Virtual Reference Feedback Tuning for Non Minimum Phase Plants

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Abstract

Model Reference control design methods fail when the plant has one or more non minimum phase zeros that are not included in the reference model, leading possibly to an unstable closed loop. This is a very serious problem for data-based control design methods, where the plant is typically unknown. In this paper, we extend the Virtual Reference Feedback Tuning method to non minimum phase plants. This extension is based on the idea proposed in [10] for Iterative Feedback Tuning. We present a very simple two-step procedure that can cope with the situation where the unknown plant may or may not have non minimum phase zeros.

Key words: Model reference control; data-based control; VRFT; non-minimum phase systems; flexible reference model.

1 Introduction

When Model Reference control design is used, it is important that the possible Non Minimum Phase (NMP) zeros of the plant to be controlled be included in the reference model. Failure to do so may even result in an unstable closed loop system. Thus, a good knowledge of the NMP zeros of the plant is essential.

In the last 15 years, a number of data-based control design methods have been proposed [6,5,3,8], where a parametrized controller structure is chosen a priori, and the controller tuning is based directly on input and output data collected on the plant without the use of a model of this plant. These data-based controller tuning methods will fail if the plant contains one or more NMP zeros that have not been included in the Reference Model. It is important to notice that this drawback is due to the reference model formulation and not to the data-based nature of the design approach. The only way to avoid it in reference model design is by including the non minimum phase zero in the reference model. Doing this a priori, when choosing the reference model, requires the exact knowledge of this zero, which in turn may be a rash hypothesis even for model-based design.

To overcome this difficulty in the case of the Iterative Feedback Tuning (IFT) method [5], a procedure was proposed in [10]. It involves an extension of the model reference formulation by adding to the classical H_2 criterion an additional term that penalizes the mean square error between the achieved output of the closed loop system and a *flexible reference model* whose poles are the same as those of the desired reference model, but whose zeros are entirely free. Actually, the numerator polynomial of this flexible reference model has all its parameters free. The global criterion is a weighted version of the standard criterion and of this *flexible criterion*; it contains the controller parameters and the coefficients of the flexible reference model. This global reference model is then minimized jointly with respect to these two sets of parameters. A convergence analysis for this modified IFT criterion is quite difficult; it was performed in [10] only for the case where the controller is tuned for step changes in the reference. However, simulations have shown that this modified scheme performs remarkably well: in the case where the plant has NMP zeros, the simulations show that the parameters of the flexible reference model

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actually converge to values that reproduce the NMP zeros of the plant.

The objective of the work reported in this paper was to examine whether a similar idea could be developed for the VRFT method [3]. The application of the flexible reference model idea to VRFT seems more difficult, because in the VRFT scheme the criterion that is minimized is different from the desired criterion; it can be made to approximate the desired criterion only by a proper prefiltering of the data. However, we show in this paper that the idea of a flexible reference model can in fact be adapted to the VRFT method of controller tuning. Just like in the case of IFT with a flexible criterion, we introduce a flexible VRFT criterion that contains a reference model whose numerator is a polynomial parametrized with a set of free parameters. Our result is thus an extension of VRFT to the case of non minimum phase plants, inspired by a similar solution proposed for IFT.

We first show that the expression appearing in this flexible H_2 criterion is a bilinear function of the parameters of the numerator of the flexible reference model and of the controller parameters. This means that the minimum of this flexible part of the criterion can be obtained using an appropriate iterative least squares procedure.

We have applied this flexible VRFT scheme to a number of simulation examples reflecting the two main situations: one where the unknown plant contains NMP zeros, one where it does not. This leads us to propose a two-step procedure that applies to these two situations. In the first step, only the flexible part of the criterion is minimized with respect to the numerator coefficients of the reference model and the controller parameters. We show that at the minimum of this flexible criterion the numerator polynomial contains the NMP zeros of the plant. Thus, the user is immediately alerted to the existence of NMP zeros of the plant, if any, and more importantly of their precise locations. The second step then proceeds as follows: (i) if the first step shows that the system contains NMP zeros, the desired reference model is modified so as to contain these NMP zeros, while the poles are kept at their desired values; (ii) if the flexible reference model has converged to a value that does not exhibit NMP zeros, the initially chosen reference model is kept. Then, the standard VRFT is used to find the controller parameters.

The paper is organized as follows. Definitions and the problem formulation are presented in Section 2. Section 3 reviews the standard VRFT method and the proposed flexible criterion for VRFT is then presented in Section 4, while Section 5 shows some examples of the application of the proposed method. In the end, we present some conclusions.

2 Preliminaries

2.1 Definitions

Consider a linear time-invariant discrete-time single-input-single-output process

$$y(t) = G_0(z)u(t) + v(t), \tag{1}$$

where z is the forward-shift operator, $G_0(z)$ is the process transfer function, $u(t)$ is the control input and $v(t)$ is the process noise. The noise is a quasi-stationary process which can be written as $v(t) = H_0(z)e(t)$ where $e(t)$ is white noise with variance σ_e^2. Both transfer functions, $G_0(z)$ and $H_0(z)$, are rational and causal and it is assumed that $G_0(z)$ has a nonzero static gain.

This process is controlled by a linear time-invariant controller which belongs to a given - user specified - class $\mathcal{C}$ of linear transfer functions. This class is such that $C(z)G_0(z)$ has positive relative degree for all $C(z) \in \mathcal{C}$; equivalently, the closed loop is not delay-free. The controller is parametrized by a parameter vector $\rho \in \mathbb{R}^n$, so that the control action $u(t)$ can be written as

$$u(t) = C(z,\rho)(r(t) - y(t)), \tag{2}$$

where $r(t)$ is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is

$$\bar{E}\left[r(t)e(s)\right] = 0 \;\forall t, s$$

where $\bar{E}[\cdot]$ is defined as

$$\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E[f(t)]$$

with $E[\cdot]$ denoting expectation [11]. The system (1)-(2) in closed loop becomes

$$\begin{aligned} y(t,\rho) &= T(z,\rho)r(t) + S(z,\rho)v(t) \\ T(z,\rho) &= \frac{C(z,\rho)G_0(z)}{1 + C(z,\rho)G_0(z)} = C(z,\rho)G_0(z)S(z,\rho) \end{aligned}$$

where we have now made the dependence on the controller parameter vector ρ explicit in the output signal $y(t,\rho)$. It is also assumed that the controller has a linear parametrization, i.e. it belongs to a controller class $\mathcal{C}$ as specified below

$$\mathcal{C} = \{C(z,\rho) = \rho^T \beta(z), \rho \in \Re^n\}, \tag{3}$$

where $\beta(z)$ is a n-column vector of fixed causal rational functions, whose poles are strictly inside the unit circle except for possible poles at $z = 1$.

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Some of the most common controller structures are indeed linearly parametrized. A PID with fixed derivative pole, for example, can be parameterized as:

$$C(z,\rho) = [k_p \; k_i \; k_d] \left[1 \; \frac{z}{z-1} \; \frac{z-1}{z} \right]^T .$$

2.2 Problem Statement

Model reference control design consists of specifying a "desired" closed loop transfer function $\bar{M}(z)$, which is known as the *reference model*, and then finding the controller that makes the closed loop behavior as close as possible to the desired one. In other words, the controller is designed by solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \tag{4}$$

$$J^{MR}(\rho) \triangleq \bar{E}\left[\left((T(z,\rho) - \bar{M}(z)) r(t) \right)^2 \right]. \tag{5}$$

The model matching controller $C_d^{MR}(z)$ is the one that allows the closed loop system to match exactly $\bar{M}(z)$ and is given by

$$C_d^{MR}(z) = \frac{\bar{M}(z)}{G_0(z)(1-\bar{M}(z))}. \tag{6}$$

Should the model matching controller $C_d^{MR}(z)$ be put in the control loop, the objective function would evaluate to zero. However, this model matching controller may not be causal, or may produce an internally unstable closed loop system through the cancellation of the NMP zero, both of which would be disastrous. Thus, the choice of the reference model $\bar{M}(z)$ must be made under some constraints to prevent these disasters; these constraints are directly verified in (6). In order for the model matching controller to be causal, the relative degree of the reference model can not be smaller than that of the plant $G_0(z)$. To prevent the unstable pole-zero cancellation, the reference model must have the same unstable zeros as the plant. So, the choice of the reference model requires the a priori knowledge of an overbound for the relative degree of the process and the exact location of the unstable zeros, if any.

When the process model is known, the reference model can be chosen to satisfy these constraints, and the model reference problem can be solved using a Linear Quadratic Regulator (LQR) and is then called Model Matching by LQR [4]. If in addition the model matching controller belongs to the controller class $\mathcal{C}$ in (3) and the process model is known, then the reference model design is trivial: it suffices to apply equation (6).

On the other hand, data-based control methods and direct adaptive control methods address the minimization of the criterion (5) directly from data collected from the system, without deriving a process model from these data [5,7,3,8]. Then it is not always possible to assume a priori knowledge of the existence of NMP zeros, and certainly not their exact positions in case they do exist, and thus the choice of an appropriate reference model is compromised. Thus most data-based design methods tend to fail when applied to non minimum phase plants.

In this paper we propose a solution to this problem, extending the VRFT [3] method to cope with NMP plants. We start by presenting the standard VRFT method.

3 The standard VRFT method

Different data-based methods exist that solve the optimization (4). Mostly, these are iterative methods which consist of estimating local quantities (first and second derivatives) of the cost function $J^{MR}(\rho)$ and applying standard optimization methods - steepest descent, quasi-Newton. A limiting factor in the application of these methods is the fact that the function being minimized is not convex, which can result - and often does - in convergence to a local minimum. This convergence problem has been studied in depth in [1], where conditions for a successful convergence to the global optimum have been derived, and manoeuvres to achieve such conditions in practice have been proposed.

The Virtual Reference Feedback Tuning Method (VRFT) presents an alternative to these iterative methods, which solves this local minima issue and at the same time requires no iterations - it is a "one shot" method, as the authors put it. VRFT consists of minimizing a different objective function, whose minimum is known to be the same as the desired one under certain ideal conditions. This new function is quadratic, and thus easy to minimize.

The VRFT method can be described as follows. Through either an open loop or a closed loop experiment, input data $u(t)$ and output data $y(t)$ are collected on the actual process. Given the measured $y(t)$, we define the *virtual reference* signal $\bar{r}(t)$:

$$\bar{M}(z)\bar{r}(t) = y(t).$$

This signal is such that, if the system were in closed loop with the model matching controller, and we apply $\bar{r}(t)$ to the reference, the resulting experiment would result in the data $y(t)$ that have been collected in the output. Should the data have been collected like this, the reference tracking error would have been given by

$$\bar{e}(t) = \bar{r}(t) - y(t).$$

This $\bar{e}(t)$ is the signal that would have fed the model matching controller in this fake experiment. We thus

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have input and output data ($\bar{e}(t)$ and $u(t)$ respectively) of the model matching controller $C_d^{MR}(z)$ and we can use these data to identify it. The identification is performed by minimizing the following criterion

$$J^{VR}(\rho) = \bar{E}\left[u(t) - C(z,\rho)\bar{e}(t)\right]^2 \\ = \bar{E}\left[u(t) - \left(C(z,\rho)\frac{1-\bar{M}(z)}{\bar{M}(z)}\right)y(t)\right]^2 \quad (7)$$

Since $C(z,\rho)$ is linear in ρ, the criterion in (7) is a quadratic function of the parameter vector ρ and hence the solution of the optimization problem can be obtained through the application of least squares, that is, by the following calculation:

$$\hat{\rho} = \bar{E}\left[\varphi(t)\varphi(t)^T\right]^{-1}\bar{E}\left[\varphi(t)u(t)\right] \quad (8)$$

where $\varphi(t) = \beta(z)\bar{e}(t)$. This is the key advantage of the VRFT criterion (7) over the MR criterion (5), and hence of VRFT over other data-based methods, like IFT or CbT, which are iterative.

Consider that the model matching controller (6) belongs to the controller set considered, that is, that the following assumption is satisfied.

Assumption 1 $C_d^{MR}(z) \in \mathcal{C}$ *or, equivalently,*

$$\exists\rho_d : C(z,\rho_d) = \rho_d^T\beta(z) = C_d^{MR}(z)$$

Under Assumption 1 the parameter value ρ_d is the global minimum of both criteria, (5) and (7), since both evaluate to zero at $\rho = \rho_d$. It is also easy to demonstrate that this global minimum is unique, for both criteria, provided that the corresponding regression vector is persistently exciting [1,3]. When Assumption 1 does not hold, the minima of the two criteria are not the same, but they can be made close to each other by proper filtering of the signals $u(t)$ and $\bar{e}(t)$. The appropriate filter is $L(z)$ defined by [3]:

$$|L(z)|^2 = |1-\bar{M}(z)|^2|\bar{M}(z)|^2\frac{\Phi_r}{\Phi_u}, \quad (9)$$

where Φ_u is the power spectrum of the signal $u(t)$ and Φ_r is the power spectrum of $r(t)$. In this case, the parameter vector ρ is estimated by

$$\hat{\rho} = \bar{E}\left[\varphi_L(t)\varphi_L(t)^T\right]^{-1}\bar{E}\left[\varphi_L(t)u_L(t)\right] \quad (10)$$

where $\varphi_L(t) = L(z)\varphi(t)$ and $u_L(t) = L(z)u(t)$.

The formulation of the VRFT method is based on signals obtained from a plant which is not affected by noise. In the presence of noise, an instrumental variable can be used instead of the standard least squares solution, in which case equation (8) is replaced by

$$\hat{\rho} = \bar{E}\left[\zeta(t)\varphi_L(t)^T\right]^{-1}\bar{E}\left[\zeta(t)u_L(t)\right] \quad (11)$$

where $\zeta(t)$ is a n-vector of instrumental variables; see [3] for details.

Now, being a model reference design in which no previous knowledge on the process is assumed, the VRFT is bound to fail for NMP plants, for the reasons discussed in Section 2. In the next section we present a modification of the optimization criterion of VRFT in order to successfully cope with NMP plants.

4 Flexible criterion for VRFT

A solution for the NMP limitation of the reference model formulation has been proposed in [10] for the IFT method. That proposal consists of using a reference model with free parameters in the numerator, so that its zeros are free. In general, such a reference model can be described as

$$M(z,\eta) = \eta^T F(z), \quad (12)$$

where $\eta \in \Re^q$ is the parameter vector and $F(z)$ is a q-vector of rational functions. This defines a class of reference models, instead of a single reference model. By leaving the numerator of the reference model to be determined, the optimization can "find" the zeros of the process, and particularly the NMP ones. It is worth noticing that in this formulation we assign the closed loop transfer function only partially. Specifically, the denominator is assigned and, if the number of free parameters q equals the order of the numerator, then the numerator is entirely free and the formulation becomes conceptually equivalent to a pole assignment design.

By using this idea in the VRFT criterion we get the following optimization criterion [2]

$$J_0^{VR}(\eta,\rho) = \bar{E}\left\{L(z)\left[u(t) - \left(\frac{1-M(z,\eta)}{M(z,\eta)}C(z,\rho)\right)y(t)\right]\right\}^2 \\ = \bar{E}\left[u_L(t) - C(z,\rho)\bar{e}_L(\eta,t)\right]^2 \quad (13)$$

where $\bar{e}_L(\eta,t) = L(z)\bar{e}(\eta,t)$. In the standard VRFT method, the model matching hypothesis - Assumption 1 - is crucial. Our analysis for this new design criterion requires a similar hypothesis. Assumption 2 below states that there exists, within the class of reference models considered, one reference model for which model matching is possible.

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Assumption 2 *There exists a pair* (η^*, ρ^*) *such that* $J_0^{VR}(\eta^*, \rho^*) = 0$*, or, equivalently,*

$$\exists \eta^*, \rho^* : \; C(z, \rho^*) = \frac{M(z, \eta^*)}{[1 - M(z, \eta^*)]G_0(z)}. \qquad (14)$$

Under Assumption 2, $\min_{\eta,\rho} J_0^{VR}(\eta, \rho) = 0$ and $\arg\min_{\eta,\rho} J_0^{VR}(\eta, \rho) = (\eta^*, \rho^*)$. It thus follows that

$$\arg\min_{\eta,\rho} J_0^{VR}(\eta, \rho) = \arg \min_{\substack{\eta,\rho \\ (\eta,\rho)\neq\{0,\,0\}}} \tilde{J}_0^{VR}(\eta, \rho) \qquad (15)$$

where

$$\tilde{J}_0^{VR}(\eta, \rho) = \bar{E}\left[LM(\eta)u(t) - LC(\rho)(1 - M(\eta))y(t)\right]^2 \qquad (16)$$

We have omitted the dependence on z in (16) for readability.

Given the linear parametrization of both the controller and the reference model, $\tilde{J}_0^{VR}(0, 0) = 0$. Thus, the multiplication by $M(z, \eta)$ has created an additional - and undesired - global minimum at the origin. This is the reason why the right hand side of (15) is subjected to a constraint whose purpose is to exclude this undesired minimum $(\eta, \rho) = \{0, 0\}$. In most control applications, a natural constraint exists which automatically does that: the reference model must have steady-state gain $M(\eta, 1) = 1$.

We now show that the minimization of (15) yields a minimum (η^*, ρ^*) such that $M(z, \eta^*)$ contains all NMP zeros of $G_0(z)$.

Theorem 1 *Let* $B(z)$ *be the least common denominator of the elements of* $\beta(z)$ *and let* $G_0(z) = \frac{n_G(z)}{d_G(z)}$ *be a coprime factorization of* $G_0(z)$*. Let Assumption 2 be satisfied. Then the NMP zeros of* $G_0(z)$ *are also zeros of* $M(z, \eta^*)$*.*

Proof 1 *From (14) we have that*

$$C(z, \rho^*) = \frac{n_M(z, \eta^*)d_G(z)}{[d_M(z) - n_M(z, \eta^*)]n_G(z)}. \qquad (17)$$

Since $G_0(z)$ *has a nonzero steady state gain,* $n_G(z)$ *has no zero at* $z = 1$*. Since the poles of* $C(z, \rho^*)$ *(i.e. the roots of* $B(z)$*) are either at* $z = 1$ *or strictly inside the unit circle, it follows that* $B(z)$ *and* $n_G(z)$ *have no common unstable roots. Therefore, since the left hand side of (17) is stable, any unstable root of* $n_G(z)$ *must be canceled by a root of* $n_M(z, \eta^*)$*.*

Now, inserting (12) in (16), one can rewrite it as

$$\begin{aligned}\tilde{J}_0^{VR}(\eta, \rho) = E\{&\eta^T F(z)[u_L(t) + \rho^T\beta(z)y_L(t)] \\ &-\rho^T\beta(z)y_L(t)\}^2\end{aligned} \qquad (18)$$

The expression in (18) is biquadratic in the parameters η and ρ. For fixed ρ, the minimization of (18) wrt η is obtained by least squares, since (18) is quadratic in the parameter η:

$$\begin{aligned}\eta^*(\rho) &\triangleq \arg\min_{\eta} \tilde{J}_0^{VR}(\eta, \rho) \\ &= \bar{E}\left\{[F(z)w(\rho,t)][F(z)w(\rho,t)]^T\right\}^{-1} \times \\ &\quad \bar{E}\left\{[F(z)w(\rho,t)][C(z,\rho)L(z)y(t)]\right\}.\end{aligned} \qquad (19)$$

where $w(\rho, t) \triangleq L(z)[u(t) + \rho^T\beta(z)y(t)]$. We note that $w(\rho, t)$ can be generated from the data, since $u(t), y(t), L(z)$ and $C(z, \rho)$ are all known. Similarly, for fixed η the minimization of (18) wrt ρ is obtained by least squares:

$$\begin{aligned}\rho^*(\eta) &\triangleq \arg\min_{\rho} \tilde{J}_0^{VR}(\eta, \rho) \\ &= \bar{E}\left\{[\beta(z)v(\eta,t)][\beta(z)v(\eta,t)]^T\right\}^{-1} \times \\ &\quad \bar{E}\left\{[\beta(z)v(\eta,t)][M(z,\eta)L(z)u(t)]\right\}.\end{aligned} \qquad (20)$$

where $v(\eta, t) \triangleq L(z)[1 - \eta^T F(z)]y(t)$.

Since the argument in (18) is bilinear in η and ρ, the minimization of $\tilde{J}_0^{VR}(\eta, \rho)$ can be treated as a sequence of least squares problems [11]:

$$\hat{\eta}^{(i)} = \arg\min_{\eta} \tilde{J}_0^{VR}(\eta, \hat{\rho}^{(i-1)}) \qquad (21)$$

$$\hat{\rho}^{(i)} = \arg\min_{\rho} \tilde{J}_0^{VR}(\hat{\eta}^{(i)}, \rho) \qquad (22)$$

where each least squares step is performed by the expressions (19)-(20) above.

This sequential least squares is guaranteed to converge at least to a local minimum [11,14].

Theorem 2 *The algorithm (21)-(22) converges to an extremum of* $\tilde{J}_0^{VR}(\eta, \rho)$*.*

Proof 2 *The proof is based on the Lyapunov theory, using the Lyapunov function* $\tilde{J}_0^{VR}(\eta, \rho)$*, that by definition is positive definite. It is clear from the very definition of the algorithm that* $\tilde{J}_0^{VR}(\eta, \rho)$ *is a strictly decreasing function of the sequence* $\hat{\eta}^{(i)}$*,* $\hat{\rho}^{(i)}$*. Then the convergence is a standard result in Lyapunov theory [9].*

When the data are collected in closed loop, it is natural to use the parameters of the controller that is in the control loop during the experiment as the initial value, but this is not the only possible choice. The algorithm also needs an initial value for the filter $L(z)$, which depends on $M(z, \eta)$. One possible choice is to use $\bar{M}(z)$ for this purpose. It is worth stressing that even though the minimization algorithm is iterative, the data from the system is collected just once, thereby keeping the "one-shot" property of the VRFT method.

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Two-step procedure

The method just presented can be applied to NMP plants, which standard VRFT can not. On the other hand, when the process is minimum phase, a full model reference design is possible, so the designer may want to stick to the standard VRFT criterion, where he/she can take advantage of its simplicity. But without previous knowledge of the process, the designer can not make the a priori decision on which method to use. For cases where it is not known a priori whether there are NMP zeros or not and how many, we propose the following two-step procedure. To simplify the presentation of this procedure and of the simulation results in the next section, we define the following generic design criterion inspired by [10]:

$$J_\lambda^{VR}(\eta,\rho) = (1-\lambda)J_0^{VR}(\eta,\rho) + \lambda J^{VR}(\rho). \tag{23}$$

where $\lambda = 0$ or $\lambda = 1$. When $\lambda = 1$, this becomes the standard VRFT criterion (7); when $\lambda = 0$ it represents the modified VRFT criterion for NMP plants (13).

The two-step procedure can then be described as follows.
Step 1. Minimize $J_\lambda^{VR}(\eta,\rho)$ in (23) with $\lambda = 0$. Call $(\hat{\eta},\hat{\rho})$ the minimizing parameters and check the zeros of $M(z,\hat{\eta})$.
Step 2. If $M(z,\hat{\eta})$ obtained in Step 1 has NMP zeros, then modify the reference model $\bar{M}(z)$ so that it contains these NMP zeros. If not, keep the initially chosen $\bar{M}(z)$. Now, apply the standard VRFT with $\bar{M}(z)$, i.e. use $\lambda = 1$.

5 Illustrative examples

In this section we present simulation studies using the flexible VRFT scheme. If the plant has NMP zeros, the proposed method estimates these zeros and then they can be included in the fixed reference model.

5.1 *Process with one non-minimum phase zero*

Suppose that we design a controller for a process whose transfer function is given by

$$G_1(z) = \frac{(z-1.2)(z-0.4)}{z(z-0.3)(z-0.8)}. \tag{24}$$

We want to control it with a PID controller

$$C(z,\rho) = \rho^T\beta(z) = [\rho_1\ \rho_2\ \rho_3]\begin{bmatrix}\frac{z^2}{z^2-z}\\ \frac{z}{z^2-z}\\ \frac{1}{z^2-z}\end{bmatrix}. \tag{25}$$

The experiment from which we get data is a closed loop experiment, where a step is applied as the reference signal, and the controller in the loop is given by

$$C_{init}(z) = \frac{-0.7(z-0.4)(z-0.6)}{z^2-z}.$$

5.1.1 *Assumption 2 is satisfied*

Consider the following flexible reference model, for which Assumption 2 is satisfied:

$$M(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.885)(z^2-0.706z+0.32)}. \tag{26}$$

For the fixed reference model we choose the same poles, but all zeros at the origin and $\bar{M}(1) = 1$,

$$\bar{M}(z) = \frac{0.07061z^2}{(z-0.885)(z^2-0.706z+0.32)}. \tag{27}$$

This is the reference model we would like to enforce in the absence of any knowledge on the NMP zero. If the standard VRFT criterion is used, with the reference model (27), the controller obtained is

$$C(z,\rho) = \frac{-2.2693(z^2-1.655z+0.7007)}{z^2-z},$$

which causes the closed loop to be unstable, due to the NMP zero present in the process but not in the reference model. So, we must abandon this fixed reference model (27).

Let us now use the proposed two step procedure. We set $\lambda = 0$ in (23) and minimize $\tilde{J}_0^{VR}(\eta,\rho)$ w.r.t η and ρ using the iterative procedure (21)-(22). The step responses of $M(z,\hat{\eta}^{(i)})$ and the closed loop $T(z,\hat{\rho}^{(i)})$ obtained at iterations 1 and 30 are presented in Fig. 2. Note that $M(z,\hat{\eta}^{(30)})$ and $T(z,\hat{\rho}^{(30)})$ are almost indistinguishable. Table 1 shows the evolution of the corresponding parameters, by means of the numerators of the controller and the flexible reference model, obtained in different iterations. The values of $M(z,\hat{\eta}^{(30)})$ and $C(z,\hat{\rho}^{(30)})$ at iteration 30 are as follows:

$$M(z,\hat{\eta}^{(30)}) = \frac{-0.59084(z-1.2)(z-0.4022)}{(z-0.885)(z^2-0.706z+0.32)},$$
$$C(z,\hat{\rho}^{(30)}) = \frac{-0.59026(z-0.8)(z-0.3004)}{z^2-z}.$$

Observe in Table 1 that $M(z,\hat{\eta}^{(30)})$ reproduces both zeros of $G_1(z)$ with a good precision, and the controller $C(z,\hat{\rho}^{(30)})$ is such that its zeros cancel the poles of the process. Note also that a good estimate of the NMP

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Table 1
Evolution of $M(z,\eta)$, $C(z,\rho)$ and $\tilde{J}_0^{VR}(\hat{\eta},\hat{\rho})$ in the iterative procedure for $G_1(z)$.

i	num$(M(z,\hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\hat{\rho}^{(i-1)})$ $(\times 10^{-6})$	num$(C(z,\hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\hat{\rho}^{(i)})$ $(\times 10^{-6})$
1	$-1.07760(z-1.182)(z-0.6404)$	1.6772616	$-0.76020(z-0.7448)(z-0.5292)$	12.928054
2	$-0.87825(z-1.186)(z-0.5686)$	6.4634327	$-0.71172(z-0.7704)(z-0.4509)$	1.8210199
10	$-0.63076(z-1.196)(z-0.4291)$	0.0104166	$-0.61096(z-0.7963)(z-0.3245)$	0.0086147
20	$-0.59617(z-1.199)(z-0.4061)$	0.0002024	$-0.59312(z-0.7996)(z-0.3038)$	0.0001776
21	$-0.59512(z-1.200)(z-0.4054)$	0.0001411	$-0.59256(z-0.7997)(z-0.3031)$	0.0001240
30	$-0.59084(z-1.200)(z-0.4022)$	0.0000064	$-0.59026(z-0.8000)(z-0.3004)$	0.0000057

zero is already present at iteration $i=21$, while convergence to the minimum phase zero is slower. This observation is consistent with the findings of [12,13] where it is shown that NMP zeros are easier to estimate than minimum phase zeros. This design is by itself satisfactory and shows the efficiency of the flexible design criterion in coping with NMP zeros. Whereas a standard VRFT design would lead to an unstable closed loop, with the proposed design approach the closed loop is stable and its behavior resembles the desired one, specified by the fixed reference model.

We can however make the closed loop behavior even closer to that of the fixed reference model. Indeed, in applying the flexible reference model we have left both zeros of the process unchanged in the closed loop transfer function. But only one of these zeros is NMP and thus needs to be there; the other closed loop zero can still be assigned by the designer. So, once we know that the process actually has a NMP zero and where it is, we change $\bar{M}(z)$ to include this NMP zero and then use (23) for $\lambda=1$, that is, standard VRFT. This new reference model is defined by

$$\bar{M}_m(z) = \frac{-0.35303(z-1.2)z}{(z-0.885)(z^2-0.706z+0.32)}, \tag{28}$$

where the gain is chosen so that $\bar{M}_m(1)=1$. Fig. 1 shows the step responses obtained at the end of Step 1 with $T(z,\hat{\rho}^{(30)}) = M(z,\hat{\eta}^{(30)})$ and at the end of Step 2 with $T(z,\hat{\rho})$ where $\hat{\rho}$ minimizes (8), as well as the step response of $\bar{M}_m(z)$. Observe that the responses of $T(z,\hat{\rho})$ and $M_m(z)$ are very similar, but that for $\lambda=1$ we obtain a smaller negative response.

5.1.2 Assumption 2 is not satisfied

In the previous example, the reference model (26) was chosen in such a way that the matching condition (14) is satisfied for some (η^*,ρ^*) pair. Since the process $G_0(z)$ is unknown, it can not be guaranteed that the designer can choose the poles of $M(z,\eta)$ such that Assumption 2 is satisfied. Let us see how the method behaves in this situation.

Suppose now that we choose for the same process (24) we choose a different fixed reference model

$$\bar{M}_f(z) = \frac{0.064z^2}{(z-0.6)^3}, \tag{29}$$

where the subscript f denotes “faster”, as well as a flexible one defined as

$$M_f(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.6)^3},$$

for which Assumption 2 is not satisfied. For $\lambda=0$ we obtain, in 30 iterations,

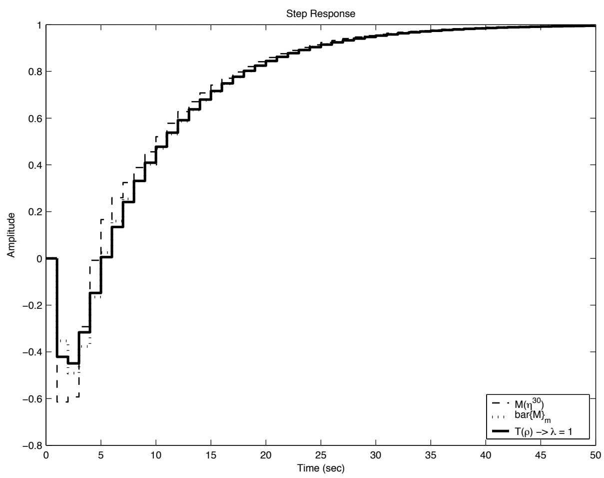

Fig. 1. Step responses obtained in Step 1: $T(z,\hat{\rho}^{(30)}) = M(z,\hat{\eta}^{(30)})$ $(M(\eta^{30}))$; and in Step 2: $T(z,\hat{\rho})$ $(T(\rho)$ -> $\lambda=1)$ with the fixed reference model (28) (bar$\{M\}_m$) for Step 2.

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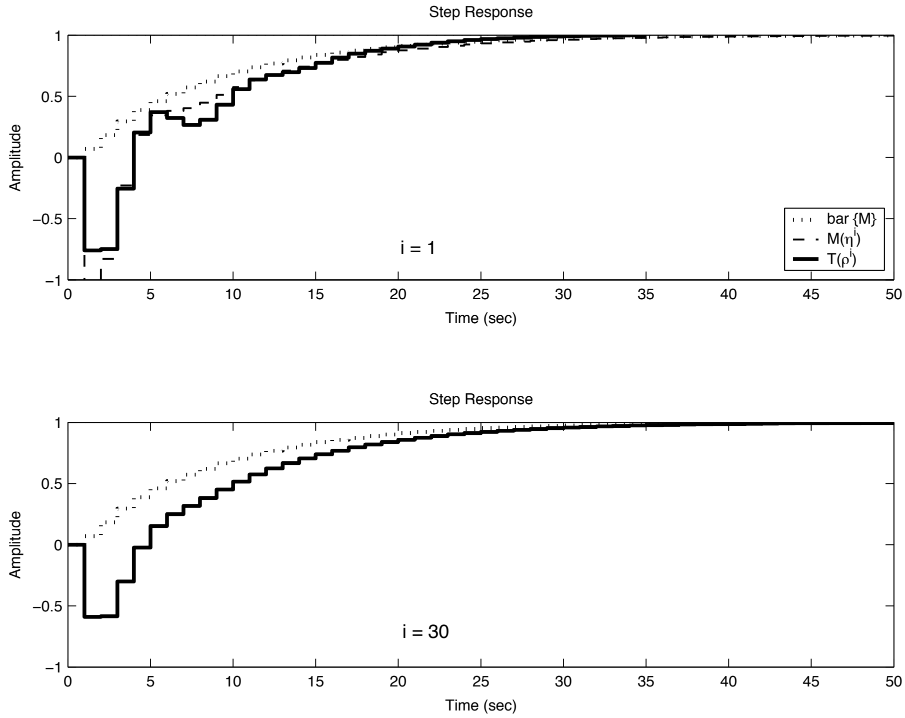

Fig. 2. Step responses of the fixed reference model (27) (bar$\{M\}$), the flexible model $M(z,\hat{\eta}^{(i)})$ $(M(\eta^i))$ and the closed loop system $T(z,\hat{\rho}^{(i)})$ $(T(\rho^i))$ for $G_1(z)$ with $\lambda = 0$ for iterations 1 and 30.

$$M_f(z,\hat{\eta}^{(30)}) = \frac{-0.54223(z-1.197)(z-0.4021)}{(z-0.6)^3},$$
$$C(z,\hat{\rho}^{(30)}) = \frac{-0.52336(z-0.7932)(z+0.0091)}{z^2-z}.$$

The step responses for iterations 1 and 30 are presented in Fig. 4. Table 2 presents the numerators of the controller and the flexible reference model, obtained in different iterations. Note that, even though Assumption 2 is not satisfied, the NMP zero is still identified with good precision by the minimization of $\tilde{J}_0^{VR}(\eta,\rho)$. Besides, the closed loop $T(z,\hat{\rho}^{(30)})$ presents a response that is not exactly, but very similar to the reference model $M_f(z,\hat{\eta}^{(30)})$ response (see Fig. 4).

We can again apply the second step of our procedure, modifying the fixed reference model to include the NMP zero just identified. The fixed reference model $\bar{M}_{f,m}(z)$ should then be defined as

$$\bar{M}_{f,m}(z) = \frac{-0.3249z(z-1.197)}{(z-0.6)^3}.$$

For $\lambda = 1$, we find the following controller

$$C(z,\rho) = \frac{-0.33598(z+0.5622)(z-0.8066)}{z^2-z}.$$

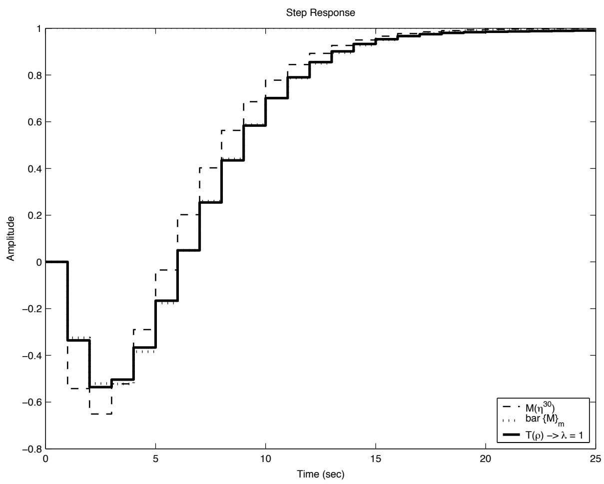

Fig. 3. Closed loop response $T(z)$ for model $G_1(z)$; controllers obtained with $\lambda = 0$ and $\lambda = 1$ with $\bar{M}_{f,m}(z)$.

Fig. 3 presents the reference models and the step responses obtained for $\lambda = 0$ and $\lambda = 1$. Again, $\bar{M}_{f,m}(z)$ allows the system to present a smaller inverse response, closer to the response specified by the original reference model $\bar{M}_f(z)$.

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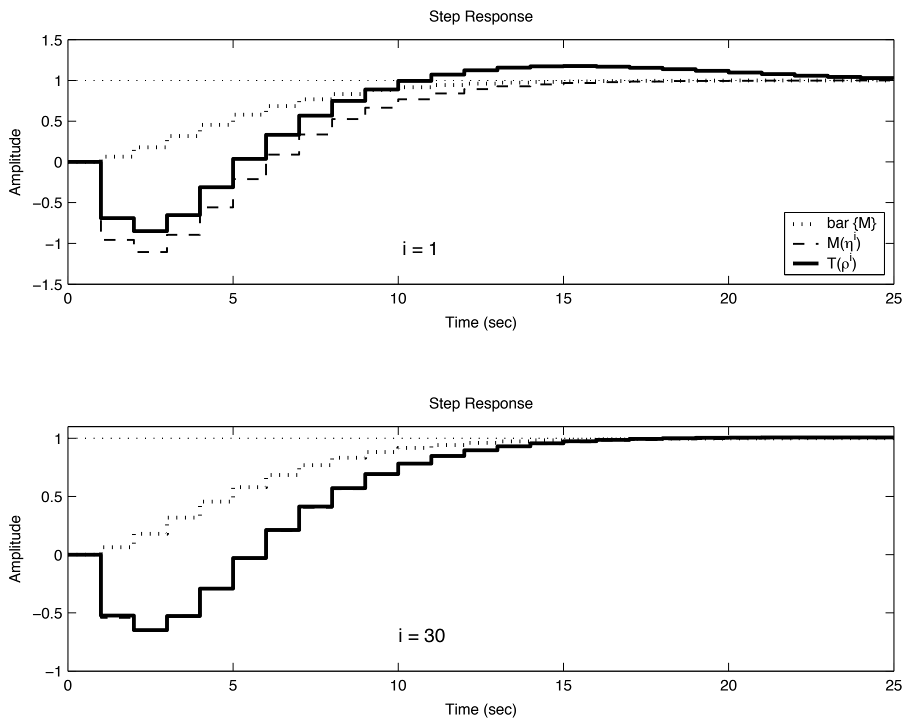

Fig. 4. Step responses of the fixed reference model (29) (bar$\{M\}$), the flexible model $M_f(z, \hat{\eta}^{(i)})$ ($M(\eta^i)$) and the closed loop system $T(z, \hat{\rho}^{(i)})$ ($T(\rho^i)$) with $G_1(z)$ at iterations 1 and 30.

Table 2
Evolution of $M_f(z, \eta)$, $C(z, \rho)$ and $\tilde{J}_0^{VR}(\hat{\eta}, \hat{\rho})$ in the iterative procedure for $G_1(z)$.

i	num$(M_f(z, \hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)}, \hat{\rho}^{(i-1)})$ ($\times 10^{-6}$)	num$(C(z, \hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)}, \hat{\rho}^{(i)})$ ($\times 10^{-6}$)
1	$-0.95696(z-1.136)(z-0.5078)$	1.3176005	$-0.69039(z-0.7452)(z-0.2133)$	9.5254229
2	$-0.83134(z-1.161)(z-0.5205)$	4.6132843	$-0.66593(z-0.7628)(z-0.1892)$	1.7675536
10	$-0.63683(z-1.194)(z-0.4821)$	0.0731824	$-0.59054(z-0.7870)(z-0.0980)$	0.0683009
20	$-0.57682(z-1.196)(z-0.4349)$	0.0240805	$-0.54914(z-0.7910)(z-0.0352)$	0.0231907
30	$-0.54223(z-1.197)(z-0.4021)$	0.0108115	$-0.52336(z-0.7932)(z+0.0091)$	0.0105319

5.2 Process with two minimum-phase zeros

Finally, we apply the method to an example in which the plant zeros are both minimum phase:

$$G_2(z) = \frac{(z+0.2)(z-0.4)}{z(z-0.3)(z-0.8)}. \quad (30)$$

It is initially in closed loop with a PID controller

$$C_{init}(z) = \frac{0.7(z-0.4)(z-0.6)}{z^2 - z},$$

which we want to retune so that the closed loop response is as close as possible to a given $\bar{M}(z)$, using a controller $C(z, \rho)$ of the form (25).

5.2.1 Assumption 2 is satisfied

In this case, the fixed reference model is given by

$$\bar{M}(z) = \frac{0.46009z^2}{(z-0.6673)(z^2+0.3063z+0.07661)},$$

and the flexible reference model is chosen as

$$M(z, \eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.6673)(z^2+0.3063z+0.07661)},$$

for which Assumption 2 is satisfied. In Step 1 the zeros of $M(z, \eta)$, estimated using (21)-(22), converge to the zeros of $G_0(z)$, but more slowly than in the case of NMP zeros: see Table 3. Since $M(z, \hat{\eta}^{(40)})$ does not present a NMP

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Table 3
Evolution of $M(z,\eta)$, $C(z,\rho)$ and $\tilde{J}_0^{VR}(\hat{\eta},\hat{\rho})$ in the iterative procedure for $G_2(z)$.

i	$\mathrm{num}(M(z,\hat{\eta}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\hat{\rho}^{(i-1)})$	$\mathrm{num}(C(z,\hat{\rho}^{(i)}))$	$\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\hat{\rho}^{(i)})$
1	$0.68335(z-0.4565)(z+0.2618)$	31.3183375	$0.68557(z-0.8190)(z-0.2473)$	5.5740771
2	$0.68326(z-0.4554)(z+0.2584)$	5.4351676	$0.68539(z-0.8189)(z-0.2494)$	5.1229686
5	$0.68315(z-0.4515)(z+0.2483)$	4.2398648	$0.68498(z-0.8180)(z-0.2559)$	3.9979539
10	$0.68309(z-0.4449)(z+0.2330)$	2.8060046	$0.68441(z-0.8155)(z-0.2661)$	2.6413208
30	$0.68210(z-0.4253)(z+0.1933)$	0.4965922	$0.68219(z-0.8074)(z-0.2930)$	0.4657815
40	$0.68152(z-0.4198)(z+0.1833)$	0.2196505	$0.68134(z-0.8052)(z-0.2999)$	0.2075660

zero, we can safely go for Step 2 and use the standard VRFT method without modifying the reference model.

5.2.2 *Assumption 2 is not satisfied*

Suppose now we choose another fixed reference model:

$$\bar{M}_f(z) = \frac{0.216z^2}{(z-0.4)^3}, \tag{31}$$

and a flexible model having the same poles as $\bar{M}_f(z)$:

$$M_f(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.4)^3}.$$

With $M_f(z,\eta)$ and the controller (25), Assumption 2 is not satisfied. Then Step 1 leads to

$$M_f(z,\hat{\eta}^{(10)}) = \frac{0.68702(z^2-0.9865z+0.3009)}{(z-0.4)^3},$$
$$C_f(z,\hat{\rho}^{(10)}) = \frac{0.67926(z-0.8134)(z-0.1744)}{z^2-z}.$$

Note that $M_f(z,\hat{\eta}^{(10)})$ is far from $\bar{M}_f(z)$ (see Fig. 5), but it does not present a NMP zero. We can then safely go to Step 2 and apply the standard VRFT with the fixed reference model (31). The controller obtained is

$$C(z,\hat{\rho}) = \frac{0.19874(z+0.5094)(z-0.7791)}{z^2-z}.$$

The closed loop response obtained with this controller is compared to the fixed reference model $\bar{M}_f(z)$ in Fig. 5.

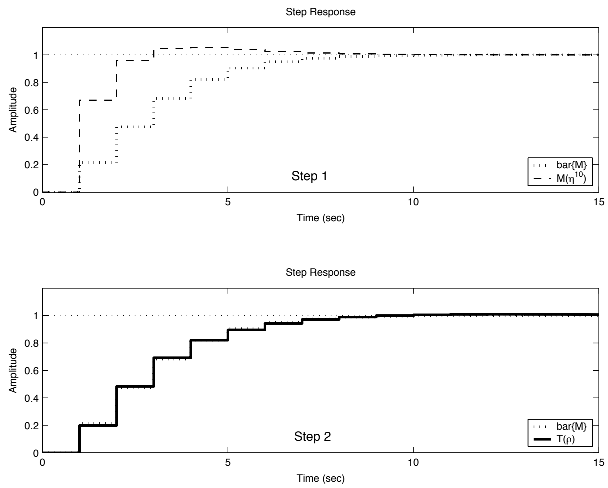

Fig. 5. Step responses of the fixed reference model (31) (bar{M}), the flexible reference model $M_f(z,\hat{\eta}^{(10)})$ ($M(\eta^{10})$) in Step 1 and of the closed loop system $T(z,\hat{\rho})$ ($T(\rho)$) in Step 2.

6 Conclusions and future work

In this paper, we have extended the VRFT design methodology to cope with NMP plants. This has been achieved through a flexible design criterion, in which the numerator of the reference model is not specified, but left free to adjust itself to the zeros of the plant. We have proposed a two-step procedure in which the possible presence of NMP zeros in the plant, as well as their location, is detected in the first step. The second step then becomes a classical VRFT, but with a criterion that takes account of the presence of these NMP zeros, if any, detected in the first step.

Extending the analysis to the case where the matching controller is not in the controller set, as well as adapting the method to deal with signals corrupted by noise, are some of the aims of our future research. Simulations have already shown good performance for the situation where the matching condition is not satisfied.

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