on System Identification 2006 (SYSID 2006), Newcastle, Australia.

⋆⋆ Current address: Laboratory of Computational Systems Biotechnology (LCSB), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH–1015 Lausanne, Switzerland.

⋆ ⋆ ⋆ The work of this author is supported by the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office.

Email addresses: ljubisa.miskovic@epfl.ch (L. Mišković), alireza.karimi@epfl.ch (A. Karimi), dominique.bonvin@epfl.ch (D. Bonvin), gevers@inma.ucl.ac.be (M. Gevers).

of accuracy. In that case, an additional excitation at one or several of the reference inputs would help achieve the desired accuracy in a shorter time. In the same line of research, the questions of identifiability and the transfer of excitation are treated in [6]. There, the authors investigate, for single-input single-output (SISO) systems, the smallest degree of richness of the external excitation that is required to achieve identifiability. Closely related questions to these appear in the context of least costly identification experiment for control [3,8].

When identifying a plant for which it is imperative to avoid the excitation of one or several reference inputs, it is of interest to establish whether there are any drawbacks compared to the identification where all reference inputs are excited. Since different experimental conditions yield different levels of accuracy of the estimated parameters, the obvious way to quantify the aforementioned drawbacks is to analyze the accuracy for the two cases.

A similar accuracy analysis in the context of open-loop identification of multi-input systems has been considered recently in [7]. More specifically, the effect of an additional input signal on the variance of the polynomial coefficients in the case of FIR, ARX, ARMAX, OE and BJ models has been investigated. Necessary and sufficient conditions on the parametrization of MISO models under which the addition of an input decreases the covariance of the parameter estimates have been provided. It has been shown that, for model structures that have common parameters in the plant and noise models, any additional independent input signal reduces the covariance of all parameters, including the parameters of the noise model and those associated with the other inputs. It has also been shown that the accuracy improvement resulting from an additional input extends beyond the case of common parameters in all transfer functions.

The present contribution is a continuation of the work presented in [7], but in the context of direct closed-loop identification. Here, a Linear Time-Invariant (LTI) system with m inputs and p outputs is to be identified using data collected in closed-loop operation. It is assumed that the following two assumptions hold:

A1) there are no common parameters between the models associated with the various outputs
A2) the disturbances acting on the different outputs are not correlated with one another

Clearly, these two assumptions restrict the class of multivariable systems for which the results of this work apply. For systems fulfilling these assumptions, a MISO model can be identified for each output separately and the resulting individual models combined into a final MIMO model [4]. The questions that this paper addresses are along the following two lines: (i) What are the possible drawbacks of not exciting all reference signals? (ii) Do the conditions on the parametrization of the MISO structures that apply to open-loop identification carry over to the case of direct closed-loop identification (with the difference that, in closed-loop operation, the external reference signals are excited instead of the inputs)?

To answer these questions, a general model structure is introduced that encompasses all commonly used parametric model structures. It is assumed that the true plant (including the noise model) is in the model set. For clarity of presentation, it is first assumed that $m = p = 2$ and the main findings are presented. Then, the results are extended to arbitrary values of m and p. An analysis of the variance of the estimated parameters, which is asymptotic in data length but not in model order, is performed for two cases of excitation: (i) a single reference signal is used to excite the closed-loop system; (ii) both references are applied simultaneously. A similar asymptotic analysis has been performed in [2], where the variances in both open and closed loop are compared for SISO systems represented by BJ model structures.

The result of this variance analysis is that, in the case of closed-loop identification, the following two situations can be distinguished:

(i) If all parameters of the noise model are present in the plant model, or if there is no noise model at all, then the accuracy of all parameter estimates is always improved by exciting both references simultaneously. For the FIR and OE structures, this result is in contrast to the open-loop case, where existence of common parameters between the plant and noise models is required to improve the accuracy of all parameter estimates.
(ii) If the noise model contains some parameters that are independent of the plant model, then simultaneous excitation of both reference signals may improve but can never worsen the quality of the parameter estimates.

The paper is organized as follows. Preliminaries concerning prediction error identification are given in Section 2. In Section 3, an expression describing the influence of the reference signals on the information matrix is derived. This expression is used for computing the variance of the parameter and transfer function estimates in Section 4. The results presented in Sections 3 and 4 are generalized to arbitrary numbers of inputs and outputs in Section 5. Section 6 illustrates the analytical results via two simulation examples. Finally, conclusions are given in Section 7.

2 Preliminaries

The following unknown 2×2 LTI "true" plant is considered:

$$\begin{aligned} \mathcal{S} : y(t) &= G(q^{-1})u(t) + H(q^{-1})\eta(t) \\ &= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} u(t) + \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix} \eta(t) \end{aligned} \tag{1}$$

where G_{11}, G_{12}, G_{21} and G_{22} are strictly causal, finite-order, rational transfer functions that are not necessarily an-

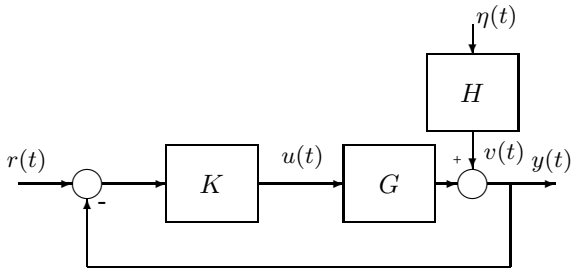

Fig. 1. Multivariable closed-loop system

alytic outside the unit circle, and H_1 and H_2 are stable and inversely stable transfer functions. The backward-shift operator q^{-1} will be omitted in the sequel whenever appropriate. The signal $y(t) \in \mathcal{R}^2$ is the output of the true plant, $u(t) \in \mathcal{R}^2$ the control signal, $r(t) \in \mathcal{R}^2$ an external reference signal and $\eta(t) \in \mathcal{R}^2$ white noise input with variance $\sigma_\eta^2 = \mathrm{diag}(\sigma_{\eta_1}^2, \sigma_{\eta_2}^2)$. The system $\mathcal{S}$ is controlled by the stabilizing 2×2 controller K as depicted in Fig. 1. The control signal $u(t)$ can be expressed as a function of $r(t)$ as follows:

$$u(t) = U\left(r(t) - H\eta(t)\right) \tag{2}$$

$$= \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \left(r(t) - H\eta(t)\right) \tag{3}$$

where the input sensitivity function U is $U = KS$, with $S = (I + GK)^{-1}$ being the output sensitivity function and $I \in \mathcal{R}^{2\times 2}$ being the identity matrix.

Under the assumptions A1 and A2, the plant (1) can be divided in the following two distinct subsystems

$$\mathcal{S}_1 : y_1(t) = G_{11}u_1(t) + G_{12}u_2(t) + H_1\eta_1(t) \tag{4}$$

$$\mathcal{S}_2 : y_2(t) = G_{21}u_1(t) + G_{22}u_2(t) + H_2\eta_2(t) \tag{5}$$

Since the identification of these two subsystems can be performed separately, only the identification of the subsystem $\mathcal{S}_1$ will be treated subsequently. By duality, the same results will hold for the subsystem $\mathcal{S}_2$.

Consider the direct closed-loop identification of the subsystem $\mathcal{S}_1$ using the following model structure:

$$\mathcal{M} = \{G_{11}(\alpha), G_{12}(\alpha,\beta), H_1(\alpha,\beta,\gamma), \\ \theta = \begin{pmatrix} \alpha^T & \beta^T & \gamma^T \end{pmatrix}^T \in D_\theta \subset \mathcal{R}^{n_\theta}\Big\} \tag{6}$$

where $G_{11}(\alpha)$, $G_{12}(\alpha,\beta)$ and $H_1(\alpha,\beta,\gamma)$ are rational transfer functions, $\theta \in \mathcal{R}^{n_\theta}$ is the vector of model parameters, and D_θ is a subset of admissible values for θ. It is assumed that the true subsystem $\mathcal{S}_1$ can be described by this model structure for some $\theta_0 = (\alpha_0^T, \beta_0^T, \gamma_0^T)^T \in D_\theta$. Note that this parametrization covers a wide range of model structures. For example, if one considers the ARMAX structure $Ay_1(t) = B_{11}u_1(t) + B_{12}u_2(t) + C_1\eta_1(t)$ then the subvector α contains the parameters of the polynomials A and B_{11}, β contains the parameters of B_{12} and γ contains the parameters of C_1. Also, in this case $H_1 = H_1(\alpha,\gamma)$.

The direct identification method gives consistent estimates of the open-loop plant if the data are sufficiently informative with respect to the adopted model structure and if the true plant, including the noise model, can be described within the chosen parametrization [11]. Here, sufficiently informative data means that the signals $u(t)$ are persistently exciting of appropriate order. In closed loop, this is ensured e.g. by a persistently exciting reference signal or by using a sufficiently complex controller. The reader is referred to [1] for more details. Using a set of input-output data of length N acquired in closed-loop operation, the estimate $\hat{\theta}_N$ is calculated via the prediction error criterion [10]:

$$\hat{\theta}_N = \begin{pmatrix} \hat{\alpha}_N \\ \hat{\beta}_N \\ \hat{\gamma}_N \end{pmatrix} = \arg\min_{\theta\in D_\theta} \frac{1}{N}\sum_{t=1}^{N}[\varepsilon(t,\theta)]^2 \tag{7}$$

where the one-step ahead prediction error $\varepsilon(t,\theta)$ for (4) is defined as:

$$\varepsilon(t,\theta) \triangleq y_1(t) - \hat{y}_1(t|t-1,\theta) \\ = H_1(\theta)^{-1}(y_1(t) - G_{11}(\theta)u_1(t) - G_{12}(\theta)u_2(t)) \tag{8}$$

and the transfer functions are written generically as functions of the parameter vector θ.

Let us assume that the parameter estimates $\hat{\theta}_N$ converge to the true parameter vector θ_0 as N tends to infinity. Then, the parameter error converges to a Gaussian random variable:

$$\sqrt{N}\left(\hat{\theta}_N - \theta_0\right) \xrightarrow{\text{dist}} \mathcal{N}(0, P_\theta) \tag{9}$$

where the covariance matrix P_θ is given by:

$$P_\theta = \sigma_{\eta_1}^2[E\psi(t,\theta_0)\psi^T(t,\theta_0)]^{-1} \triangleq \sigma_{\eta_1(t)}^2 M^{-1} \tag{10}$$

with $\psi(t,\theta) \triangleq \frac{\partial\varepsilon(t,\theta)}{\partial\theta}$ and M the information matrix. Typically, to compute approximate expressions for the covariance of the parameter vector estimates, the asymptotic covariance formulas (9)-(10) are used:

$$cov(\hat{\theta}_N) \approx \frac{1}{N}P_\theta = \frac{\sigma_{\eta_1(t)}^2}{N}M^{-1}. \tag{11}$$

In the next section, an expression for M is derived that shows the dependence of this matrix on the external excitation signals $r_1(t)$ and $r_2(t)$. In turn, this expression will help analyze the dependence of the covariance of the parameter estimate $\hat{\theta}_N$ on $r_1(t)$ and $r_2(t)$.

3 Expression for the information matrix M

Combining (3), (4) and (8), the gradient of the prediction error with respect to the parameters at $\theta = \theta_0$ can be expressed as follows:

$$\begin{aligned}
&\psi(t,\theta_0) = {H_1}^{-1}\left[\left(g_{11}^{\theta}U_{11} + g_{12}^{\theta}U_{21}\right)r_1(t)\right.\\
&+ \left(g_{11}^{\theta}U_{12} + g_{12}^{\theta}U_{22}\right)r_2(t)\\
&+ \left(h_1^{\theta} - g_{11}^{\theta}U_{11}H_1 - g_{12}^{\theta}U_{21}H_1\right)\eta_1(t)\\
&\left. - \left(g_{11}^{\theta}U_{12}H_2 + g_{12}^{\theta}U_{22}H_2\right)\eta_2(t)\right]\\
&\triangleq \Pi_1^r r_1(t) + \Pi_2^r r_2(t) + \Pi_1^{\eta}\eta_1(t) + \Pi_2^{\eta}\eta_2(t) \qquad (12)
\end{aligned}$$

where

$$g_{11}^{\theta} = \left.\frac{\partial G_{11}(\theta)}{\partial\theta}\right|_{\theta=\theta_0};\; g_{12}^{\theta} = \left.\frac{\partial G_{12}(\theta)}{\partial\theta}\right|_{\theta=\theta_0}$$

and

$$h_1^{\theta} = \left.\frac{\partial H_1(\theta)}{\partial\theta}\right|_{\theta=\theta_0}. \qquad (13)$$

The quantities Π_1^r, Π_2^r, Π_1^{η} and Π_2^{η} are introduced in (12) for the sake of simplicity of notation.

From (10)-(13), and using Parseval's theorem and the fact that $r_1(t)$, $r_2(t)$, $\eta_1(t)$ and $\eta_2(t)$ are not correlated, the information matrix can be rewritten as:

$$\begin{aligned}
M &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\Pi_1^r\Pi_1^{r*}\Phi_{r_1} + \Pi_2^r\Pi_2^{r*}\Phi_{r_2} + \right.\\
&\qquad \left. + \Pi_1^{\eta}\Pi_1^{\eta*}\sigma_{\eta_1^2} + \Pi_2^{\eta}\Pi_2^{\eta*}\sigma_{\eta_2^2}\right\}d\omega\\
&\triangleq M(r_1) + M(r_2) + M(\eta_1) + M(\eta_2) \qquad (14)
\end{aligned}$$

where $(.)^*$ is used to denote the complex conjugate transpose.

Consider now the partition of the parameter vector θ in (6). The sensitivities of the transfer functions G_{11}, G_{12} and H_1 with respect to θ read:

$$\begin{aligned}
&g_{11}^{\theta} = \begin{pmatrix} g_{11}^{\alpha} & 0 & 0\end{pmatrix}^T,\quad g_{12}^{\theta} = \begin{pmatrix} g_{12}^{\alpha} & g_{12}^{\beta} & 0\end{pmatrix}^T\\
&\text{and } h_1^{\theta} = \begin{pmatrix} h_1^{\alpha} & h_1^{\beta} & h_1^{\gamma}\end{pmatrix}^T \qquad (15)
\end{aligned}$$

where the definition of the components of g_{11}^{θ}, g_{12}^{θ} and h_1^{θ} is analogous to that in (13). It follows from (12), (13) and (15) that the quantities Π_1^r, Π_2^r, Π_1^{η} and Π_2^{η} reduce to:

$$\begin{aligned}
&\Pi_k^r = {H_1}^{-1}\begin{pmatrix} g_{11}^{\alpha}U_{1k} + g_{12}^{\alpha}U_{2k} & g_{12}^{\beta}U_{2k} & 0\end{pmatrix} \qquad k = 1,2\\
&\Pi_1^{\eta} = H_1^{-1}\begin{pmatrix} h_1^{\alpha} - \sum_{l=1}^{2} g_{1l}^{\alpha}U_{l1}H_1 & h_1^{\beta} - g_{12}^{\beta}U_{21}H_1 & h_1^{\gamma}\end{pmatrix}\\
&\Pi_2^{\eta} = H_1^{-1}\begin{pmatrix} -\sum_{l=1}^{2} g_{1l}^{\alpha}U_{l2}H_2 & -g_{12}^{\beta}U_{22}H_2 & 0\end{pmatrix} \qquad (16)
\end{aligned}$$

Consequently, the contribution of $r_1(t)$, $r_2(t)$ and $\eta_2(t)$ to M, i.e. $M(r_1)+M(r_2)+M(\eta_2)$, can formally be expressed as:

$$M(r,\eta_2) = \begin{pmatrix} M_{11}(r,\eta_2) & M_{12}(r,\eta_2) & 0\\ M_{21}(r,\eta_2) & M_{22}(r,\eta_2) & 0\\ 0 & 0 & 0\end{pmatrix}. \qquad (17)$$

Similar calculations provide expression for $M(\eta_1)$

$$M(\eta_1) = \begin{pmatrix} M_{11}(\eta_1) & M_{12}(\eta_1) & M_{13}(\eta_1)\\ M_{21}(\eta_1) & M_{22}(\eta_1) & M_{23}(\eta_1)\\ M_{31}(\eta_1) & M_{32}(\eta_1) & M_{33}(\eta_1)\end{pmatrix}, \qquad (18)$$

from which the information matrix M becomes:

$$M = \begin{pmatrix} M_{11}(r,\eta) & M_{12}(r,\eta) & M_{13}(\eta_1)\\ M_{21}(r,\eta) & M_{22}(r,\eta) & M_{23}(\eta_1)\\ M_{31}(\eta_1) & M_{32}(\eta_1) & M_{33}(\eta_1)\end{pmatrix}. \qquad (19)$$

In expressions (17)-(19), if a submatrix of M carries the argument r, this means that this particular submatrix depends on the statistics of *both* $r_1(t)$ and $r_2(t)$. The same holds for the argument η. Otherwise, the submatrices of M carry as argument only the particular component on which they depend, for example $M_{33}(\eta_1)$ depends only on the statistics of $\eta_1(t)$.

In the sequel, the effect of the presence or absence of the second external reference signal $r_2(t)$ on the variance of the elements of the parameter vector estimate is analyzed. Note that, for a given model structure, the presence or absence of a particular external reference signal does not change the structure of the information matrix M since, in closed-loop operation, both inputs $u_1(t)$ and $u_2(t)$ are excited by both reference signals.

4 Effect of the second reference signal

Consider the matrix M given in (19). All the possible model structures that correspond to the parametrization (6) can be classified in two groups:

$\mathcal{A}$) The model structures that have no noise model or where the subvector γ of the vector θ is empty (there are no parameters in the noise model H_1 that are independent of the plant model). This group includes the classical FIR, ARX and OE model structures.

$\mathcal{B}$) The model structures whose noise model contains some (not necessarily all) parameters that are independent of the plant model. This group includes the ARMAX and BJ model structures.

In order to study the effect of $r_1(t)$ and $r_2(t)$ on the accuracy of the parameter estimates of α, β and γ, we introduce:

$$C \triangleq M^{-1} = \begin{pmatrix} C_\alpha & C_{\alpha\beta} & C_{\alpha\gamma} \\ C_{\beta\alpha} & C_\beta & C_{\beta\gamma} \\ C_{\gamma\alpha} & C_{\gamma\beta} & C_\gamma \end{pmatrix} \tag{20}$$

where C_α, C_β and C_γ are diagonal sub-matrices of M^{-1} related to the covariances of α_N, β_N and γ_N as $cov(\hat{\alpha}_N) \approx \frac{\sigma_{\eta_1}^2}{N} C_\alpha$, $cov(\hat{\beta}_N) \approx \frac{\sigma_{\eta_1}^2}{N} C_\beta$, and $cov(\hat{\gamma}_N) \approx \frac{\sigma_{\eta_1}^2}{N} C_\gamma$. The off-diagonal submatrices represent the cross-covariances of α_N, β_N and γ_N and are not used in the sequel. Furthermore, the variance of the identified plant models $G_{11}(\hat{\theta}_N)$ and $G_{12}(\hat{\theta}_N)$ and the identified noise model $H_1(\hat{\theta}_N)$ can be calculated using Gauss' approximation formula [10]. For a large number of data N and by using (15) for g_{11}^θ, g_{12}^θ and h_1^θ, one obtains:

$$\begin{aligned} var\left(G_{11}(e^{j\omega}, \hat{\theta}_N)\right) &\approx \frac{\sigma_{\eta_1}^2}{N} \left(g_{11}^\alpha\right)^* C_\alpha \, g_{11}^\alpha \\ var\left(G_{12}(e^{j\omega}, \hat{\theta}_N)\right) &\approx \frac{\sigma_{\eta_1}^2}{N} \left\{ \left(g_{12}^\alpha\right)^* C_\alpha g_{12}^\alpha \right. \\ &\quad \left. + \left(g_{12}^\beta\right)^* C_\beta \, g_{12}^\beta \right\} \\ var\left(H_1(e^{j\omega}, \hat{\theta}_N)\right) &\approx \frac{\sigma_{\eta_1}^2}{N} \left\{ \left(h_1^\alpha\right)^* C_\alpha h_1^\alpha \right. \\ &\quad \left. + \left(h_1^\beta\right)^* C_\beta \, h_1^\beta + \left(h_1^\gamma\right)^* C_\gamma \, h_1^\gamma \right\}. \end{aligned} \tag{21}$$

In the sequel, the analysis is performed separately for the two groups $\mathcal{A}$ and $\mathcal{B}$, and thus the corresponding covariance matrices C and their elements will carry the appropriate subscripts "$\mathcal{A}$" and "$\mathcal{B}$", respectively. Furthermore, the block-diagonal elements C_α, C_β, C_γ, and the matrices C and M will carry the superscript "(1)" when only reference signal $r_1(t)$ is applied and "(2)" when both reference signals are applied simultaneously.

4.1 Main result

For a structure of group $\mathcal{A}$, when the vector γ is empty and both excitation signals $r_1(t)$ and $r_2(t)$ are present, the information matrix M in (19) reduces to

$$M_{\mathcal{A}}^{(2)} = \begin{pmatrix} M_{11}(r,\eta) & M_{12}(r,\eta) \\ M_{21}(r,\eta) & M_{22}(r,\eta) \end{pmatrix}. \tag{22}$$

When $r_1(t)$ alone is excited, the corresponding information matrix reads:

$$M_{\mathcal{A}}^{(1)} = \begin{pmatrix} M_{11}(r_1,\eta) & M_{12}(r_1,\eta) \\ M_{21}(r_1,\eta) & M_{22}(r_1,\eta) \end{pmatrix}. \tag{23}$$

The matrix $M_{\mathcal{A}}^{(2)}$ can be written as:

$$M_{\mathcal{A}}^{(2)} = M_{\mathcal{A}}^{(1)} + \tilde{M}_{\mathcal{A}} \tag{24}$$

with

$$\tilde{M}_{\mathcal{A}} \triangleq \begin{pmatrix} M_{11}(r_2) & M_{12}(r_2) \\ M_{21}(r_2) & M_{22}(r_2) \end{pmatrix}. \tag{25}$$

Consider next a structure of group $\mathcal{B}$. When only $r_1(t)$ is excited, the information matrix $M_{\mathcal{B}}^{(1)}$ has the following form:

$$M_{\mathcal{B}}^{(1)} = \begin{pmatrix} M_{11}(r_1,\eta) & M_{12}(r_1,\eta) & M_{13}(\eta_1) \\ M_{21}(r_1,\eta) & M_{22}(r_1,\eta) & M_{23}(\eta_1) \\ M_{31}(\eta_1) & M_{32}(\eta_1) & M_{33}(\eta_1) \end{pmatrix}. \tag{26}$$

When both $r_1(t)$ and $r_2(t)$ are present, the information matrix $M_{\mathcal{B}}^{(2)}$ is given by expression (19). $M_{\mathcal{B}}^{(1)}$ and $M_{\mathcal{B}}^{(2)}$ are related as follows:

$$M_{\mathcal{B}}^{(2)} = M_{\mathcal{B}}^{(1)} + \tilde{M}_{\mathcal{B}} \tag{27}$$

with

$$\tilde{M}_{\mathcal{B}} = \begin{pmatrix} M_{11}(r_2) & M_{12}(r_2) & 0 \\ M_{21}(r_2) & M_{22}(r_2) & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

Next, the following result can be established.

Theorem 1 *Consider the closed-loop identification of the parameter vector θ of the model structures $\mathcal{A}, \mathcal{B} \subset \mathcal{M}$. Let the excitation signals $r_1(t)$ and $r_2(t)$ be independent and persistently exciting of sufficient order. Then:*
(i) for model structures of group $\mathcal{A}$, the covariance matrices of the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$ decrease by addition of the second excitation $r_2(t)$, i.e.

$$C_{\alpha,\mathcal{A}}^{(2)} < C_{\alpha,\mathcal{A}}^{(1)} \quad \text{and} \quad C_{\beta,\mathcal{A}}^{(2)} < C_{\beta,\mathcal{A}}^{(1)}. \tag{29}$$

(ii) for model structures of group $\mathcal{B}$, the covariance matrices of the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$ cannot increase by addition of the second excitation $r_2(t)$, i.e.

$$C_{\alpha,\mathcal{B}}^{(2)} \leq C_{\alpha,\mathcal{B}}^{(1)} \quad \text{and} \quad C_{\beta,\mathcal{B}}^{(2)} \leq C_{\beta,\mathcal{B}}^{(1)}. \tag{30}$$

In addition, the covariance matrices of $\hat{\gamma}$ are strictly smaller by addition of the second excitation $r_2(t)$, i.e.

$$C_{\gamma,\mathcal{B}}^{(2)} < C_{\gamma,\mathcal{B}}^{(1)}. \tag{31}$$

Proof. Part (i) follows as an immediate consequence of the expression (24) and the fact that $\tilde{M}_{\mathcal{A}} > 0$:

$$\begin{aligned} C_{\mathcal{A}}^{(1)} - C_{\mathcal{A}}^{(2)} &= C_{\mathcal{A}}^{(2)} \left(M_{\mathcal{A}}^{(2)} - M_{\mathcal{A}}^{(1)} \right) C_{\mathcal{A}}^{(1)} \\ &= \left(M_{\mathcal{A}}^{(1)} \tilde{M}_{\mathcal{A}}^{-1} M_{\mathcal{A}}^{(2)} \right)^{-1} \\ &= \left(M_{\mathcal{A}}^{(1)} \tilde{M}_{\mathcal{A}}^{-1} M_{\mathcal{A}}^{(1)} + M_{\mathcal{A}}^{(1)} \right)^{-1} > 0. \end{aligned} \tag{32}$$

Part (ii): the matrix $\tilde{M}_{\mathcal{B}}$ is positive semi-definite (note the non-negative contribution of $r_2(t)$ to the elements of M in (14)). Consequently,

$$\begin{aligned} &C_{\mathcal{B}}^{(1)} - C_{\mathcal{B}}^{(2)} = C_{\mathcal{B}}^{(2)} \tilde{M}_{\mathcal{B}} C_{\mathcal{B}}^{(1)} \\ &= \left(M_{\mathcal{B}}^{(1)} \tilde{M}_{\mathcal{B}}^{-1} M_{\mathcal{B}}^{(1)} + M_{\mathcal{B}}^{(1)} \right)^{-1} \geq 0. \end{aligned} \tag{33}$$

Now, the expression (30) follows from the fact that any principal submatrix of a positive semi-definite matrix is positive semi-definite. Also, it follows from (33) that $C_{\gamma,\mathcal{B}}^{(2)} \leq C_{\gamma,\mathcal{B}}^{(1)}$. However, this inequality can be strengthened as follows. When $r_1(t)$ alone is present, by straightforward calculation of the inverse of the $(3,3)$ element of $M_{\mathcal{B}}^{(1)}$, one obtains:

$$\begin{aligned} C_{\gamma,\mathcal{B}}^{(1)} &= \left(M_{33}(\eta_1) - \left(M_{31}(\eta_1) \; M_{32}(\eta_1) \right) \right. \\ &\times \left. \begin{pmatrix} M_{11}(r_1,\eta) & M_{12}(r_1,\eta) \\ M_{21}(r_1,\eta) & M_{22}(r_1,\eta) \end{pmatrix}^{-1} \times \begin{pmatrix} M_{13}(\eta_1) \\ M_{23}(\eta_1) \end{pmatrix} \right)^{-1}. \end{aligned} \tag{34}$$

Similarly, when both $r_1(t)$ and $r_2(t)$ are applied:

$$\begin{aligned} C_{\gamma,\mathcal{B}}^{(2)} &= \left(M_{33}(\eta_1) - \left(M_{31}(\eta_1) \; M_{32}(\eta_1) \right) \right. \\ &\times \left(\begin{pmatrix} M_{11}(r_1,\eta) & M_{12}(r_1,\eta) \\ M_{21}(r_1,\eta) & M_{22}(r_1,\eta) \end{pmatrix} + \tilde{M}_{\mathcal{A}} \right)^{-1} \\ &\left. \times \left(M_{13}(\eta_1) \; M_{23}(\eta_1) \right)^T \right)^{-1} \end{aligned} \tag{35}$$

where the matrix $\tilde{M}_{\mathcal{A}} > 0$ is given in (25). By comparing expressions (34) and (35), the expression (31) follows immediately. □

Comments on part (i)

1) For a structure of group $\mathcal{A}$, the simultaneous excitation of $r_1(t)$ and $r_2(t)$ reduces the covariance of the estimates of the parameter vectors α and β compared to the case where $r_1(t)$ alone is excited.

2) If the variance of $r_2(t)$ tends to infinity, $M_{\mathcal{A}}^{(2)}$ and $\tilde{M}_{\mathcal{A}}$ also tend to infinity and consequently $C_{\mathcal{A}}^{(2)}$ tends to zero. The intuition is that α and β become perfectly known when the power of $r_2(t)$, and therefore also the power of $u_1(t)$ and $u_2(t)$, tend to infinity.

3) The presence of $r_2(t)$ reduces the variance of all transfer function estimates. If the power of $r_2(t)$ grows unbounded, the variances of $G_{11}(\hat{\theta}_N)$, $G_{12}(\hat{\theta}_N)$ and $H_1(\hat{\theta}_N)$ tend to zero.

Comments on part (ii)

1) For a structure of group $\mathcal{B}$, the presence of a second reference signal $r_2(t)$ does not increase the covariance of the estimates of the parameter vectors α, β and reduces the covariance of the estimates of γ. This statement is valid also for model structures with independent parametrization of the plant and noise models such as BJ.

2) If the energy of $r_2(t)$ grows unbounded, expressions (35) and (25) reveal that $C_{\gamma,\mathcal{B}}^{(2)}$ tends to $M_{33}^{-1}(\eta_1)$. At the same time, using (19), it is straightforward to show that $C_{\alpha,\mathcal{B}}^{(2)}$ and $C_{\beta,\mathcal{B}}^{(2)}$ tend to zero. This can be explained as follows: when $r_2(t)$ goes to infinity, $u_1(t)$ and $u_2(t)$ also go to infinity, and the parameters α and β become perfectly known; then, the estimation of γ corresponds to the identification of the unknown parameters of the Moving Average (MA) model $y(t) = H_1(q^{-1})\eta_1(t)$ (note that some parameters of H_1 might already be known as they are part of α and/or β).

3) The excitation $r_2(t)$ never impairs, and in most cases improves, the accuracy of all transfer function estimates: see (30), (31) and (21). When the power of $r_2(t)$ goes to infinity, the variances of $G_{11}(\hat{\theta}_N)$ and $G_{12}(\hat{\theta}_N)$ tend to zero.

4) Even when the plant and noise models are parameterized independently, there is a strong correlation between the parameter estimates due to closed-loop operation. A smaller variance of the plant parameter estimates implies a smaller variance of the parameter estimates associated with the noise model and vice versa.

It follows from Theorem 1 that, regardless of the parametrization, the addition of the external signal $r_2(t)$ never increases (and typically reduces) the variance of the parameter estimates obtained via direct closed-loop identification. This conclusion holds for any controller K that guarantees informative experiments in closed loop. Furthermore, it follows from (14) that, for direct closed-loop identification and for both groups $\mathcal{A}$ and $\mathcal{B}$, the contribution of the noise is never detrimental to the precision of the parameter estimates.

5 Extension to general multivariable systems satisfying assumptions A1 and A2

In this section, the analysis presented in Sections 3 and 4 for the case of two inputs and two outputs is generalized to multivariable systems with arbitrary numbers of inputs and outputs that satisfy the assumptions $A1$ and $A2$. Let us consider the m-input 1-output subsystem $\mathcal{S}_{m1}$ of an m-input

p-output system $\mathcal{S}_{mp}$:

$$\mathcal{S}_{m1}: y_1(t) = G_{11}u_1(t) + G_{12}u_2(t) + \cdots + G_{1m}u_m(t) + H_1\eta_1(t) \quad (36)$$

and suppose that $\mathcal{S}_{m1}$ is to be identified using the following model structure:

$$\mathcal{M}_{m1} = \{G_{11}(\alpha), G_{12}(\alpha,\beta), \cdots, G_{1m}(\alpha,\beta,\cdots,\delta), H_1(\alpha,\beta,\cdots,\delta,\gamma),\ \theta = \begin{pmatrix} \alpha^T & \beta^T & \cdots & \delta^T & \gamma^T \end{pmatrix}^T\} \quad (37)$$

with $\theta \in D_\theta \subset \mathcal{R}^{n_\theta}$. Here $G_{11}(\alpha)$, $G_{12}(\alpha,\beta)$,$\cdots$, $G_{1m}(\alpha,\beta,\cdots,\delta)$ and $H_1(\alpha,\beta,\cdots,\delta,\gamma)$ are rational transfer functions. Observe that the partition of θ in the subvectors $\alpha,\beta,...$ will be different for each of the outputs. It is assumed that $\mathcal{S}_{mp}$ is controlled by the $m \times p$ controller K. The control signal $u(t) \in \mathcal{R}^m$ can be expressed as in (2) with $r(t) \in \mathcal{R}^p$ and $\eta(t) \in \mathcal{R}^p$. The one-step ahead prediction error $\varepsilon_{m1}(t,\theta)$ for (36) reads:

$$\varepsilon_{m1}(t,\theta) = H_1(\theta)^{-1}\Big(y_1(t) - \sum_{k=1}^{m} G_{1k}u_k(t)\Big) \quad (38)$$

From (3), (36) and (38) the gradient of ε_{m1} with respect to the parameters at $\theta = \theta_0$ can be expressed as:

$$\begin{aligned}\psi_{m1}(t,\theta_0) &= {H_1}^{-1}\left[\left(\sum_{k=1}^{m} g_{1k}^{\theta}U_{k1}\right)r_1(t)\right.\\ &+ \left(\sum_{k=1}^{m} g_{1k}^{\theta}U_{k2}\right)r_2(t) + \cdots + \left(\sum_{k=1}^{m} g_{1k}^{\theta}U_{km}\right)r_m(t)\\ &+ \left(h_1^{\theta} - \sum_{k=1}^{m} g_{1k}^{\theta}U_{k1}H_1\right)\eta_1(t) - \left(\sum_{k=1}^{m} g_{1k}^{\theta}U_{k2}H_2\right)\eta_2(t)\\ &- \cdots - \left.\left(\sum_{k=1}^{m} g_{1k}^{\theta}U_{km}H_m\right)\eta_m(t)\right]\\ &\triangleq \sum_{k=1}^{m}\Pi_k^r r_k(t) + \sum_{k=1}^{m}\Pi_k^{\eta}\eta_k(t) \end{aligned} \quad (39)$$

where the sensitivities g_{1k}^{θ}, $k = 1,m$ are defined analogously as in (13). Recall that $\theta_0 = [\alpha_0^T, \beta_0^T, \ldots, \delta_0^T, \gamma_0^T]^T$ represents the values of the model parameters that exactly describe the true subsystem $\mathcal{S}_{m1}$. A calculus similar to the one that led to (14) and (16) yields:

$$\begin{aligned} M_m &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\sum_{k=1}^{m}\Pi_k^r\Pi_k^{r*}\Phi_{r_k} + \sum_{k=1}^{m}\Pi_k^{\eta}\Pi_k^{\eta *}\sigma_{\eta_k^2}\right\}d\omega\\ &\triangleq \sum_{k=1}^{m} M_m(r_k) + \sum_{k=1}^{m} M_m(\eta_k) \end{aligned} \quad (40)$$

and

$$\begin{aligned}\Pi_k^r &= H_1^{-1}\Big(\sum_{l=1}^{m} g_{1l}^{\alpha}U_{lk} \quad \sum_{l=2}^{m} g_{1l}^{\beta}U_{lk} \quad \cdots \quad g_{1m}^{\delta}U_{mk} \quad 0\Big)\\ \Pi_1^{\eta} &= H_1^{-1}\Big(h_1^{\alpha} - \sum_{l=1}^{m} g_{1l}^{\alpha}U_{l1}H_1 \quad h_1^{\beta} - \sum_{l=2}^{m} g_{1l}^{\beta}U_{l1}H_1 \quad \cdots \quad h_1^{\delta} - g_{1m}^{\delta}U_{m1}H_1 \quad h_1^{\gamma}\Big)\\ \Pi_k^{\eta} &= H_1^{-1}\Big(-\sum_{l=1}^{m} g_{1l}^{\alpha}U_{lk}H_k \quad -\sum_{l=2}^{m} g_{1l}^{\beta}U_{lk}H_k \quad \cdots \quad -g_{1m}^{\delta}U_{mk}H_k \quad 0\Big)\end{aligned} \quad (41)$$

where, in the equation for Π_k^r, the index k ranges from 1 to m, while in that for Π_k^{η}, k ranges from 2 to m. Using (41) in (40) gives the information matrix M_m in the following form:

$$M_m = \begin{pmatrix} M_{11}(r,\eta) & \cdots & M_{1m}(r,\eta) & M_{1(m+1)}(\eta_1)\\ \vdots & \ddots & \vdots & \\ M_{m1}(r,\eta) & \cdots & M_{mm}(r,\eta) & \vdots\\ M_{(m+1)1}(\eta_1) & \cdots & & M_{(m+1)(m+1)}(\eta_1)\end{pmatrix}$$

The contribution of a component of $r(t)$, say $r_1(t)$, to M_m reads

$$M_m(r_1) = \begin{pmatrix} M_{11}(r_1) & \cdots & M_{1m}(r_1) & 0\\ \vdots & \ddots & \vdots & \\ M_{m1}(r_1) & \cdots & M_{mm}(r_1) & \vdots\\ 0 & & \cdots & 0\end{pmatrix}.$$

Observe that M_m and $M_m(r_1)$ have exactly the same structure as M in (19) and $M(r,\eta_2)$ in (17), respectively. Hence, the results of Theorem 1 apply mutatis mutandis to the general multivariable structures satisfying A1 and A2.

6 Simulation Results

In order to illustrate the analytical results for both groups $\mathcal{A}$ and $\mathcal{B}$, two 2×2 simulated plants are considered. Both plants are controlled by the same 2×2 controller:

$$K(q^{-1}) = \frac{0.8(1-0.3q^{-1})}{(1-0.4q^{-1})}\begin{pmatrix} 1 & 0.25\\ 0.25 & -1\end{pmatrix} \quad (42)$$

The controller is designed so as to stabilize both plants without other performance consideration [1].

[1] Note that the direct closed-loop identification approach can be used for identification of unstable plants provided the predictor

A Monte-Carlo simulation is performed to compare the case where the reference signal $r_1(t)$ alone is excited with the case where the two reference signals are applied simultaneously. The reference signals $r_1(t)$ and $r_2(t)$ are PRBS generated by a 10-bit shift register with data length $N = 1023$ and standard deviations $\sigma_{r_1} = 0.4$ and $\sigma_{r_2} = 1$. The disturbance signals $\eta_1(t)$ and $\eta_2(t)$ are white noises with standard deviations $\sigma_{\eta_1} = \sigma_{\eta_2} = 0.4$. The signals $r_1(t)$, $r_2(t)$, $\eta_1(t)$ and $\eta_2(t)$ are mutually independent. This way, the assumptions of Theorem 1 are verified.

Simulation 1: Group $\mathcal{A}$
The following FIR plant is considered:

$$y_1(t) = B_{11}u_1(t) + B_{12}u_2(t) + \eta_1(t)$$
$$y_2(t) = B_{21}u_1(t) + B_{22}u_2(t) + \eta_2(t)$$

with $B_{11} = 0.5q^{-1} + 0.15q^{-2}$, $B_{12} = 0.26q^{-1} + 1.6q^{-2}$, $B_{21} = 0.06q^{-1} + 0.45q^{-2}$ and $B_{22} = 0.7q^{-1} + 0.2625q^{-2}$. The variance of the parameter estimates is computed for both cases of excitation. In these simulated examples, we compute the parameter estimates corresponding to both outputs of the system; thus $\theta = (b^1_{11}, b^2_{11}, b^1_{12}, b^2_{12}, b^1_{21}, b^2_{21}, b^1_{22}, b^2_{22})^T$. When $r_1(t)$ alone is excited, the asymptotic variances of the elements of θ computed by 1000 Monte-Carlo runs are:

$$var(\hat{\theta}^{(1)}) = 10^{-4}(\,3.546 \;\; 2.777 \;\; 9.115 \;\; 12.49$$
$$3.712 \;\; 2.494 \;\; 8.736 \;\; 12.11)$$

The asymptotic variances of θ computed when both $r_1(t)$ and $r_2(t)$ are excited simultaneously are:

$$var(\hat{\theta}^{(2)}) = 10^{-4}(\,1.103 \;\; 0.749 \;\; 1.549 \;\; 2.621$$
$$1.202 \;\; 0.778 \;\; 1.619 \;\; 2.585)$$

The variances are reduced by addition of the second excitation, which is due to the additional energy in both $u_1(t)$ and $u_2(t)$ caused by the extra signal $r_2(t)$. Note that, in the case of open-loop identification of FIR models, the asymptotic accuracy of the estimates of the b^j_{11} coefficients is totally independent of the presence of $u_2(t)$.

Simulation 2: Group $\mathcal{B}$
The following ARMAX structure is considered:

$$A_1y_1(t) = B_{11}u_1(t) + B_{12}u_2(t) + C_1\eta_1(t)$$
$$A_2y_2(t) = B_{21}u_1(t) + B_{22}u_2(t) + C_2\eta_2(t)$$

with $A_1 = 1 - 0.45q^{-1}$, $B_{11} = q^{-1}$, $B_{12} = 0.6q^{-1}$, $C_1 = 1 - 0.8q^{-1}$, $A_2 = 1 - 0.55q^{-1}$, $B_{21} = 0.75q^{-1}$, $B_{22} = 0.8q^{-1}$ and $C_2 = 1 - 0.7q^{-1}$. The parameter vector $\theta = (a_1, a_2, b^1_{11}, b^1_{12}, b^1_{21}, b^1_{22}, c_1, c_2)^T$ is considered. The Monte-Carlo simulations provide the following variances:

$$var(\hat{\theta}^{(1)}) = 10^{-3}(0.653 \;\; 32.19 \;\; 0.742 \;\; 0.191$$
$$2.810 \;\; 32.41 \;\; 0.518 \;\; 1.23).$$

and the closed-loop system are stable [10].

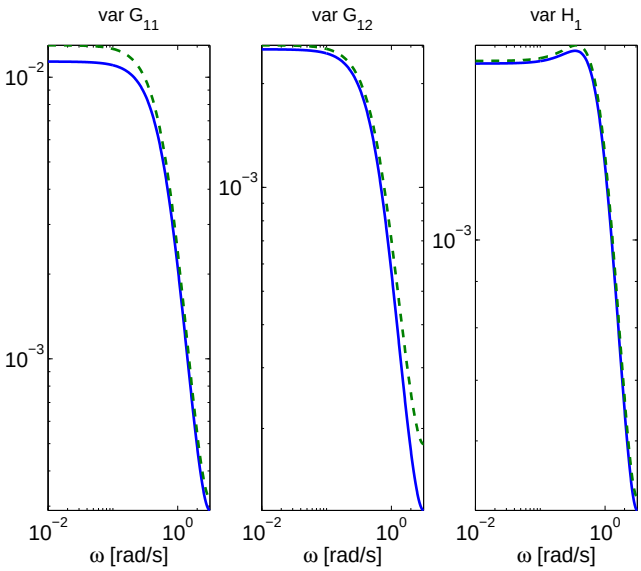

Fig. 2. Variance of the transfer function estimates: $G_{11}(q^{-1})$ (left), $G_{12}(q^{-1})$ (middle) and $H_1(q^{-1})$ (right), for the ARMAX model with 2 reference inputs (solid line) and one input (dashed line).

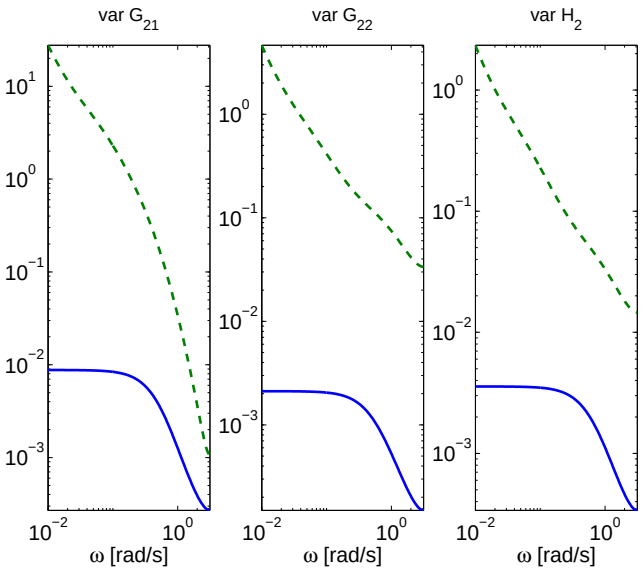

Fig. 3. Variance of the transfer function estimates: $G_{21}(q^{-1})$ (left), $G_{22}(q^{-1})$ (middle) and $H_2(q^{-1})$ (right), for the ARMAX model with 2 reference inputs (solid line) and one input (dashed line).

$$var(\hat{\theta}^{(2)}) = 10^{-3}(0.551 \;\; 0.185 \;\; 0.686 \;\; 0.134$$
$$0.751 \;\; 0.204 \;\; 0.478 \;\; 0.726).$$

As expected, the presence of $r_2(t)$ improves the precision of all estimated coefficients. The corresponding variances of the transfer function estimates $G_{11}(q^{-1})$, $G_{12}(q^{-1})$, $G_{21}(q^{-1})$, $G_{22}(q^{-1})$, $H_1(q^{-1})$ and $H_2(q^{-1})$ are computed at 500 frequency points for the two cases of excitation and compared in Figs. 2 and 3. As expected, the accuracy of the six transfer function estimates is improved.

7 Conclusions

In this contribution, the effect of not exciting some of the references is quantified for the case of closed-loop identification. A variance analysis of the identified parameters has been performed for two situations: (i) when a reference input is not excited; (ii) when all reference signals are excited simultaneously. It follows from this analysis that, regardless of the parametrization, the non-excitation of one or several references almost always impairs the quality of the parameter estimates. This result might not surprise the reader, especially if one considers that an additional reference signal increases the energy of all inputs, which in turn results in improved accuracy of the plant model parameters. However, before this work was undertaken, it was not clear: (i) how an additional reference signal affects the parameters of the noise model; (ii) whether the improved accuracy of the plant model parameters occurs for any (arbitrary) model structure? Observe that the result presented here contrasts with the situation of open-loop identification, where an additional input improves the accuracy of the estimated parameters only for the model structures that have common parameters between the different transfer functions [7].

The assumption of having the true plant and noise models in the model set $\mathcal{S}_1 \in \mathcal{M}$ is rarely met in practice. In particular, it is well known in the literature concerned with direct closed-loop identification [5,9,10] that a noise model not covering the true noise characteristics introduces a bias. It is shown in [10], Section 13.4, expression (13.53), that the bias term is proportional to Φ_u^η/Φ_u, where Φ_u denotes the input signal spectrum and Φ_u^η is the contribution of the noise $\eta(t)$ to Φ_u. It is easy to see that, when all references are excited simultaneously, the input signal spectrum Φ_u is larger than the one prevailing when one or more reference signals are non-excited, whereas Φ_u^η remains constant in both cases. Hence, the quantity Φ_u^η/Φ_u is reduced in the case of simultaneous excitation. In other words, when $\mathcal{S}_1 \notin \mathcal{M}$, simultaneous excitation of all references is to be preferred to the other excitation scenarios both in terms of the bias error and the variance error.

References

[1] A. S. Bazanella, M. Gevers, and L. Mišković. Closed-loop identification of MIMO systems: a new look at identifiability and experiment design. In *CD-ROM Proc. of European Control Conference*, pages 5694–5699, Kos, Greece, July 2007.

[2] X. Bombois, M. Gevers, and G. Scorletti. Open-loop versus closed-loop identification of Box-Jenkins models: A new variance analysis. In *CD-ROM Proc. of CDC-ECC 2005*, pages 3117–3122, Sevilla, Spain, December 2005.

[3] X. Bombois, G. Scorletti, M. Gevers, P. M. J. Van den Hof, and R. Hildebrand. Least costly identification experiment for control. *Automatica*, 42:1651–1662, 2006.

[4] B. S. Dayal and J. F. MacGregor. Multi-output process identification. *Journal of Process Control*, 7(4):269–282, August 1997.

[5] U. Forssell and L. Ljung. Closed-loop identification revisited. *Automatica*, 35(7):1215–1241, July 1999.

[6] M. Gevers, A.S. Bazanella, and L. Mišković. Identifiability and informative experiments in open and closed-loop identification. In *'Modeling, Estimation and Control: Festschrift in honor of Giorgio Picci on the occasion of his sixty-fifth Birthday' A. Chiuso, A. Ferrante and S. Pinzoni Eds.*, Lecture Notes in Control and Information Sciences, pages 167–187. Springer Verlag, 2007.

[7] M. Gevers, L. Mišković, D. Bonvin, and A. Karimi. Identification of multi-input systems: Variance analysis and input design issues. *Automatica*, 42(4):559–572, April 2006.

[8] H. Jansson and H. Hjalmarsson. Optimal experiment design in closed loop. In *16th IFAC World Congress on Automatic Control, paper 04528*, July 2005.

[9] A. Karimi and I. D. Landau. Comparison of the closed loop identification methods in terms of the bias distribution. *Systems and Control Letters*, 34(4):159–167, July 1998.

[10] L. Ljung. *System Identification - Theory for the User*. Prentice Hall, Upper Saddle River, NJ, II edition, 1999.

[11] L. Ljung and U. Forssell. An alternative motivation for the indirect approach to closed-loop identification. *IEEE Transactions on Automatic Control*, AC-44(11):2206–2209, November 1999.