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CAUTION IN ITERATIVE MODELING AND CONTROL DESIGN [1]

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Abstract: The Vinnicombe metric defining the gap between two plants, or two controllers, and its related robust stability results, are used as a tool to understand the need for cautious iterations (i.e. small controller modifications and, possibly, small model adjustments) that has been observed to be useful in iterative identification and control design. By the same token, these gap metric results allow one to compute controller updates that are smaller than would result from the optimal design based on the nominal design, and that guarantee stability of the actual closed loop system.

Keywords: identification for control, robust control

1. INTRODUCTION

In this paper we attempt to give a theoretical justification for the need for caution in iterative modeling and control design. A number of iterative schemes have been proposed over the last few years for model-based iterative controller redesign. A common feature of these schemes is that iterations are performed of model updates (by identification with the most recent controller applied to the actual plant) and of model-based controller updates (the controller design being based on the most recent model). Representative examples of these schemes can be found in (Lee *et al.*, 1993), (van den Hof *et al.*, 1995), (Zang *et al.*, 1995). There is no guarantee in any of these schemes that the succession of designed controllers stabilizes the actual plant. Experience has shown that stability robustness is enhanced by applying cautious steps of plant modification and controller modification. The first aim of the research reported in this paper was to provide a theoretical justification and understanding for this need for caution, by using the robust stability results based on the ν-gap metric between two plants introduced by G. Vinnicombe (Vinnicombe, 1993). In pursuing this aim, we have been led to derive some useful inequalities between the H_∞ measure of the difference between two closed loop transfer functions, and the ν-gap between the corresponding plants. The application of these ν-gap based stability results to iterative model-based controller tuning does indeed show why small controller ad-

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justments may be required to guarantee the closed loop stability of the actual closed loop system with a controller computed from an identified nominal model. Using Youla-Kucera parametrizations, we propose a practical way to reduce the controller modification while satisfying a ν-gap based robust stability constraint.

The paper is organized as follows. In Section 2, we present the main robust stability results based on the Vinnicombe metric. In Section 3, we establish some connections and bounds between the H_∞ distance between two closed loop systems (P, C) and (P_1, C), the stability bounds of these two systems, and the Vinnicombe distance between P and P_1. These connections and bounds are then exploited in the context of iterative model and controller design, in view of introducing closed loop stability guarantees. This leads to guidelines for cautious controller updates in Section 4. In Section 5 we show how the model update step may also need to be altered on the basis of the robust stability results developed in this paper. A numerical example in Section 6 illustrates the ideas.

2. THE VINNICOMBE METRIC AND ITS STABILITY RESULT

We consider the following unity feedback system.

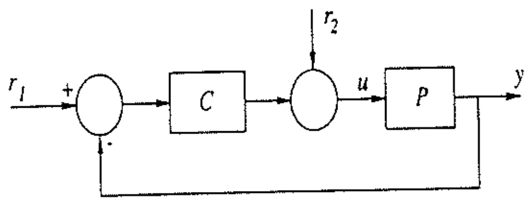

Fig. 1. The unity feedback closed loop system

The transfer function from $\begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$ to $\begin{pmatrix} y \\ u \end{pmatrix}$ is given by

$$T(P,C) = \begin{pmatrix} \frac{PC}{1+PC} & \frac{P}{1+PC} \\ \frac{C}{1+PC} & \frac{1}{1+PC} \end{pmatrix}. \quad (1)$$

For the actual loop, the transfer function will then be $T(P, C)$ and for the design loop $T(\hat{P}, C)$, where $\hat{P}$ is some model of the plant P. We define the following generalized stability margin for the closed loop system (P, C):

$$b_{P,C} = \|T(P,C)\|_\infty^{-1} \quad \text{if } (P,C) \text{ is stable}, \quad (2)$$
$$= 0 \quad \text{otherwise}. \quad (3)$$

where $\|G\|_\infty = \sup_\omega \bar{\sigma}(G(j\omega))$. We shall also use the ν-gap metric introduced by Vinnicombe (Vinnicombe, 1993) that defines a distance between two plants P_1 and P_2. For rational plants the ν-gap is defined as follows:

$$\delta_\nu(P_1, P_2) = \quad (4)$$
$$\|(I + P_2P_2^*)^{-1/2}(P_2 - P_1)(I + P_1^*P_1)^{-1/2}\|_\infty < 1$$
$$\text{if } det(I + P_2^*P_1)(j\omega) \neq 0 \ \forall\omega, \text{ and}$$
$$wno \ \ det(I + P_2^*P_1) + \eta(P_1) - \bar{\eta}(P_2) = 0 \quad (5)$$

where $\eta(P)$ denotes the number of open Right Half Plane (RHP) poles of P, $\bar{\eta}(P)$ denotes the number of closed Right Half Plane (RHP) poles of P, and $wno(g)$ denotes the winding number about the origin of $g(s)$ as "s" follows the standard Nyquist contour indented around the imaginary axis poles of P_1 and P_2. If the condition (5) is not satisfied, then $\delta_\nu(P_1, P_2) = 1$.

If the condition (5) is fulfilled, this gap metric is the gap between the $\mathcal{L}_2$ graph spaces of P_1 and P_2, which just depends on their frequency response. The stability results of (Vinnicombe, 1993) that will be of interest to us in our study of cautious iterations are as follows.

Proposition 2.1. (1) Given a nominal plant P_1 and a compensator C, then (P_2, C) is stable for all plants P_2 satisfying $\delta_\nu(P_1, P_2) \leq \beta$ if and only if $b_{P_1,C} > \beta$.
(2) Given a nominal plant P and a stabilizing compensator C_1, then (P, C_2) is stable for all compensators C_2 satisfying $\delta_\nu(C_1, C_2) \leq \beta$ if and only if $b_{P,C_1} > \beta$.

Part 1 of Proposition 2.1 implies that among the plants P_2 for which $\delta_\nu(P_1, P_2) = b_{P_1,C}$ one at least will not give closed-loop stability. *It does not mean* that all P_2 with $\delta_\nu(P_1, P_2) \geq b_{P_1,C}$ will yield an unstable closed-loop system. [Indeed, V. Blondel has pointed out to us the example $P_1 = s(s+1)^{-1}, P_2 = (s+1)s^{-1}$, both of which are stabilized by $C = 1$, while also $\delta_\nu(P_1, P_2) = 1$]. Thus the sole reliance on such robust stability results for the design of new controllers may lead to conservative designs. Using Youla-Kucera parametrizations amounts to perturbing some initial P_1 with arbitrary large perturbations in a certain controller-dependent direction without destroying closed loop stability.

We now derive a number of bounds and inequalities that will prove useful in establishing the need for cautious model and controller adjustments.

3. CONNECTING VINNICOMBE DISTANCE AND PERFORMANCE MEASURES

We consider two stable closed loop sytems (P, C) and (P_1, C) such that $\delta_\nu(P, P_1) < 1$. One could, for example, view P as the plant controlled by the stabilizing controller C, and P_1 as a model of P obtained by identification with data obtained on the (P, C) loop. A classical measure of fit between these two closed loop systems is the infinity norm of the difference between the two closed loop transfer functions :

$$||T(P, C) - T(P_1, C)||_\infty \qquad (6)$$

Indeed, the minimization of this measure over some set of parametrized models $P_1(\theta)$ has been advocated as an identification criterion in identification for control (Schrama, 1992).
The following result, established in (Vinnicombe, 1993) relates the performance measure (6) to the ν-gap $\delta_\nu(P_1, P_2)$ and to the corresponding stability measures.

Proposition 3.1. Consider the two closed loop systems (P, C) and (P_1, C) with $\delta_\nu(P, P_1) < 1$. Then

$$\delta_\nu(P, P_1) \leq ||T(P, C) - T(P_1, C)||_\infty \leq \frac{\delta_\nu(P, P_1)}{b_{P,C} b_{P_1,C}} \qquad (7)$$

If the stability margin of the nominal closed loop $b_{P_1,C}$ is large (i.e. close to 1) and the distance $\delta_\nu(P, P_1)$ between the model and the plant is much smaller than $b_{P_1,C}$, the upper bound can be approximated by $\frac{\delta_\nu(P,P_1)}{b^2_{P_1,C}}$. In such case, the bounds on $||T(P, C) - T(P_1, C)||_\infty$ provided by Proposition 3.1 are tight.

We shall also use the dual of Proposition 3.1, expressed as the following corollary.

Corollary 3.1. Consider the two closed loop systems (P, C_1) and (P, C_2) with $\delta_\nu(C_1, C_2) < 1$. Then

$$\delta_\nu(C_1, C_2) \leq ||T(P_1, C_1) - T(P_1, C_2)||_\infty \leq \frac{\delta_\nu(C_1, C_2)}{b_{P,C_1} b_{P,C_2}} \qquad (8)$$

We also want to relate the difference between the stability margins of two closed loop systems (P, C) and (P_1, C) to the ν-gap between P and P_1. To do so, we first establish the following technical Lemma.

Lemma 3.1. Denote $T = T(P, C)$ and $T_1 = T(P_1, C)$ and let $||T - T_1|| < \varepsilon$. Then

$$|||T||^{-1} - ||T_1||^{-1}| < \varepsilon ||T||^{-1} ||T_1||^{-1} \qquad (9)$$

Proof. The triangle inequality yields

$$||T - T_1|| \geq ||T|| - ||T_1||$$

and

$$||T - T_1|| \geq ||T_1|| - ||T||.$$

Therefore

$$||T|| - \varepsilon \leq ||T_1|| \leq ||T|| + \varepsilon$$

Multiplying by $||T||^{-1}||T_1||^{-1}$ yields

$$\begin{aligned} &||T_1||^{-1} - \varepsilon ||T||^{-1} ||T_1||^{-1} \\ &\leq ||T||^{-1} \\ &\leq ||T_1||^{-1} + \varepsilon ||T||^{-1} ||T_1||^{-1} \end{aligned} \qquad (10)$$

which proves the desired result.

The following theorem is an immediate consequence of this Lemma.

Theorem 3.1. Consider the two closed loop systems (P, C) and (P_1, C). Then

$$|b_{P_1,C} - b_{P,C}| \leq \delta_\nu(P, P_1) \qquad (11)$$

Proof. The proof is an immediate consequence of Proposition 3.1 and Lemma 3.1 : substitute $\frac{\delta_\nu(P,P_1)}{b_{P,C} b_{P_1,C}}$ for ε.

Comment : Expressions (7) and (11) show that the distance between the stability measures $b_{P,C}$ and $b_{P_1,C}$ is always smaller than the distance (measured in H_∞ norm) between $T(P, C)$ and $T(P_1, C)$.

4. ITERATIVE DESIGN: CONTROLLER ADJUSTMENT

In an iterative identification and control design scheme, one typically designs a succession of model-based controllers, and one identifies a succession of models obtained from closed loop data with the most recent controller acting on the plant. Let, at some stage of the iterations, C_i be the acting controller and $\hat{P}_j$ be the nominal plant model. It is assumed that the closed loop systems (P, C_i) and $(\hat{P}_j, C_i)$ are both stable. Two possible situations can arise:

(1) The model $\hat{P}_j$ is satisfactory, in that the closed loops (P, C_i) and $(\hat{P}_j, C_i)$ are "close". We then want to make a controller adjustment based on $\hat{P}_j$ from C_i to C_{i+1} with, possibly, stability guarantees for the (P, C_{i+1}) loop.
(2) The model $\hat{P}_j$ is no longer satisfactory, in that $||T(P, C_i) - T(\hat{P}_j, C_i)||_\infty$ is large. We then want to compute a new model that will allow us to compute a better controller C_{i+1}.

In this section, we examine some robust stability considerations related to the controller adjustment step; we shall examine the model adjustment step in the next section.

First, by the "closeness" between the loops (P, C_i) and $(\hat{P}_j, C_i)$ we mean that $T(P, C_i) \simeq T(\hat{P}_j, C_i)$, and hence $b_{P,C_i} \simeq b_{\hat{P}_j,C_i}$. The model $\hat{P}_j$ is then considered to be sufficiently accurate to be used for the design of the next controller, C_{i+1}. Without introducing caution, this new controller would typically be computed by minimizing some control performance criterion $J(\hat{P}_j, C)$. The resulting controller, C_{i+1}, would of course stabilize $\hat{P}_j$, but there would be no guarantee that it stabilizes the actual plant P. Assume now that we modify the computation of the new controller as follows: define C_{i+1} such that

$$C_{i+1} = \arg\min_C J(\hat{P}_j, C)$$

under the constraint

$$\delta_\nu(C_{i+1}, C_i) < k b_{\hat{P}_j,C_i}, \quad (12)$$

where k is a safety constant in $(0,1)$ to account for the error between b_{P,C_i} and $b_{\hat{P}_j,C_i}$. Under our assumption that $b_{P,C_i} \simeq b_{\hat{P}_j,C_i}$, it follows that $\delta_\nu(C_{i+1}, C_i) < b_{P,C_i}$, and therefore the stability of the actual closed loop follows from the dual of part 1 of Proposition 2.1.

Comment: Using the inequalities derived in the previous section one can obtain a more precise - albeit not computable - upper bound on the allowable controller movement $\delta_\nu(C_{i+1}, C_i)$ as a function of the nominal stability margin $b_{\hat{P}_j,C_i}$ and of the distance between the actual system and the nominal model $\hat{P}$. Indeed, by Theorem 3.1,

$$|b_{P,C_i} - b_{\hat{P}_j,C_i}| < \delta_\nu(P, \hat{P}_j).$$

Therefore $\delta_\nu(C_{i+1}, C_i) < b_{P,C_i}$ if

$$\delta_\nu(C_{i+1}, C_i) < b_{\hat{P}_j,C_i} - \delta_\nu(P, \hat{P}_j).$$

Even though this last quantity depends on the unknown system, it provides some interesting insight. We observe that the larger the nominal stability margin $b_{\hat{P}_j,C_i}$ and the smaller the distance $\delta_\nu(P, \hat{P}_j)$ between the plant and the model, the larger a change in controller is allowed with guaranteed closed loop stability on the actual system. In the previous discussion, $\delta_\nu(P, \hat{P}_j)$ can be replaced by its upper bound $\|T(P, C_i) - T(\hat{P}_j, C_i)\|_\infty$, which is another measure of the distance between the two closed loop systems. However, this will give a more conservative result, since this measure is always larger than $\delta_\nu(P, \hat{P}_j)$.

The inequality (12) tells us what, from the point of view of stability, is an acceptable controller C_{i+1} to replace C_i. However, if our goal is to achieve a certain closed-loop performance, by minimizing a performance index say, then we need to understand which C_{i+1} we should choose in a set defined like (12).

To fix ideas, we shall postulate that the design goal is to obtain a stabilizing compensator C to minimize a performance index $J(P, C)$, and that we have a model $\hat{P}_j$ of P such that $T(\hat{P}_j, C_i) \simeq T(P, C_i)$. Let $\hat{P}_j$ have a right coprime realization ND^{-1} and C_i a right coprime realization UV^{-1}. The set of all stablizing compensators of $\hat{P}_j$ is given by

$$\mathcal{C} = \{C(Q) : C(Q) = (U - DQ)(V + NQ)^{-1}\}, \quad (13)$$

where Q (the Youla-Kucera parameter) is an arbitrary stable proper transfer function. Let us make a further assumption that is certainly fulfilled in the H_2 and H_∞ problems.

Assumption 4.1. The performance index $J(\hat{P}_j, C(Q))$ for $C \epsilon \mathcal{C}$ depends on Q in a convex manner.

We can then find C_{i+1} in the following way. Suppose

$$C^*_{i+1} = \arg\min_{C(Q)} J(\hat{P}_j, C(Q)). \quad (14)$$

(When the minimum has to be replaced by an infimum, there is a minor adjustment to these calculations.) Let Q^* be such that

$$C^*_{i+1} = (U - DQ^*)(V + NQ^*)^{-1}. \quad (15)$$

To avoid trivialities, suppose that $Q^* \neq 0$, i.e. C_i does not minimize $J(\hat{P}_j, C)$.

If

$$\delta_\nu(C_i, C^*_{i+1}) \leq k b_{\hat{P}_j,C_i}, \quad (16)$$

with k the constant introduced at the end of the last section. choose

$$C_{i+1} = C^*_{i+1}. \quad (17)$$

Otherwise, consider the set

$$C(\alpha Q^*) = (U - \alpha DQ^*)(V + \alpha NQ^*)^{-1}, \quad \alpha \epsilon [0,1]. \quad (18)$$

Observe that $\alpha = 0$ corresponds to C_i, $\alpha = 1$ corresponds to C^*_{i+1}, and for all $\alpha \epsilon [0,1]$, $C(\alpha Q^*)$ is stabilizing. Choose $\alpha \epsilon (0,1)$ so that

$$\delta_\nu(C_i, C(\alpha Q^*)) = k b_{\hat{P}_j,C_i}. \quad (19)$$

Such an α exists, since δ_ν is a smooth function of α, taking values at $\alpha = 0$ of 0 and at $\alpha = 1$ of something in excess of $k b_{\hat{P}_j,C_i}$. Also, take

$$C_{i+1} = C(\alpha Q^*). \quad (20)$$

Evidently, this choice moves the controller *in the direction of* C^*_{i-1}, but not necessarily all the way; in fact, the movement is such as to retain the bound on δ_ν. In addition, we now show that it improves the performance index.

Theorem 4.1. Suppose that the stable transfer function Q^* minimizes the performance index $J(\hat{P}_j, C(Q))$ which satisfies Assumption 4.1. Let $\alpha\epsilon(0,1)$. Then

$$J(\hat{P}_j, C(Q^*)) \leq J(\hat{P}_j, C(\alpha Q^*)) < J(\hat{P}_j, C_i),$$

(where C_i corresponds to $\alpha = 0$).

Proof: The left hand inequality follows by optimality of Q^*. For the right hand inequality observe that by the convexity property of J,

$$J(\hat{P}_j, C(\alpha Q^*)) \leq (1-\alpha)J(\hat{P}_j, C_i) + \alpha J(\hat{P}_j, C(Q^*)) \quad (21)$$

$$< (1-\alpha)J(\hat{P}_j, C_i) + \alpha J(\hat{P}_j, C_i) \quad (22)$$

$$= J(\hat{P}_j, C_i). \quad (23)$$

In (Lee *et al.*, 1993) a performance index of the type described above was not used to determine the controller. Rather, the so called IMC design method was used, where one seeks a controller to achieve a standard closed-loop transfer function in which a single parameter, the bandwidth, appears.

This means that the controller which, in conjunction with a model $\hat{P}_j$, achieves a particular bandwidth is parameterisable by that bandwidth. It is again straightforward to compute a Vinnicombe distance between two such controllers and to set a limit on the change of bandwidth, in terms of $b_{\hat{P}_j,C_i}$.

Let us note that if C_i has been chosen to secure a closed-loop bandwidth exceeding that of the open loop plant P, the entry $C(1+PC)^{-1}$ of $T(P,C)$ will become large, in fact $O[\|P^{-1}\|]$ outside the plant bandwidth, and accordingly $b_{\hat{P}_j,C_i}$ will be small. This will limit the scope for further bandwidth expansion.

5. ITERATIVE DESIGN: MODEL ADJUSTMENT

In this section we examine the situation where the closed loop transfer functions $T(P,C_i)$ and $T(\hat{P}_j,C_i)$ are significantly different. In particular, we can no longer assume that $b_{P,C_i} \simeq b_{\hat{P}_j,C_i}$. The mismatch between $T(P,C_i)$ and $T(\hat{P}_j,C_i)$ indicates that the model $\hat{P}_j$ cannot be used for the design of the new controller, and this indicates the need for the identification of a new model from closed loop data obtained on the present system (P,C_i).

Without consideration for stability robustness, this model would be obtained by minimizing some identification criterion $V(P,\hat{P}(\theta),C_i)$ over a parametrized set of models $\{\hat{P}(\theta)\}$. We shall assume here that the closed loop H_∞ identification criterion $V(P,\hat{P}(\theta),C_i) = \|T(P,C_i) - T(\hat{P}(\theta),C_i)\|_\infty$ has been adopted. Thus, without any consideration for stability robustness, the identification step would be

$$\hat{P}_{j+1} = arg \;\; min_{\hat{P}} V(P,\hat{P},C_i) \quad (24)$$

with the minimization performed over a parametrized set of models.

We now develop some stability robustness considerations that may lead us to inject some additional requirements on the estimation of $\hat{P}_{j+1}$. For simplicity of notations we shall set $j=1$, i.e. $\hat{P}_1$, is the present model, $\hat{P}_2$ is the new model to be identified, while P is still used for the true plant and C_i for the present controller. We now consider the following design objective for the identification of $\hat{P}_2$.

Design Objective : Estimate a new model $\hat{P}_2$ that minimizes $V(P,\hat{P}_2,C_i)$ while at the same time increasing the set $\delta_\nu(C_i,C_{i+1})$ of admissible controllers $C_{i+1} = C_{i+1}(\hat{P}_2)$ that guarantee the stability of the (P,C_{i+1}) loop.

The stability of the (P,C_{i+1}) loop is guaranteed for all controllers C_{i+1} such that

$$\delta_\nu(C_i,C_{i+1}) < b_{P,C_i}. \quad (25)$$

We now derive two alternative lower bounds

$\xi(P,\hat{P}_1,C_i)$ and $\xi(P,\hat{P}_2,C_i)$ for b_{P,C_i}

Observe that

$$\|T(P,C_i)\|_\infty \leq \|T(\hat{P}_j,C_i)\|_\infty + \|T(P,C_i) - T(\hat{P}_j,C_i)\|_\infty \quad (26)$$

for $j = 1,2$, and denote

$$\xi(P,\hat{P}_j,C_i) \triangleq [\|T(\hat{P}_j,C_i)\|_\infty + \|T(P,C_i) - T(\hat{P}_j,C_i)\|_\infty]^{-1} = [b^{-1}_{\hat{P}_j,C_i} + V(P,\hat{P}_j,C_i)]^{-1} \quad j = 1,2 \quad (27)$$

It then follows from (26) and (27) that

$$b_{P,C_i} \geq max(\xi(P,\hat{P}_1,C_i), \xi(P,\hat{P}_2,C_i)) \quad (28)$$

Therefore we have established the following result.

Theorem 5.1. Consider the plant P, the present controller C_i, and two alternative models: the present model $\hat{P}_1$ and the new model $\hat{P}_2$. Then a new controller C_{i+1} designed from $\hat{P}_2$ will deliver a stable closed loop with P if

$$\delta_\nu(C_{i+1},C_i) < max(\xi(P,\hat{P}_1,C_i), \xi(P,\hat{P}_2,C_i)).$$

In order for the new model $\hat{P}_2$ to allow for a larger set of stabilizing controllers $\delta_\nu(C_i,C_{i+1})$ than the present model $\hat{P}_1$, we would thus want the new model $\hat{P}_2$ to be such that

$$\xi(P,\hat{P}_2,C_i) \geq \xi(P,\hat{P}_1,C_i).$$

Alternatively:

$$V(P,\hat{P}_1,C_i) - V(P,\hat{P}_2,C_i) \hat{\hat{=}}$$
$$\|T(P,C_i) - T(\hat{P}_1,C_i)\|_\infty - \|T(P,C_i) - T(\hat{P}_2,C_i)\|_\infty$$
$$\geq b^{-1}_{\hat{P}_2,C_i} - b^{-1}_{\hat{P}_1,C_i} \quad (29)$$

The examination of (29) leads to some interesting observations.

Comments

- First notice that, even though the terms $V(P,\hat{P}_1,C_i)$ and $V(P,\hat{P}_2,C_i)$ are not known, they can be estimated. As for the stability margins $b_{\hat{P}_1,C_i}$ and $b_{\hat{P}_2,C_i}$ they can be computed exactly.

- Clearly, if $\hat{P}_2$ is estimated without any stability robustness consideration (i.e. $\hat{P}_2 = argmin_{\hat{P}}$ $V(P,\hat{P}_2,C_i)$) then the left hand side (LHS) of (29) is positive. It will even be a "large" positive number given that the motivation for identifying a new model is that the present model $\hat{P}_1$ has been judged to be no longer acceptable. The right hand side (RHS) of (29) can have either sign, and we therefore examine each case separately.

Case 1: $b_{\hat{P}_2,C_i} \geq b_{\hat{P}_1,C_i} \Leftrightarrow$ RHS(29) ≤ 0.
The stability margin with the new model is larger than the stability margin with the previous model. In such case, the inequality (29) is satisfied with a model $\hat{P}_2$ obtained by minimizing $V(P,\hat{P},C_i)$, and the new model delivers a larger set $\delta_\nu(C_i,C_{i+1})$ of stabilizing controllers C_{i-1} than did $\hat{P}_1$. Observe that the condition $b_{\hat{P}_2,C_i} \geq b_{\hat{P}_1,C_i}$ is checkable. Since $(\hat{P}_2,C_i)$ is closer to (P,C_i) than $(\hat{P}_1,C_i)$, this will typically be the case if the achieved stability margin (of the (P,C_i) loop) is better than the designed stability margin (of the $(\hat{P}_1,C_i)$ loop).

Case2: $b_{\hat{P}_2,C_i} \leq b_{\hat{P}_1,C_i} \Leftrightarrow$ RHS(29) ≥ 0.
In such case condition (29) will be satisfied if

$$|b^{-1}_{\hat{P}_2,C_i} - b^{-1}_{\hat{P}_1,C_i}| < V(P,\hat{P}_1,C_i) - V(P,\hat{P}_2,C_i). \quad (30)$$

To achieve (30) may require a cautious movement from $\hat{P}_1$ to $\hat{P}_2$. Indeed, observe that, by the proof of Lemma 3.1, we have

$$|b^{-1}_{\hat{P}_2,C_i} - b^{-1}_{\hat{P}_1,C_i}| \leq \|T(\hat{P}_2,C_i) - T(\hat{P}_1,C_i)\|_\infty$$
$$\leq \frac{\delta_\nu(\hat{P}_1,\hat{P}_2)}{b_{\hat{P}_1,C_i} b_{\hat{P}_2,C_i}} \quad (31)$$

Thus, to insure that condition (30) is satisfied, it may be required to force $\hat{P}_2$ to be close to $\hat{P}_1$ in the sense of making the difference between the two closed loop transfer functions $T(\hat{P}_2,C_i)$ and $T(\hat{P}_1,C_i)$ small in the H_∞ sense, which is also accomplished by making $\delta_\nu(\hat{P}_1,\hat{P}_2)$ small. One way of accomplishing this is to follow the procedure of Section 4 for the controller update, using a dual Youla-Kucera parametrization to move only a fraction of the way from the previous model $\hat{P}_1$ to the model $\hat{P}_2$ that results from the minimization of the unconstrained identification criterion $V(P,\hat{P},C_i)$.

6. EXAMPLE

We now provide an example that illustrates the calculations that are required in an iterative identification and controller design in which the stability robustness bounds presented in this paper are checked at every step of the procedure.

Let the true plant be $P(s) = \frac{1}{(1+s)(1+0.1s)}$. We consider reduced order models parametrized as $P(\theta) = \frac{K}{s+a}$. For the control design, we use the following LQG regulation criterion:

$$J(P,C) = \int_{-\infty}^{\infty} \frac{(1+\lambda|C|^2)}{|1+PC|^2}|H|^2 d\omega$$

where $H(s) = \frac{0.1s+1}{10s+1}$.

Initial calculations
Let the initial model be $\hat{P}_1(s) = \frac{1}{1+s}$. The corresponding optimal controller is

$$C_1 = arg\ \ min_C J(\hat{P}_1,C)$$
$$= \frac{0.0054s + 0.0654}{0.01s^2 + 0.1141s + 0.0076}$$

With this controller we get the following stability margins: $b_{P,C_1} = 0.7835$, $b_{\hat{P}_1,C_1} = 0.8248$. The closed loop modeling performance measure is $\|T(P,C_1) - T(\hat{P}_1,C_1)\|_\infty = 0.1309$. Thus our new estimate for the stability margin, based on $\hat{P}_1$, is

$$\xi(P,\hat{P}_1,C_1) = [b^{-1}_{\hat{P}_1,C_1} + 0.1309]^{-1} = 0.7444$$

Observe that $\xi(P,\hat{P}_1,C_1)$ is a first lower bound for b_{P,C_1}.

First model update
We first compute the optimal $\hat{P}_2$ and then compute $\xi(P,\hat{P}_2,C_1)$ to check whether this $\hat{P}_2$ provides a tighter bound for b_{P,C_1} than $\hat{P}_1$ did.

$$\hat{P}_2 = arg\ \ min_\theta \|T(P,C_1) - T(P(\theta),C_1)\|_\infty$$
$$= \frac{0.8135}{s+0.6676}$$

With this $\hat{P}_2$ we get $b_{\hat{P}_2,C_1} = 0.8145$ and $\|T(P,C_1) - T(\hat{P}_2,C_1)\|_\infty = 0.0999$, yielding $\xi(P,\hat{P}_2,C_1) = 0.7532$. Observe that $\xi(P,\hat{P}_2,C_1) > \xi(P,\hat{P}_1,C_1)$. Therefore the model $\hat{P}_2$ will allow a larger movement $\delta_\nu(C_2,C_1)$ between the present controller and the "to be designed controller" than the model $\hat{P}_1$. Recall that, by Theorem 5.1, any C_1 such that $\delta_\nu(C_2,C_1) < \xi(P,\hat{P}_2,C_1)$ is guaranteed to stabilize the true plant P, since $\xi(P,\hat{P}_2,C_1) < b_{P,C_1}$.

First controller update
The optimal (unconstrained) controller C_2 is

$$C_2 = arg \;\; min_C J(\hat{P}_2, C) = \frac{0.0699s + 0.0467}{0.01s^2 + 0.1105s + 0.0484}$$

We compute $\delta_\nu(C_1,C_2) = 0.0573 < \xi(P,\hat{P}_2,C_1)$. Therefore C_2 is guaranteed to stabilize P. With the new controller C_2 we compute $b_{P,C_2} = 0.7706$, $b_{\hat{P}_2,C_2} = 0.8068$, $\|T(P,C_2) - T(\hat{P}_2,C_2)\|_\infty = 0.1039$, and $\xi(P,\hat{P}_2,C_2) = 0.7444$.

Second model update
The optimal model $\hat{P}_3$ over the same parametrized set is

$$\hat{P}_3 = arg \;\; min_\theta \|T(P,C_2) - T(P(\theta),C_2)\|_\infty = \frac{0.8148}{s + 0.6624}$$

This yields $b_{\hat{P}_3,C_2} = 0.8064$ and $\|T(P,C_2) - T(\hat{P}_3,C_2)\|_\infty = 0.1035$, and $\xi(P,\hat{P}_3,C_2) = 0.7443$. Observe that these values are very close to those obtained in the previous iteration. The new model $\hat{P}_3$ yields a slightly improved fit with the true closed loop system ($V(P,\hat{P}_3,C_2) = 0.1035 < V(p,\hat{P}_2,C_2) = 0.1039$), but it does not improve on the allowable movement $\delta_\nu(C_3,C_2)$ between C_2 and the new controller C_3, since we are in a situation where $\xi(P,\hat{P}_3,C_2) < \xi(P,\hat{P}_2,C_2)$. However, this does not matter, since the unconstrained optimal controller computed from $\hat{P}_3$ is

$$C_3 = arg \;\; min_C J(\hat{P}_3, C) = \frac{0.0701s - 0.0465}{0.01s^2 + 0.1105s + 0.0479}$$

for which $\delta_\nu(C_2,C_3) = 0.0030 << \xi(P,\hat{P}_2,C_2) < b_{P,C_2}$. Thus, controller C_3 is guaranteed to stabilize the true P.

7. SOME FINAL COMMENTS

Using the Vinnicombe gap metric and its corresponding robust stability result as our major tool, we have attempted to rationalize the need for small controller adjustments in iterative identification and control design. We should warn that, even though the stability results of Proposition 2.1 are formulated as necessary and sufficient conditions, their blind application may lead to conservative results. Indeed, in Proposition 2.1 the set $\delta_\nu(P_1,P_2) \geq \beta$ defines a ball of models P_2 around P_1, all of which are stabilized by C. Thus these constraints are non-directional. By using directional information, such as is done in the Youla-Kucera parametrization one can easily construct models P_2 that violate the condition $\delta_\nu(P_1,P_2) < b_{P_1,C}$ and yet are stabilized by C.

8. REFERENCES

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