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ISSUES IN MODELING FOR CONTROL [1]

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Abstract

This paper highlights the role of feedback in the identification and validation of a model, when that model is to be used for control design. Feedback reduces the uncertainty of the estimated model in frequency bands that are critical for control design. Thus, in the presence of noise, closed loop identification for control leads to less conservative robust control designs than open loop identification of validated full order models, followed by controller design.

1 Introduction

This paper discusses a number of issues in the problem of modeling and identification for control design. We restrict the analysis to linear models and linear controllers. We provide insights and partial answers to the following central question: 'How should we identify a model $\hat{P}$ that is *good for control design* ?'

A reasonable qualification of a *good model* $\hat{P}$ *for control design* is

(i) the controller $C(\hat{P})$ designed from this model stabilizes the model $\hat{P}$ and the plant P (simultaneous stabilization), and

(ii) the achieved performance, on the $(P, C(\hat{P}))$ loop, is close to the designed performance, on the $(\hat{P}, C(\hat{P}))$ loop.

The characterization of all models that satisfy (i), for a model reference control design criterion, was studied in [4]-[3].

Here P is a symbol used to denote a description of the plant, possibly with its disturbance characteristics; similarly $\hat{P}$ may denote an input-output model, possibly with a disturbance model and an uncertainty description; finally $C(\hat{P})$, or C for short, denotes a controller designed from the model $\hat{P}$, which could possibly be a two-degree-of-freedom controller. We observe that the problem of modeling for control involves three players: the plant P, the model $\hat{P}$ and the '*to be designed controller*' $C(\hat{P})$.

Identification for control often involves one or several steps of closed loop identification, by which we mean identification of a model $\hat{P}$ of the plant P with data collected on the closed loop system formed by the feedback connection of P and some controller C_{id}. We denote this closed loop system by (P, C_{id}). Closed loop identification also involves three players: the plant P, the model $\hat{P}$ and the controller C_{id}. These three players are not the same as before, because the controllers are different, although they overlap. As a result, closed loop identification for control involves four players: the plant P, the model $\hat{P}$, the controller C_{id} applied during identification, and the 'to be designed controller' C. The focus of our discussion is on the interplay between these four players. The typical context is one in which a plant P is presently under closed loop control, with a low order controller C_{id}, and where it is desired to estimate a model $\hat{P}$ with the view of designing a new low order controller C that should achieve better performance on the plant P while providing stability robustness guarantees.

In particular we address the following issues.
• We examine the role of the controller in changing the experimental conditions.
• We compare the effects of open loop and closed loop identification in terms of bias and variance errors in the

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context of identification for control.
• We compare iterative design using closed loop data with the alternative of first identifying a high order model (preferably in open loop) that is validated by the data, and of then using this model for the design of a low order controller using model or controller reduction techniques. The claimed advantage of this alternative is that the low order model comes with an uncertainty description.
• We motivate the need for cautious controller adjustments in iterative design.

2 The role of feedback

It is now well known that one can find two plants whose Nyquist diagrams or impulse responses are practically indistinguishable, and a controller for the two plants for which the closed-loop behaviours are enormously different - even unstable in one instance and stable in the other. Conversely one can attach a stabilizing controller to two plants and observe what appear to be identical closed-loop behaviours, when the open-loop behaviours of the plants are enormously different. These facts were vividly illustrated by Skelton [11], who pointed out, for example, that the plants $P_1 = \frac{1}{s+1}$ and $P_2 = \frac{1}{s}$ have remarkably different open loop behaviours, while an output feedback $u = -Ky$ will make their closed loop behaviours almost indistinguishable for large K.

We return now to the modeling game with three players described in the introduction. Since the model $\hat{P}$ is used for the design of C, what is required of $\hat{P}$ is that the stability of the $(\hat{P}, C)$ loop implies that of the (P, C) loop (stability robustness), and that the closed loop transfer functions of these two loops are close to one another (performance robustness).

Now one of the main aims of feedback (historically) is to reduce the effects of model uncertainty on the open loop plant. Feedback often has a sensitivity reduction objective. Therefore any sensible feedback design will have the effect that the closed loop systems (P, C) and $(\hat{P}, C)$ behave much closer to one another than P and $\hat{P}$.

What are the consequences of these observations on identification for control? The issue here is change of experimental conditions. Models can only have their quality evaluated for a particular set of experimental conditions. Changing from open-loop to closed-loop operation with a specified controller is of course one change of experimental conditions. *So is any change of a controller a change of experimental conditions.* Unless a plant model is exact, high accuracy under one set of experimental conditions (e.g. open loop) does not guarantee its efficacy under changed experimental conditions. It is also intuitively clear that small changes of controller should probably avoid, or ameliorate, the problem of possible loss of efficacy of a model under changed experimental conditions.

As a consequence, the best way to evaluate the quality of the model $\hat{P}$ is to test it under the experimental conditions under which the plant P is due to operate, i.e. in closed loop with the 'to be designed controller' C. For the same reason, it should ideally be identified under those same feedback conditions. This is of course impossible since knowledge of the model is required to design the controller C. The philosophy behind iterative design of models and controllers is to approximate these experimental conditions in successive steps. This allows one to successively reduce the uncertainty in the frequency bands of importance for the design of the next controller, even with reduced order models.

The alternative, advocated in [8]-[9], is to identify a model of sufficiently high order that it is validated by the data, and to subsequently perform a step of model or controller reduction. This will lead to much more conservative designs because a validated model (typically of high order) will have an uncertainty distribution that is not shaped for control design. In addition, this uncertainty cannot be reduced by order reduction, whether frequency shaped or not. These points will be illustrated in Sections 4 and 5.

3 Discussion of bias errors

For the purposes of analysis, we shall from now on consider that there exists a 'true system' :

$$y = Pu + v \tag{1}$$

where P is a linear time-invariant transfer function, u is the control input, y is the output and v denotes additive perturbations at the output. We consider that this system is possibly connected to a unity feedback controller C :

$$u = C(r - y). \tag{2}$$

If we denote by $\bar{y}$ the output of the feedback connection of a model $\hat{P}$ and the same controller C, then the closed loop expressions for the outputs y and $\bar{y}$ are

$$\begin{aligned} y &= \frac{PC}{1+PC}r + \frac{1}{1+PC}v \\ \bar{y} &= \frac{\hat{P}C}{1+\hat{P}C}r \end{aligned} \tag{3}$$

The mean square error between the actual output and the nominal output is therefore given by[1]

$$E|y-\bar{y}|^2 = \int \left| \frac{P-\hat{P}}{1+\hat{P}C} \right|^2 |S|^2|C|^2\Phi_r + \int |S|^2\Phi_v. \quad (4)$$

Here S is the actual sensitivity function: $S = \frac{1}{1+PC}$.

Consider now that a parametrized model $\hat{P}(\theta)$ is identified from N input-output data obtained on the true plant, possibly connected to a feedback controller C_{id}, using a prediction error method. Define, as usual,

$$\begin{aligned} \epsilon_F(t,\theta) &= L(\theta)[y(t) - \hat{P}(\theta)u(t)] \\ V_N(\theta) &= \sum_1^N \epsilon_F^2(t,\theta) \\ \hat{\theta}_N &= \arg\min V_N(\theta). \end{aligned} \quad (5)$$

Here $L(\theta)$ is a prefilter that may possibly be parametrized by the parameter vector θ. It then follows easily that

$$\hat{\theta}_N \to \theta^* = \arg\min V(\theta) \quad \text{as} \quad N \to \infty$$

where

$$\begin{aligned} V(\theta) &= \int |P - \hat{P}(\theta)|^2 |L(\theta)|^2 \Phi_u^r \\ &+ \int |1+\hat{P}(\theta)C_{id}|^2 |S|^2 |L(\theta)|^2 \Phi_v. \end{aligned} \quad (6)$$

Here Φ_u^r is the spectral density of that part of the control signal that originates from the reference input. Indeed one can split the input spectrum into

$$\Phi_u = \Phi_u^r + \Phi_u^v \quad \text{with} \quad \Phi_u^r = |\frac{C_{id}}{1+PC_{id}}|^2\Phi_r. \quad (7)$$

Observe now that if one takes a parametrized prefilter $L(\theta) = \frac{1}{1+\hat{P}(\theta)C}$, then the minimizing argument of $V(\theta)$ in (6) is the same as the minimizing argument of

$$\bar{V}(\theta) = \int \left| \frac{P-\hat{P}(\theta)}{1+\hat{P}(\theta)C_{id}} \right|^2 |S|^2|C_{id}|^2\Phi_r \quad (8)$$

Thus the identification will tend to a model that realizes the best compromise between fitting $\hat{P}(\theta)$ to P and making the model sensitivity $\frac{1}{1+\hat{P}(\theta)C_{id}}$ small. Additionally, the weighting term in (8) contains the actual sensitivity S. Thus, the fit between $\hat{P}(\theta)$ and P will be tight where either the actual sensitivity S or the model sensitivity is large, i.e. around the corresponding crossover frequencies.

[1] All integrals in the following expressions are to be seen as integrals over the frequency argument ω. For simplicity - and reasons of space - we have omitted this frequency argument.

This last observation has been the main heuristic motivation for performing closed loop identification when low order models are used for the purpose of designing new controllers with higher performance. The heuristic reasoning is that, since the model is necessarily biased by the very nature of its low orderness, one should organize the bias distribution such that one has low error around the cross-over frequency of the present controller. A small uncertainty around this cross-over frequency then allows one to design a new controller that will expand the existing bandwidth.

This heuristic motivation can be given more solid support by considering our observations of Section 2 on the assessment of the quality of models. Ideally the model $\hat{P}$ should be evaluated by how well it mimics the behaviour of the actual system when both are connected in feedback with the 'to be designed controller' C. In other words, a model $\hat{P}$ is judged good if the quantity $E|y-\bar{y}|^2$ in (4) is small when the controller C operates in the closed loops (3). We now observe that, if $C_{id} = C$, then the minimization of the asymptotic identification criterion (8) also minimizes the performance criterion (4). Thus, when low order models are used for control design, then the bias should be minimized in the closed loop sense of the criterion $\bar{V}(\theta)$ (see (8)). This is achieved by closed loop identification with the 'to be designed controller' in the loop: $C_{id} = C$. Of course, this is impossible since the 'to be designed controller' C is unknown.

The concept of iterative identification and control design is an attempt to approach these ideal experimental conditions by slow adjustments from one controller to the next one. By slowly increasing the required performance (i.e. the designed bandwidth) the 'to be designed controller' is never very different from the controller C_{id} that is presently operating, and therefore the experimental conditions are always close to the optimal ones. This philosophy has been called the 'windsurfer approach' in [7]. The reason for introducing small steps of controller adjustments is not just to ensure that the frequency weighting during identification is close to the desired frequency weighting with the new controller. It is, more importantly, justified by stability robustness arguments: see [2] and [1].

In this section we have justified the use of iterative closed loop identification and control on the basis of *bias considerations*. We observe that the matching of the criteria (4) and (8) is impossible if the data are collected in open loop, even if the 'to be designed controller' C were known because both criteria contain the actual sensitivity S which depends on the unknown true system. *This bias justification only holds when reduced order models are used. Full order models can be* *identified without bias, and there is then no theoretical reason to prefer closed loop to open loop identification.*

In the context of modeling for control, it has therefore been argued that a (wiser?) alternative to closed loop identification of low order models is to identify the plant in open loop using higher order unbiased models that can be *validated*, and to then apply a model or controller reduction step if a low order controller is desired. This approach has recently been strongly advocated in [8]. In a statistical framework a model is called validated if the estimated uncertainty region around the estimated model contains the true system with some prescribed probability level (say 95%). For validated models, the variance error dominates the bias error [10]. In the next section, we explain why such strategy may lead to very conservative controller designs because it produces larger uncertainty regions on the estimated transfer functions than those obtained with low order models; in addition these uncertainty regions are essentially equally distributed over the frequency range.

4 Respect the uncertain, but take advantage of feedback

In this section we focus on the variance errors in the estimated transfer functions. It was shown in [5] for minimum variance control design, and in [6] for other control design criteria that closed loop identification is optimal for the minimization of the variance error on a 'to be designed controller' when the system is in the model set. For some criteria (minimum variance, model reference) the optimal design consists in closed loop identification with the 'to be designed controller' in the loop during data collection, (i.e. $C_{id} = C$), a similar conclusion to that reached in the previous section on the basis of bias considerations. Needless to say, this optimal design is again not applicable because computation of the optimal 'to be designed controller' requires knowledge of the system being identified. However, the results give heuristic support to iterative schemes.

There are two important limitations to the relevance of the optimal design results of [5] and [6]. The first and most important is that they assume the system to be in the model set, i.e. only variance errors are considered. The second is that these results were based on performance considerations only, without regard for robust stability considerations. Here we focus on the robust stability issue, and we explain why, on the basis of variance considerations (but bias errors are allowed too), closed loop identification with a controller that is as close as possible to the 'to be designed controller' is to be preferred over open loop identification of a validated model.

We first present the validation procedure suggested by Ljung [8] for models identified with prediction error methods. We consider again that there exists a true linear time-invariant system described by (1). We evaluate a model $\hat{P}$, by constructing the residuals formed by the difference between the measured and simulated output:

$$\eta(t) = y(t) - \hat{P}u(t) = \tilde{P}u(t) + v(t) \qquad (9)$$

The residuals contain two contributions: one comes from the input and contains information on the model error, and one comes from the noise and contains no such information. Equation (9) is referred to as the *model error model.* The problem of model validation is to decide which part of the residual can be explained by model errors, and which can be attributed to the noise. The interesting observation made in [8] is that the data contain information about this split between the two contributions to η. Indeed, the elements of the crosscorrelation function between $\eta(t)$ and a finite vector of past $u(.)$'s can be taken as a Finite Impulse Response estimate of $\tilde{P}$. The model validation strategy proposed in [8] is to accept an estimated model $\hat{P}$ if the corresponding estimate of $\tilde{P}$ and its uncertainty region (at a specified probability level) contains 0 at all frequencies. This means that, at that probability level, the true P is contained in the uncertainty region around the estimated $\hat{P}$, i.e. it is not falsified.

This strategy is very appealing as a model validation strategy. However, it leads to (full order) models with high variance and, more importantly, with a variance error that is - roughly speaking - equally distributed over the frequency range if the input signal to noise ratio is more or less equally distributed. It is easy to understand that, if a low order controller is computed on the basis of such uncertainty regions, then stability robustness guarantees will require conservative designs because of the large - and unfocused - uncertainty regions. Instead, an identification using similarly noisy data collected in closed loop with a controller that is close to the 'to be designed controller', will produce low variance errors in the frequency range of interest for the control design, leading to less conservative designs. Two issues are involved here that both contribute to the larger uncertainty in the regions of importance for control design: (1) the uncertainty distribution is not tuned towards the control design objective, and (2) a model that is validated in all frequency regions is of higher order (thus has higher variance) than one that only needs validation in a smaller frequency region. Observe also that subsequent order reduction of a validated model will not reduce the uncertainty.

5 Simulation

Although these new insights on variance errors are still preliminary and need to be confirmed by theoretical work, we have developed a simulation in order to test their validity. Assume that the true system P is described by the following ARX model:

$$(1-1.42z^{-1}+0.45z^{-2})y(t) = z^{-1}(1+0.25z^{-2})u(t)+e(t)$$

where e is unit variance white noise. The experiment consists of identifying the model and, using the variance of the parameter estimates, of computing the maximal gain of a proportional output feedback controller that leaves the closed loop poles of all possible models within the stability region, with a probability level of 95%. We performed the three identification experiments described below, and for each of these we computed the maximum gain K_{lim} that would guarantee closed loop stability for all models in the corresponding uncertainty regions delivered by 2σ intervals around the estimated model parameters. The three experiments, all with 1000 data, were as follows.

Experiment 1 : open loop identification with unit variance white noise;
Experiment 2 : closed loop identification, with a proportional controller $u = r - y$; the reference signal r is white noise with variance 20, leading to the same output variance as in the open loop experiment; the parameters of the open loop model P are estimated;
Experiment 3 : closed loop identification, with a proportional controller $u = r - y$; the reference signal r is as in experiment 2; the parameters of the closed loop model from r to y are estimated.

The idea of running the third experiment is that, instead of identifying the open loop system and computing the controller from $\hat{P}$ and its uncertainty region, one can alternatively identify the model of the present closed loop system and compute the new controller gain as a correction to the previous one. We expected that the closed loop models might give tighter uncertainty regions.

Results
Each of these 3 experiments was run 100 times, in order to get conclusions that would not depend on a particular noise realization. For the true system, the smallest destabilizing gain for the proportional controller is $K_{lim} = 2.2$. The estimated limit gains $\hat{K}_{lim}$ obtained by averaging the 100 Monte Carlo 100 runs with the three successive experiments described above were 1.53, 2.05 and 2.08, respectively. The variance around these estimates were 0.024, 0.022 and 0.002, respectively. A new set of 100 simulations were performed of the closed loop identification experiments with the gain $K = 2.07$ replacing the initial gain of $K = 1$. Here experiment 3 produced a new estimated $K_{lim} = 2.18$ with very low variance, while experiment 2 failed.

6 Conclusions

In identification for control, one should respect the uncertain, but use feedback to one's advantage. One of the roles of feedback is to reduce uncertainty where this is needed.

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