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# ON THE IDENTIFICATION OF A FEEDBACK CONTROLLED SYSTEM

M.R. Gevers
Laboratoire d'Automatique
Université Catholique de Louvain
Bâtiment Maxwell
B-1348 Louvain-La-Neuve

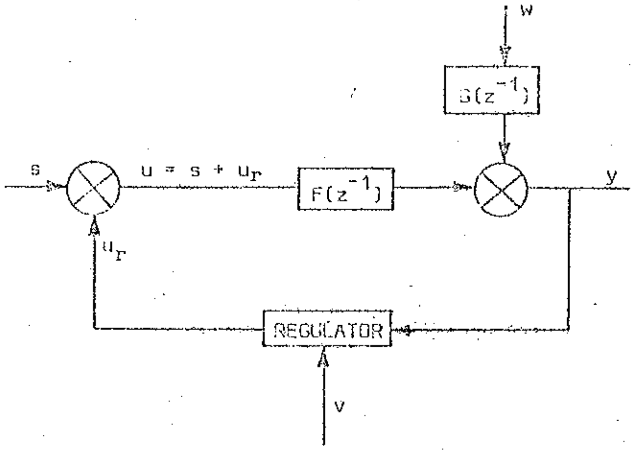
## Abstract

Various recent identifiability results for feedback systems are discussed and compared. It is shown that the use of a white noise perturbation input leads to robust methods, in that they require a minimum amount of information about the structure of the system. Such a method, using a combined stochastic approximation and covariance factorization algorithm, is briefly presented.

## I. Introduction

The identification of linear systems operating under feedback control will be considered. In most practical cases the dynamics of the regulator is unknown (e.g. a manual operator) and the control cannot be disconnected during identification. In addition, the system and the regulator are usually pertubed by unknown noise processes.

A closed loop system with linear open loop dynamics can in general be represented by the following configuration



where u is the measurable input signal, s is a measurable externally applied, y is the output signal and w and v are unmeasurable external noise sources that can be assumed white.

Various assumptions can be made about the regulator : the dynamics can be known or unknown, the regulator can be linear or nonlinear, constant or time-varying. Notice also that s and v may or may not be present.

The synthesis of an optimal regulator requires the identification of both the open-loop transfer function $F(z^{-1})$ and the noise dynamics $G(z^{-1})$ of the feedback controlled system.

## II. Existing results on the identifiability of feedback systems

In order to study the applicability of identification methods to practical situations the question of identifiability has to be tackled first. As Gustavsson et al.(1) have shown, identifiability depends upon the structure chozen for the model, the identification method, and the experimental conditions (e.g. choice of input signal, regulator structure, etc...).

They consider basically two different identification methods :

1°) Indirect identification : A model for the global feedback system is identified first, and the open loop model is subsequently derived assuming knowledge of the regulator. This technique is rarely suitable since in most practical situations the regulator is unknown and often noise corrupted.

2°) Direct identification : A prediction error method (such as the maximum likelihood method) is applied using the input u and the output y just as if the system were operating in open loop.

For this second method, Gustavsson et al. have shown that the system is identifiable without knowledge of the true system structure provided the chozen model structure is able to reproduce the exact system transfer function and noise dynamics, and one of the following conditions hold

a) the signal s can be chozen to be persistently exciting of high order

b) the signal v is independent of w and is persistently exciting (independently demonstrated in (2) )

c) the regulator is nonlinear and nondegenerate

d) the regulator is persistently time varying (see (1))

In all these cases direct open loop methods can be used. Notice that Wellstead (2) and Gustavsson et al. (1) require that there be at least one lag in the loop.

Another indirect method, not considered in(1), has been proposed recently. It does not require the exact knowledge of the regulator, except that the regulator be linear and noise corrupted. In this method, a new vector stochastic process z is definded made up of the input and output variables u and y, and an innovations model is obtained for the z process by a maximum likelihood method (3)-(5). The feedforward and feedback transfer functions, as well as the noise models are then derived from the matrix transfer function representation of this global model by matrix manipulations.

To obtain a unique model Chan(4) and Phadke (5) chooze a particular "canonical representation" for the global model. However the choice of a "canonical representation" is rather arbitrary. Indeed different causal and causally invertible innovations models can be defined for vector processes, that can be obtained from one another by similarity transformations, but that do not necessarily lead to the same open loop transfer functions.

In order to correctly identify the open loop dynamics, some knowledge is therefore necessary about the structure of the model.

## III. Critical comments

The major drawback of most identifiability results advocating direct open loop methods is that theyrequire some knowledge about the structure of the system and / or the regulator. In the best possible case, one must be sure that the chozen model structure, and in particular the parametrization, are such that there exits at least one value $\theta_o$ of the parameter vector $\theta$ for which the model and the system have the same open loop transfer function and noise dynamics.

Maximum likelihood estimation techniques, unfortunately, are such that the parametrization has to be decided a priori. If the estimated model is not correct a new parametrization has to be chozen and the minimization has to be started anew. In addition, maximum likehood methods usually require a good initial guess for the parameter estimate. Therefore, even for open loop systems, correlation or covariance factorization methods are often used as a first step.

One way of making the choice of the structure less critical is to add a measurable white perturbation input s, whereby the open loop input-output dynamics can be identified through a stochastic approximation method using an impulse response model, which is not as structure-dependent.

Saridis and Lobbia(6) have proposed such a method ; however they did not identify the noise dynamics, which were assumed known. Defalque, Installe and the author (7) have proposed a two-stage method for the identification of both the input-output dynamics and the noise dynamics. This method will now be briefly described.

## IV. Identification with an extra perturbation input

In (7) the system is identified in state-variable form under the assumptions that it is stationary, single-input single-output, and that it is pertubed by a colored Gauss-Markov process noise of unknown dynamics. No assumption is made about the structure of the unknown regulator.

Using a measurable white pertubation the input-output model is identified first through a stochastic approximation method that operates on the data in real time. The rank of the Hankel matrix of estimated impulse response elements is tested for the order of the model.

The estimated effect of the known input is then substracted from the system ouptut and a state-variable innovations model is obtained for the resulting noise process, which can be shown to converge to the noise dynamics model. A covariance factorization method is used, whereby the order of the model is easily determined.

If the order of the noise process is larger than that of the input-output model, (i.e. if the process noise is colored) a coordinate transformation is necessary to combine the two stages of the identification.

The reader is referred to (7) for more details and for a numerical simulation.

## V. Conclusion

Various recent identifiability results for feedback systems have been discussed. In several cases, direct maximum likelhood methods can be used ignoring the presence of feedback. It has been argued, however, that when not much is known about the structure of the system the use of methods using an external perturbation input may be preferable.

## References

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