C64 CDC 30, 1991

H_2 Iterative Model Refinement and Control Robustness Enhancement

Zhuquan Zang† Robert. R. Bitmead† Michel Gevers‡

† Department of Systems Engineering
Research School of Physical Sciences and Engineering
Australian National University
GPO Box 4, Canberra, ACT 2601
Australia

‡ Department of Automatic Control
University of Louvain
Place Du Levant 3, B-1348 Louvain-La-Neuve
Belgium

ABSTRACT: Many practical applications of control system design based on input-output measurements permit the repeated application of a system identification procedure operating on closed loop data together with successive refinement of the designed controller. Here we develop a paradigm for such an iterative design. The key to the procedure is to account for evaluated modelling error in the control design and, equally, to allow for the requirements of the closed loop controller in performing the identification. With an H_2 control problem, this is achieved by frequency weighting the LQ control criterion and by filtering the identifier signals in a logical fashion.

1 Introduction

Theory exists (robust stability theory and robust control) which allows the inclusion of modelling error into controller design. For example, in the worst case analysis of H_∞ optimal control the H_∞ norm of the modelling uncertainty is included into the design to derive a weighted H_∞ design. Similarly gain and phase margins may be specified in LQG control and LTR design methods. There is a problem apparent with this in that the modelling error (or at least a bound) is assumed to have been provided *ex deus.* In more realistic situations the model error is the result of system identification with physical modelling and this does not conform to the standard picture of robust control because of the availability of design variables for the identifier. Moreover, in many circumstances the modelling itself is selectable by the designer and admits the prospect of being tuned to suit the particular control law schema. In this paper we shall follow through the development of a combined iterative control system design which couples the separate stages of model identification using frequency weighted Least Squares with controller design from this model using frequency weighted LQG methods. Several successive passes through these distinct operations will generate our iterative design. Such a design is not truly an Adaptive Controller but is closely related in spirit because our control law resulting will be shaped by the measured data and not just by a priori assumption.

We are describing a quasi-adaptive control scheme in which the simultaneous control design and parametric identification is replaced by an iteration of the identification stage with fixed controller and the control design stage based on the identified model. In many respects this concurs with the averaging approach of [2] or the periodic adjustment of [3]. In [1] a procedure was presented for modifying least squares plant estimation to accommodate the control law requirements. This involved linear filtering of all the data prior to estimation. The novelty of our approach here will be to utilize both parameter and variance data from the identification phase in the specification of the control objective also. In this way a more cogent approach to the identification and control of linear systems will be developed.

In very many practical applications of modern control, it is the case that an initial controller may be refined using on-line performance measurements and that, further, the amount of such data is effectively unlimited. In such circumstances one may use these measurements to generate more appropriate models and, subsequently, better feedback control laws, as opposed to, say, a once-off robust design which does not utilize process performance measurements. Our aim here will be to develop a strategy for successive improvement of control laws using system data.

The paper is organized as follows. In Section 2, we consider the H_2 (least squares) system identification methods and the corresponding H_2 (LQG) control laws. In Section 3 we develop a computational example and give some comments and discussions about the results obtained. Finally, in Section 4 we give some concluding remarks. Throughout this paper, all the results will be derived for the simple case of single input single output system. The extension to the multivariable case is reasonably straightforward but messy.

2 Iterative LS Identification and Frequency Weighted LQ Control Design

In this section we shall develop a procedure for iterative plant identification and LQ control design. Here the novelty of our H_2 approach is to account for evaluated modelling error in an LQ optimal control design and to allow for the requirements of the closed loop controller in performing the least squares identification. This is achieved by frequency weighting the LQ control criterion using a filter which contains identification information and by filtering the identifier signals using a filter which contains LQ optimal control design information. For other ways of identifying a plant model or estimating model error for the purpose of robust control design see, e.g., [7], [9]-[12].

2.1 Problem Formulation

Here our global objective is to identify a model $\hat{P}$ and then, based on $\hat{P}$, to design a controller. With the designed controller and the identified model the following *global* LQ performance criterion is considered

$$J^* = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [(y_t - r_t)^2 + \lambda u_t^2] \tag{2.1}$$

where y_t is the real plant output, u_t is the control signal to be designed based on an identified plant model to force the output y_t to track a given reference trajectory r_t as closely as possible, where r_t is modelled as the output of a reference model driven by a white noise n_t, i.e. $r_t = R(z)n_t$, where $R(z)$ is a rational transfer function chosen by designer to reflect the reference spectrum. Our aim is to minimize J^* by applied the designed controller to the real plant.

2.2 Problem Solution

2.2.1 Modified LQ Control Design

We follow here the basic error modelling formulation of Ljung [4]. Suppose we are given a real plant with an input-output relationship described by the following equality.

$$y_t = P(z)u_t + v_t \tag{2.2}$$

where $P(z)$ is a strictly proper rational transfer function, u_t is the input, v_t is an unmeasurable disturbance acting on the output y_t. Also we are given a parametrized model set

$$\{\hat{P}(z,\theta), \hat{H}(z,\theta), \quad \theta \in D_\theta \subset R^d\}, \tag{2.3}$$

A particular model in that model set will be described by

$$\tilde{y}_t(\theta) = \hat{P}(z,\theta)u_t + \hat{H}(z,\theta)q_t \tag{2.4}$$

for a specific value of θ, where q_t is a white process and $\hat{P}$ and $\hat{H}$ are, respectively, strictly proper and proper transfer functions. Also we suppose the poles of both $\hat{H}$ and $\hat{H}^{-1}$ are strictly inside the unit circle. Associated with the model (2.4) is the following one step ahead predictor.

$$\hat{y}_t(\theta) = \hat{H}^{-1}(z,\theta)\hat{P}(z,\theta)u_t + (1 - \hat{H}^{-1}(z,\theta))y_t \tag{2.5}$$

see, Ljung [4]. In particular, if we model the real plant by using a disturbance free model ($\hat{H} = 0$) or assuming the disturbance acting on the model output is a white noise, it is easy to see that the one step ahead predictor corresponding to these two special models is

$$\hat{y}_t(\theta) = \hat{P}(z,\theta)u_t \tag{2.6}$$

Combining either one of the above two predictors with the real plant (2.2) we therefore can define a prediction error

$$\epsilon_t(\theta) = y_t - \hat{y}_t(\theta)$$

and its spectrum (assuming closed loop stability and quasistationary exogeous signals, see [4]).

$$\Phi_{y-\hat{y}}(\omega) = \sum_{\tau=-\infty}^{\infty} \bar{E}(\epsilon_t(\theta)\epsilon_{t-\tau}(\theta))e^{-j\omega\tau}$$

where $\bar{E}$ denotes the appropriate average or expected value. We suppose that a plant model $\hat{P}$ is provided together with the information of the prediction error spectrum $\Phi_{y-\hat{y}}(\omega)$. We then consider the specification of a frequency weighted LQ tracking problem based on this model.

Since the controller u_t is designed based on a known plant model, we need to transform the above tracking problem (2.1) into separate but interacting identification and control problems. The aim will be to take account of the identification of the model in the formulation of the control law and of the feedback control objective in performing the identification. These modifications need to reflect the overall control objective (2.1).

We proceed as follows. Instead of following the traditional formulation of an LQ tracking problem by considering minimizing the following performance criterion $\hat{J}$

$$\hat{J} = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N}\{(\hat{y}_t^c - r_t)^2 + \lambda'(u_t^c)^2\} \tag{2.7}$$

we begin with the following frequency weighted LQ tracking problem.

$$J = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N}\{[F_1(\hat{y}_t^c - r_t)]^2 + \lambda'(F_2 u_t^c)^2\} \tag{2.8}$$

where $\hat{y}_t^c$ is the output of an identified model operating in closed loop, F_1 and F_2 are weighting functions (linear filters) to be chosen, λ' is a constant to be decided. We shall study how the modelling information might be incorporated into the local control objective (2.8) to reflect the global objective (2.1) through the judicious choice of F_1 and F_2. Recall that control design only may be performed relying on the identified model.

Suppose through identification, we obtain a plant model. Its input u_t^c and output $\hat{y}_t^c$ are related by

$$\hat{y}_t^c = \hat{P}u_t^c + v_t' \tag{2.9}$$

where the disturbance v_t' is not necessary equal to v_t—the unmeasurable plant disturbance. The above system is depicted in Fig. 1 with the controller denoted by C_1, C_2 yet to be designed. Notice that here we consider a two-degree-of-freedom controller design problem. This form of controller ($u_t = C_1 n_t - C_2 y_t$) is consistent with that obtained in LQ optimal control design, whether the design is based on a state space model or on an input output model, see, [1], [8].

By direct comparison we select $F_1(z)$ and $F_2(z)$ as

$$F_1 = \left(\frac{\Phi_{y-r}}{\Phi_{\hat{y}^c-r}}\right)^{1/2}, \quad F_2 = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2} \tag{2.10}$$

and λ' as λ to make the frequency weighted tracking objective (2.8)

$$J = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\left\{\left[\left(\frac{\Phi_{y-r}}{\Phi_{\hat{y}^c-r}}\right)^{1/2}(\hat{y}_t^c - r_t)\right]^2 + \lambda\left[\left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}u_t^c\right]^2\right\}.$$

This should be compared to (2.1) and interpretted as a distortion of the local control objective function for the identified model to reflect the global criterion, as was motivated by the H_∞ frequency weighted control design method derived in Section 2.

The question now is: How do we evaluate these filters F_1 and F_2? This is done directly from the identification experiment itself (in which $y_t - r_t$ and u_t are measured signals) and from the previous control design stage and the identified model using the relationships

$$\hat{y}_t^c - r_t = \frac{\hat{P}(C_1 - RC_2) - R}{1 + \hat{P}C_2}n_t + \frac{1}{1 + \hat{P}C_2}v_t'. \tag{2.11}$$

and

$$\hat{u}_t^c = C_1 n_t - C_2\hat{y}_t^c = \frac{C_1}{1 + \hat{P}C_2}n_t - \frac{C_2}{1 + \hat{P}C_2}v_t'. \tag{2.12}$$

A specific example of such a calculation of these spectra is given in the next section and relies simply on the processing of closed loop experimental data.

Remarks:

- Since F_1 and F_2 act as weighting functions in the local LQ optimal control design objective (2.8), both C_1, C_2, and $\hat{P}$ need to be fixed and available before the starting of each specific design stage. Obviously, the $\hat{P}$ component in the weighting functions comes from the identification stage, C_1 and C_2 are derived from an earlier LQ control design stage.

- The selection of the particular factorizations in (2.10) is the stable, minimum phase spectral factors. This admits the recasting of the frequency weighted LQ tracking problem as a non-weighted LQ problem with transfer functions, $\bar{P} = \hat{P}F_1F_2^{-1}$, $\bar{C}_1 = F_2C_1$, $\bar{C}_2 = F_2F_1^{-1}C_2$, and $\bar{R} = RF_1$.

- If we analyse more closely the choice in (2.10), we see that, using modelling information in the form of F_1 and F_2, the control criterion is being modified to penalise more heavily both control and output signal deviation in frequency bands where the model fit is poor.

Now the above choice of the weighting functions allows us to conclude that designing a controller C_1, C_2 based on an identified model such that the performance criterion (2.1) is minimized is approximately equivalent to minimizing the performance criterion (2.8) with F_1, F_2 and λ' chosen as above. In this fashion, sensible modelling information modifies the local control objective.

2.2.2 Modified Least Squares Identification

Now the remaining task is to identify iteratively the plant model and then design a controller so that in each identification and control design step the criterion (2.8) is reduced.

First we need to specify an identification criterion which suits our interacting identification and control design implementation. For the prediction error $\epsilon_t(\theta) = y_t - \hat{y}_t(\theta)$ we define $V_N(\theta)$ as

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}[\epsilon_t^f(\theta)]^2$$

where $\epsilon_t^f(\theta)$ denotes the prediction errors filtered through a stable linear filter with transfer function $D(z)$:

$$\epsilon_t^f(\theta) = D(z)\epsilon_t(\theta).$$

Our identification criterion is then defined as

$$V(\theta) = \lim_{N\to\infty} V_N(\theta), \tag{2.13}$$

Using (2.2), (2.6) and the definition of $V(\theta)$, the above criterion can be written (in the frequency domain) as

$$V(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\left|\frac{(P-\hat{P})C_1}{1+PC_2}\right|^2 + \left|\frac{(1+\hat{P}C_2)}{1+PC_2}\right|^2\Phi_v\right\}\left|\frac{D}{\hat{H}}\right|^2 d\omega \tag{2.14}$$

Note that here we have used the whiteness of n_t and the uncorrelatedness of n_t with v_t. Here, without loss of generality, we can assume that $\hat{H} = 1$. The exertion of influence over the Least Squares identification criterion is through the choice of filter D, so that the (local) identification criterion (2.14) is commensurate with the global criterion (2.1).

In selecting the filter, $D(z)$, to be applied to affect the identification criterion we draw the distinction between modelling for closed loop control and prediction error minimization.

Two systems are depicted in Fig. 2 and 3, defining signals y_t, the true plant output, and $\hat{y}_t^c$, the model output, operating in closed loop with the feedback controller and identical input sequence n_t

Our identification-for-control aim is to select a plant model $\hat{P}$ from some admissible class so that $\hat{y}_t^c$ is closest to y_t in a least squares sense. More exactly, we try to select a plant model from the model set (2.3) through identification such that the following performance criterion is minimized.

$$\gamma = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}[(y_t - \hat{y}_t^c)^2 + \lambda(u_t - u_t^c)^2] \tag{2.15}$$

This should be compared to the prediction error minimization depicted in Fig. 4, which is an open-loop identification. We may now ask how these two minimizations are connected.

Clearly from Fig. 2 and 3 we have

$$y_t = \frac{PC_1}{1+PC_2}n_t + \frac{1}{1+PC_2}v_t. \tag{2.16}$$

$$\hat{y}_t^c = \frac{\hat{P}C_1}{1+\hat{P}C_2}n_t. \tag{2.17}$$

Thus, in the case when $\lambda = 0$, $v_t' = 0$, γ (we use γ_0 to denote γ in this case) can be written as (in the frequency domain)

$$\gamma_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\frac{|(P-\hat{P})C_1|^2}{|(1+PC_2)(1+\hat{P}C_2)|^2} + \frac{1}{|(1+PC_2)|^2}\Phi_v\right\}d\omega \tag{2.18}$$

Note that here we have used the whiteness of n_t and the uncorrelatedness of n_t with v_t. A comparision between (2.18) and (2.14) immediately suggests that to achieve a minimization of closed-loop identification error the open-loop signal be filtered through

$$D(z) = (1+\hat{P}(z)C_2(z))^{-1} \tag{2.19}$$

Since the identified plant model $\hat{P}$ appears in this frequency weighting, it would normally only be feasible to adjust this filter using an earlier estimate of $\hat{P}$.

This choice of $D(z)$ is suited to the minimization of the squared error between y_t and $\hat{y}_t^c$, which is an aim consistent with minimum variance control, i.e. $\lambda = 0$ in (2.1). More exactly, we have the following

Theorem 2.1 *Suppose that the plant P, the model $\hat{P}$, and the controller C_1, C_2 are given as above. Also suppose that the controller C_1, C_2 stabilizes both the plant and the plant model. Then with the filter $D(z)$ chosen as in (2.19) we have*

$$V(\theta) = \gamma_0$$

$$\gamma_0 \geq \left[\left(\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}y_t^2\right)^{1/2} - \left(\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}(\hat{y}_t^c)^2\right)^{1/2}\right]^2$$

To extend this line of reasoning from minimum variance control through to the more general criterion J^*, we proceed as follows. Notice that by using (2.12), (2.16), (2.17), and Parseval's identity, in the frequency domain, (2.15) can be written as

$$\gamma = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\frac{|(P-\hat{P})C_1|^2(1+\lambda|C_2|^2)}{|(1+PC_2)(1+\hat{P}C_2)|^2} + \frac{(1+\lambda|C_2|^2)}{|(1+PC_2)|^2}\Phi_v\right\}d\omega \tag{2.20}$$

Now it is easy to see from (2.20) and (2.14) that if we define $G(z)$ as a stable filter obtained from the following factorization problem

$$G(z)G^*(z^{-1}) = 1 + \lambda C_2(z)C_2^*(z^{-1}) \tag{2.21}$$

by selecting

$$D(z) = G(z)(1+\hat{P}(z)C_2(z))^{-1} \tag{2.22}$$

we have $V(\theta) = \gamma$. In fact, we can prove the following

Theorem 2.2 *Suppose the assumptions in Theorem 2.1 are satisfied with G, D chosen as in (2.21) and (2.22), then we have*

$$V(\theta) = \gamma \tag{2.23}$$

$$\gamma \geq ((J^*)^{1/2} - (\hat{J})^{1/2})^2 \tag{2.24}$$

where J^* is defined as before and $\hat{J}$ is the unfiltered version of J, and defined in (2.7)

Now we conclude from the above theorem that by selecting the filter D as in (2.22) we therefore achieve, with open-loop identification, a model fit which causes the closed loop performance to be as close as possible to J^*. Note that as in the derivation of the control design filter F_1 and F_2, here the identification filter $D(z)$ needs to be fixed in each step of identification. Obviously the $\hat{P}$ component in the filter is derived from an earlier identification stage and the C_1, C_2 components are derived from the local LQ control design stage. Another important point to note is that if we assume $H = \hat{H}$, the preceding implementation for the selection of the filter $D(z)$ is still valid. In the case when $H \neq \hat{H}$ and $\hat{H} \neq 0$, extra effort needs to be taken in the selection of the filter D. However, the underlying ideas are completely the same as we have developed here. We are now in a position to state our H_2 iterative identification and control algorithm.

2.2.3 The Algorithm

We propose an identification and control schema as follows

Step 1. (Initial Closed Loop Identification:) Suppose there exists a controller C_1^0, C_2^0 which stabilizes the real closed loop system depicted in Fig. 4. (In the case when the plant itself is stable, we choose $C_1 = 1$ and $C_2 = 0$.) We use this controller to begin our closed loop identification. That is, we choose a plant model $\hat{P}_0$ from the model set (2.2) to minimize $V(\theta)$ with $D_0(z) = 1$. The identifier, together with C_1^0, C_2^0 provides $\hat{P}_0(z)$, F_1^0, and F_2^0. Where

$$F_1^0 = \left(\frac{\Phi_{y-r}}{\Phi_{\hat{y}^c-r}}\right)^{1/2}, \quad F_2^0 = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}$$

Step 2. (Initial LQ Control Design:) From Step 1 we get $\hat{P}_0$, F_1^0,and F_2^0. We then use this information to design a new controller so that (2.8) is minimized. That is, the following weighted LQ tracking criterion $J_{local,1}$ is minimized.

$$J_{local,1} = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N}\{[F_1^0(z)(\hat{y}_t^c - r_t)]^2 + \lambda[F_2^0(z)u_t^c]^2\}$$

This defines a new controller $C_1^1(z)$, $C_2^1(z)$.

Step 3. (Identification:) Using the newly obtained controller component C_2^1 we solve

$$G_1(z)G_1^*(z^{-1}) = 1 + \lambda C_2^1(z)C_2^{*1}(z^{-1})$$

for stable minimum phase $G_1(z)$, select

$$D_1(z) = G_1(z)(1 + \hat{P}_0(z)C_2^1(z))^{-1}$$

and perform the identification stage to minimize $V(\theta)$. This, together with $C_1^1(z)$, $C_2^1(z)$, provides us with $\hat{P}_1(z)$, F_1^1 and F_2^1. Where

$$F_1^1 = \left(\frac{\Phi_{y-r}}{\Phi_{\hat{y}^c-r}}\right)^{1/2}, \quad F_2^1 = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}$$

Step 4. Continue as in Step 2.
Step 5. Continue as in Step 3.
Remarks:

- The selection of the D-filter in least squares identification as in (2.22) is a feature countenanced in [1] but which here includes the explicit appearance of the global LQ control objective. An analysis of the form of $D(z)$ shows how the prediction error identification is modified to reflect better just those properties required of a model to perform well in closed loop. Specifically, it is clear that in the frequency region about the gain crossover point, D will have a significant magnitude which will emphasize fit. Similarly, with higher weightings on the control penalty, greater importance is attributed to modelling errors where the control gain is high. Thus, it would appear that this style of D concurs with some sensible intuitions.

- The question of convergence of the global performance criterion, J^*, is moot however. It is not clear that J^* itself reduces at each pass. Rather, at each control design, J^* is reduced for the particular model and then, at the identification, the model is fitted in concert with the applied control to cause the performance to conform. Clearly, questions remain and the issue of preservation of stability has not been resolved. Nevertheless, we believe that the approach augurs well for the logical refinement of controllers.

3 Design Example

3.1 Computer Experiment Setup

In this section, simulation results are presented to show the effectiveness of the theoretical approach of our iterative model identification and LQ control design. The iterative design strategy was performed on the following example.

The real plant is chosen to be fifth order of the form

$$y_t = \frac{B(z)}{A(z)}u_t + v_t \tag{3.1}$$

where

$$\begin{aligned} A(z) &= 1 - 1.25z^{-1} + 0.4575z^{-2} + 0.0279z^{-3} \\ &\quad -0.0491z^{-4} + 0.0077z^{-5} \\ B(z) &= z^{-1} - 1.2z^{-2} - 0.3z^{-3} + 0.156z^{-4} + 0.0845z^{-5} \end{aligned}$$

which is stable and non-minimum-phase with single delay. The plant model to be identified is assumed to be third order with a single delay, i.e. of the form

$$\hat{y}_t^c = \hat{P}(z)u_t^c + v_t' \tag{3.2}$$

and the reference model is chosen to be second order

$$R(z) = \frac{0.1311 + 0.2622z^{-1} + 0.1311z^{-2}}{1 - 0.7478z^{-1} + 0.2722z^{-2}} \tag{3.3}$$

In the whole experiment both the plant disturbance v and the model disturbance v' are the same and assumed to be white noise. The LQ control cost $\lambda = 0.4$ and the Kalman filter is designed with process noise of unit variance entering through the input channel and measurement noise of variance $\rho = 0.8$ (Roughly an LQG/LTR strategy.) The closed loop signal spectra Φ_{y-r}, Φ_{y^c-r}, Φ_u, Φ_{u^c} are modeled from measured signals using third order AR models. The closed loop identification was performed on 2048 samples per iteration.

3.2 Simulation Results

3.2.1 Open loop identification and control design

Figure 5 shows the frequency responses magnitudes of the true plant (solid line) and the open-loop-identified plant model (dashed line). Figure 6 shows the frequency response magnitudes of the closed loop sensitivity functions (solid line for the achieved sensitivity function and dashed line for the designed one) with the controller obtained from the LQG optimal control design based on the open-loop identified plant model. By the end of first iteration of plant model identification and control design we have $J^* = 0.1413$ and $\hat{J} = 0.1181$. That is, the open loop plant model and its corresponding unweighted controller yield this figure of merit according to the global criterion (2.1).

3.2.2 First and second iteration of the plant identification and LQG control design

Figure 7 gives the simulation results for the frequency response magnitudes of the identifier filters $G(z)$ (in dashed line) and $D(z)$ (in solid line). Both filters $G(z)$ and $D(z)$ are derived from model 1 (open-loop-indentified model) and control design 1 experiments.
Figure 8 gives the frequency response magnitudes of model 2 and the real plant.
Figure 9 gives the frequency reponse magnitudes of the weighting functions $F_1(z)$ (in solid line) and $F_2(z)$ (in dashed line) which we obtained from model 2 and control design 1

experiment.

Figure 10 shows the simulation results for the frequency response magnitudes of the closed loop sensitivity function obtained from model 2 and control design 2 experiment. In this round of the iterative plant identification and control design we have reduced the costs to $J^* = 0.1156$ and $\hat{J} = 0.1295$ for the achieved closed loop and designed closed loop respectively.

3.2.3 Further iterative plant identification and control design

Further iterations lead to very slight modifications with slight decrease of J^*. By the end of third iteration we have the performance criteria $J^* = 0.1114$ and $\hat{J} = 0.1285$. Most improvement was achieved from the first and second iterations. After second iteration the filters G, D, the weighting functions F_1, F_2 and the achieved and designed closed loop sensitivity functions are all stabilized.

3.3 Summary

This design example performed by computer demonstrates several points.

- The most crucial observation is that the achieved closed loop performance with the true plant improves from step to step as measured by the J^* global criterion, even though the local control criterion is not decreasing.
- (With later iterations sometimes a slight diminishment of performance is seen temporarily.)
- An inspection of the iterated closed loop sensitivities shows two features: the achieved sensitivity function is reduced in both an L_2 and an L_∞ sense, the designed sensitivity approaches the achieved sensitivity more closely.
- The weighting function, F_1 and F_2, and the identification filters, G and D, stabilize to fixed values as the iterations progress.
- (With the choice of particular control criterion, i.e. λ small, the effect the LQ criterion on the identification is slight. That is there is little benefit in using the G filter.)

4 Conclusions

We have developed an iterative identify-then-control paradigm. The focus of the approach is to consider a single global control objective and then to perform a sequence of
1. frequency weighted system identification,
2. frequency weighted control design,
each with their respective local objective functions. These criteria (embodied in the frequency weighting) for each case reflect the current (local) circumstances but are turned to address minimization of the global objective. Methods were presented for the H_2 (LQ and Least Squares) control design and plant identification.

To implement the methods requires that the identification stage provide not only a best fitting model but also a measure of model error. In the H_2 case the prediction error spectrum is used from the identification experiment. The model error is introduced to modify the local control law specification. In a complementary fashion, the global control objective and local controller are used to adjust the identifier's frequency weighting.

There exists a considerable amount of work to establish more detailed properties of such mehtods and to extend their validity fully to adaptive control. This work is on-going but it is clearly of interest to establish the connection between the applicability of these schemes and the provision of a priori plant information. In terms of practical applications, however, the methodology advanced here goes a long way towards adressing the questions of how to adjust and improve existing controllers using current on-line experimental data.

References

[1] R.R. Bitmead M. Gevers and V. Wertz, *Adaptive Optimal Control–Thinking Man's GPC*, Prentice-Hall, Englewood Cliffs NJ, USA, 1990.

[2] B.D.O. Anderson, R.R. Bitmead, C.R. Johnson Jr, P.V. Kokotovic, R.L. Kosut, I.M.Y. Mareels, L. Praly, and B.D. Riedle, *Stability of adaptive systems: Passivity and Averaging Analysis*, MIT Press, Cambridge MA, USA 1986.

[3] H. Elliott, R. Cristi, and M. Das, "Global Stability of Adaptive Pole Placement Algorithm", *IEEE Transaction on Automatic Control* vol. AC-30 pp. 348-356, 1985.

[4] L. Ljung, *System Identification: Theory fo the User*, Prentice-Hall, Englewood Cliffs NJ, USA,1987.

[5] B. Wahlberg and L. Ljung, "Design Variables for Bias Distribution in Transfer Function Estimation, " *IEEE Trans. Automat. Cont.*, vol. AC-31, pp. 134-144, 1986.

[6] M. Morari and E. Zafiriou, *Robust Process Control*, Prentice-Hall, Englewood Cliffs NJ, USA, 1989.

[7] G. Goodwin, M. Gevers, and B. Ninness, "Optimal Model Order Selection and Estimation of Model Uncertainty for Identification with Finite Data." Technical Report, December 1990, Dept. of Electrical and Computer Engineering, The University of Newcastle, Australia.

[8] B.D.O. Anderson and J.B. Moore, *Optimal Control: Linear Quadratic Methods*, Prentice-Hall, Englewood Cliffs NJ, USA, 1990.

[9] F.R. Hansen, G.F. Franklin and R. Kosut, "Closed-loop Identification via the Fractional Representation: Experiment Design."*Proc. ACC*, Pittsburgh, PA, 1422-1427.

[10] R.J.P. Schrama,"An Open-loop Solution to the Approximate Closed-loop Identification Problem", 1991 IFAC, Budapest, Hungary.

[11] R.L. Kosut, M.K. Lau and S.P. Boyd, "System Identification for Robust Control Design", *European Control Conference*, Grenoble, France, July 1991.

[12] B. Wahlberg and L. Ljung, "On Estimation of Transfer Function Error Bounds" *European Control Conference*, Grenoble, France, July 1991.

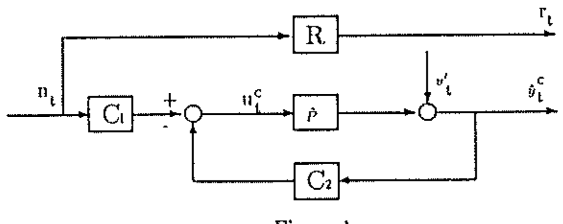

Figure 1

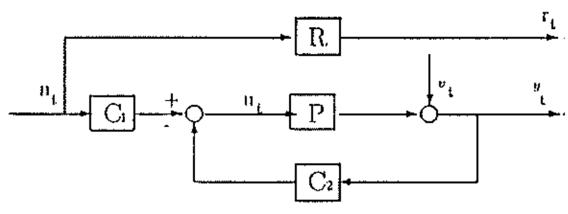

Figure 2

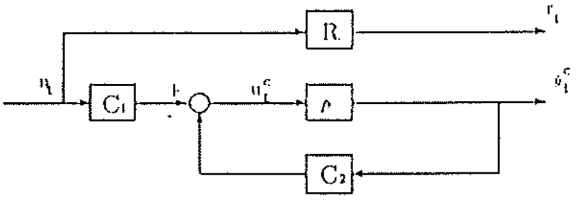

Figure 3

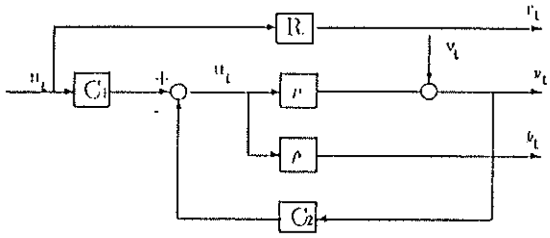

Figure 4

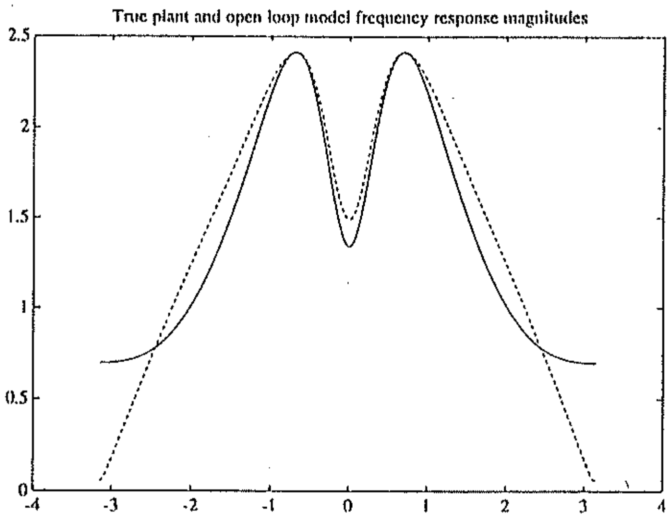

Figure 5

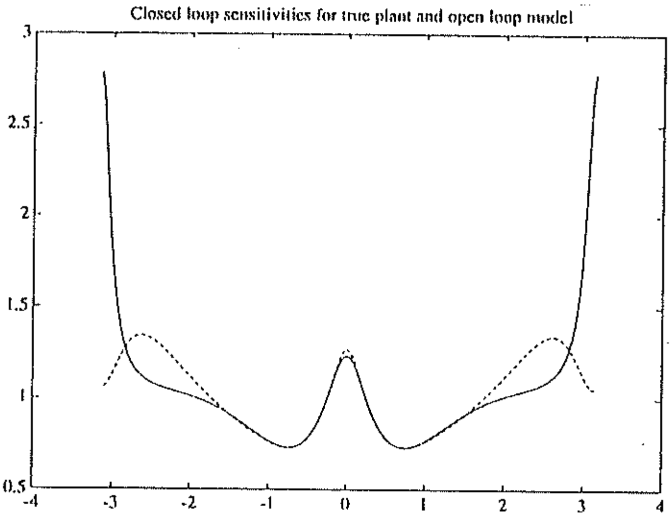

Figure 6

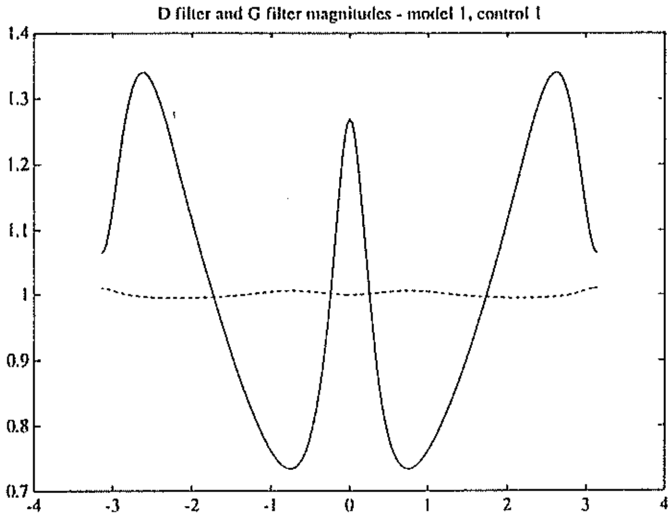

Figure 7

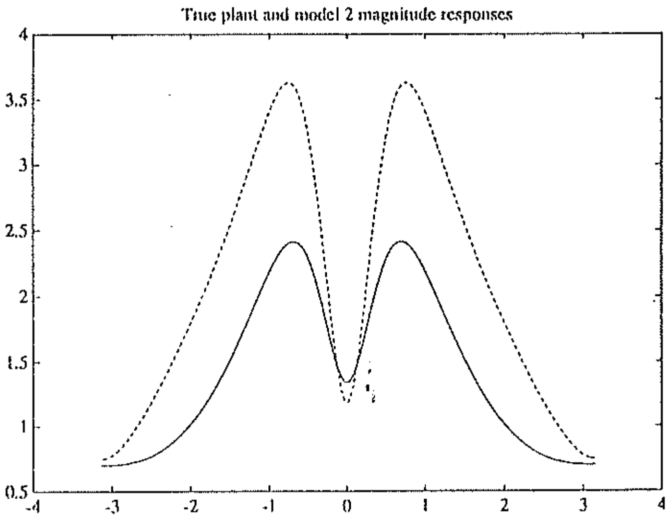

Figure 8

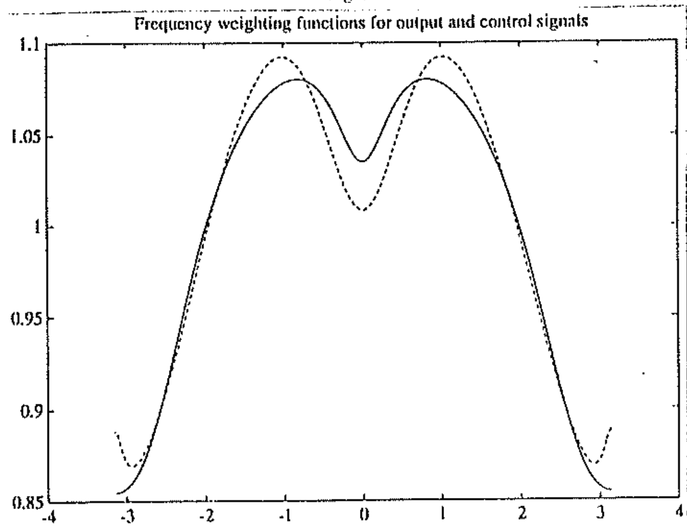

Figure 9

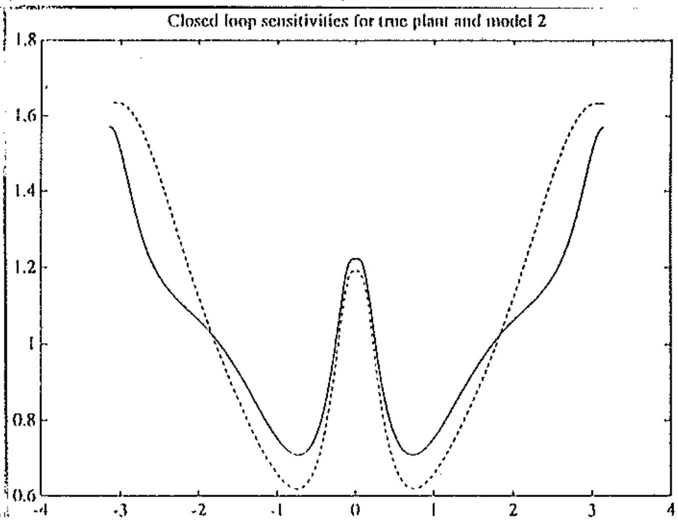

Figure 10