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STABILIZATION OF NONLINEAR SYSTEMS BY MEANS OF STATE ESTIMATE FEEDBACK

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ABSTRACT

We are concerned with the class of single input single output (SISO) nonlinear systems that are locally observable and exponentially stabilizable by state feedback. An extended Luenberger observer is considered for which stability of the closed loop is preserved when the state is replaced by the state estimate : a convergence domain is characterized by means of a Lyapunov analysis.

I. INTRODUCTION

Over the past fifteen years, a number of research studies have dealt with the problem of extending observation techniques to nonlinear systems. In this field, a particular attention has been paid to the so called "exponential observers" (asymptotic state estimator with exponentially fast decaying error). Introduced by Kou et al. [7], this notion is related to the development of specific observer design methods possessing certain linearity properties. Several techniques have been proposed : immersion in a linear system [8], linearization by change of coordinates and output injection [2], [5], [6], [13], [14], and extended linearization [1]. The basic properties of exponential observers have only recently been examined by Xia et al. [12].

Although these observers are usually designed for regulation or control purposes, not much effort has been deployed for the analysis of the closed loop when the state is replaced by its estimate in the feedback law. In [1] and [12], local results have been obtained according to the well-known eigenvalue separation property.

In this paper, we are concerned with the class of exponentially stabilizable and locally observable nonlinear systems. For these systems, we first consider an extended Luenberger observer design and then use the observed state to regulate the nonlinear system with a suitable stabilizing input law (certainty equivalence principle). In order to examine the global behavior of the closed loop, a Lyapunov function is obtained which specifies a convergence domain of the equilibrium point (in which both the state trajectory and the state observation error trajectory are guaranteed to be bounded).

The paper is organized as follows. The structural conditions characterizing the class of nonlinear systems under consideration are established in section 2. In section 3 and 4, the dynamics of the state observer and of the closed loop system are successively examined. The Lyapunov analyis is performed in section 5 and our approach is illustrated by a simulation example in section 6. Finally, section 7 contains conclusions and possible extensions.

II. SYSTEM DESCRIPTION

We consider single input single output (SISO) systems described by

$$\dot{x} = f(x,u) \tag{2.1.a}$$

$$y = h(x) \tag{2.1.b}$$

The model (2.1) is assumed to satisfy the following conditions $^{+}$:

[A1] There exists an open convex set $Xo \times Uo \subset R^n \times R$ containing the origin $(x,u) = (0,0)$ and wherein the functions $f(x,u)$ and $h(x)$ are of class C^2.

[A2] The unforced system has an equilibrium point at the origin, i.e. $f(0,0) = 0$.

[A3] There exists a state feedback law $u(x)$ of class C^2 on Xo such that $u(0) = 0$, $u(x) \in Uo$ on Xo and the linearized closed loop system at the origin is asymptotically stable, i.e. $\mathrm{Re}\ \lambda\ (df/dx(0,0)) < 0$.

[A4] The pair of matrices $(\delta h/\delta x,\ \delta f/\delta x(x,u(x)))$ is observable on Xo.

+ Notations : $\dfrac{\partial h}{\partial x}$ $\left(\dfrac{\partial^T h}{\partial x}\right)$ is the row (column) vector gradient of the scalar function $h(x)$

$$f(x,u) = (f1,\ldots,fn)^T(x,u)$$

$$\left(\frac{\partial f}{\partial x}\right)_{i,j} = \frac{\partial f_i}{\partial x_j} \quad (1 \le i, j \le n)$$

$$\left(\frac{df}{dx}\right)_{i,j} = \frac{\partial f_i}{\partial x_j} + \frac{\partial f_i}{\partial u}\frac{\partial u}{\partial x_j}$$

Assumption [A1] will allow us to use Taylor's theorem (see appendix) for the analysis of our regulation scheme. Assumptions [A2] and [A3] are exponential stabilizability conditions of the origin. The observability assumption [A4] will be useful to assign the exponential observation error dynamics. Note that in [7], the detectability of the linearized (open loop) system at the origin has been proved to be a necessary condition for a nonlinear system to admit an exponential observer. However, detectability is not sufficient to freely assign the poles of the linearized system.

III. DYNAMICS OF THE OBSERVER

Consider the observer

$$\dot{\hat{x}} = f(\hat{x},u) + k(\hat{x})(y - \hat{y}) \qquad (3.1.a)$$

$$\hat{y} = h(\hat{x}) \qquad (3.1.b)$$

where $k(\hat{x})$ is a n-dimensionnal column vector. This form is consistent with the one that an exponential observer necessarily takes [12] because the particular solution we propose here to regulate the system consists in driving the system (2.1) with the feedback regulation law (whose existence is assumed in [A3]) where the state x has been replaced by its estimate $\hat{x}$, i.e. $u = u(\hat{x})$.

Let $e = x-\hat{x}$ be the state observation error and define the n-dimensional row vector

$$w_y(\hat{x},e,\eta) = \frac{1}{2}\, e^T \frac{\partial^2 h}{\partial x \partial x^T}(\hat{x}+\eta e)$$

where $\eta \in [0,1]$ (see the appendix for the choice of η). According to [A2] and to Taylor's theorem at order 1 or 2, two alternative and equivalent equations are obtained for the observer :

$$\dot{\hat{x}} = \frac{df}{dx}(\eta\hat{x},u(\eta\hat{x}))\hat{x} + k(\hat{x})\,\frac{\partial h}{\partial x}(\hat{x}+\eta e)e \qquad (3.2)$$

or

$$\dot{\hat{x}} = f(\hat{x},u(\hat{x})) + k(\hat{x}).\left(\frac{\partial h}{\partial x}(\hat{x}) + w_y(\hat{x},e,\eta)\right)e \qquad (3.3)$$

IV. DYNAMICS OF THE CLOSED LOOP SYSTEM

The behavior of the closed loop is described by the equations (3.2) (or 3.3)), those of the system (2.1), and the regulation law $u(\hat{x})$. Note that

$$\dot{x} = f(x,u(\hat{x}))$$

$$= f(x,u(x)) - (f(x,u(x)) - f(x,u(\hat{x})))$$

or

$$\dot{x} = f(\hat{x},u(\hat{x})) + f(x,u(\hat{x})) - f(\hat{x},u(\hat{x}))$$

Let $W_x(x,\eta)$ and $W_e(x,e,\eta)$ be nxn matrices whose i th lines are defined by

$$(W_x(x,\eta))_i = \frac{1}{2}\, x^T \frac{d^2 f_i}{dx dx^T}(\eta x,u(\eta x)) \quad (1 \le i \le n)$$

$$(W_e(\hat{x},e,\eta))_i = \frac{1}{2}\, e^T \frac{\partial^2 f_i}{\partial x \partial x^T}(\hat{x}+\eta e.\ u(\hat{x}))$$

With the Taylor expansion about the origin or about the current observed state, we get two alternative expressions for the dynamics of the state :

$$\dot{x} = \left(\frac{df}{dx}(0,0) + W_x(x,\eta)\right)x - \frac{\partial f}{\partial u}(x,u(\hat{x}+\eta e))\frac{\partial u}{\partial x}(\hat{x}+\eta e)e \qquad (4.1)$$

or

$$\dot{x} = f(\hat{x},u(\hat{x})) + \left(\frac{\partial f}{\partial x}(\hat{x},u(\hat{x})) + W_e(\hat{x},e,\eta)\right)e \qquad (4.2)$$

Then, with

$$\Gamma(x) \overset{\Delta}{=} \frac{\partial f}{\partial x}(x,u(x)) - k(x)\frac{\partial k}{\partial x}(x)$$

the observation error dynamics is (from (3.3) and (4.2)) :

$$\dot{e} = (\Gamma(\hat{x}) + W_e(\hat{x},e,\eta) - k(\hat{x})w_y(\hat{x},e,\eta))e \qquad (4.3)$$

V. LYAPUNOV ANALYSIS OF THE CLOSED LOOP SYSTEM

According to [A4], the eigenvalues of $\Gamma(x)$ may be assigned to prespecified values on Xo. To perform the extended Luenberger observer design, let k(x) be such that for all $x \in$ Xo, Re $\lambda(\Gamma(x))$ (possibly depending on x) are negative. Consequently, there exists a nxn symmetric and positive definite matrix $P_e(x)$ which verifies

$$\Gamma^T(x)P_e(x) + P_e(x)\Gamma(x) = -Q_e$$

for any nxn constant symmetric and positive definite matrix Q_e. On the other hand, by assumption [A3], there exists a nxn constant symmetric and positive definite matrix P_x such that

$$\frac{d^T f}{dx}(0,0)P_x + P_x\frac{df}{dx}(0,0) = -Q_x$$

for any nxn constant symmetric and positive definite matrix Q_x.

Let Q_e and Q_x be arbitrary fixed positive definite matrices, and consider now the candidate Lyapunov function

$$V(x,e) = V_x(x) + \alpha.V_e(x,e)$$

with $\alpha > 0$

$$V_x(x) = x^T P_x.x$$

$$V_e(x,e) = e^T \frac{P_e(x-e)}{|P_e(x-e)|}.e$$

where $|P_e(x-e)|$ is the Euclidian norm of $P_e(x-e)$ defined by

$$|P_e(x-e)| = \sqrt{\sum_{i,j}(P_e(x-e))^2_{i,j}}$$

The derivatives of $V_x(x)$ and $V_e(x,e)$ take the form (from (4.1))

$$\frac{d}{dt}V_x(x) = -x^T.Q_x.x + 2x^T P_x\left(-\frac{\partial f}{\partial u}(x,u(\hat{x}+\eta e))\frac{\partial u}{\partial x}(\hat{x}+\eta e)e + W_x(x,\eta)x\right)$$

and (from (4.3) and (3.2))

$$\frac{d}{dt}V_e(x,e) = \frac{1}{|P_e(\hat{x})|}(-e^T Q_e e + e^T(2P_e(\hat{x})(-k(\hat{x})w_y(\hat{x},e,\eta) + W_e(\hat{x},e,\eta)) + D(\hat{x},e,\eta))e)$$

where $D(\hat{x},e,\eta)$ is the following nxn matrix :

$$(D(\hat{x},e,\eta))_{i,j} = |P_e(\hat{x})| \frac{d}{dt} \frac{(P_e(\hat{x}))_{i,j}}{|P_e(\hat{x})|}$$

$$= \left(\frac{\partial (P_e)_{i,j}}{\partial x}(\hat{x}) - \frac{(P_e(\hat{x}))_{i,j}}{|P_e(\hat{x})|} \frac{\partial |P_e|}{\partial x}(\hat{x}) \right)$$

$$\left(\frac{df}{dx}(\eta\hat{x},u(\eta\hat{x}))\hat{x} + k(\hat{x})\frac{\partial h}{\partial x}(\hat{x}+\eta e)e \right)$$

Define the 2nx2n matrix $M(x,e,\eta)$ by

$$M(x,e,\eta) = \begin{pmatrix} M_{11}(x,\eta) & M_{12}(x,e,\eta) \\ 0 & M_{22}(x,e,\eta) \end{pmatrix}$$

with

$$M_{11}(x,\eta) = -Q_x + 2P_x W_x(x,\eta)$$

$$M_{12}(x,e,\eta) = -2P_x \frac{\partial f}{\partial u}(x,u(x+(\eta-1)e))\frac{\partial u}{\partial x}(x+(\eta-1)e)$$

$$M_{22}(x,e,\eta) = \frac{\alpha}{|P_e(x-e)|}(-Q_e$$

$$+ 2P_e(x-e)(-k(x-e)w_y(x-e,e,\eta) + W_e(x-e,e,\eta)) + D(x-e,e,\eta))$$

The derivative of $V(x,e)$ may then be written

$$\frac{d}{dt}V(x,e) = (x^T,e^T)\, M(x,e,\eta)\, (x,e)^T$$

$$= \frac{1}{2}(x^T,e^T)\, (M(x,e,\eta) + M^T(x,e,\eta))\, (x,e)^T$$

Find now a sufficently high constant α and a sufficently small positive constant c such that

$$0 < V(x,e) \le c \Rightarrow x \text{ and } x-e \in Xo$$

$$\text{and} \qquad (5.1)$$

$$\frac{d}{dt}V(x,e) < 0$$

Such constants can always be found : as Q_x and Q_e are positive definite, a smaller c (restricting the size of allowable (x,e)) yields $M_{11}(x,\eta)$ and $M_{22}(x,e,\eta)$ negative definite ; a higher α (restricting the size of allowable e) then yields $M(x,e,\eta)$ negative definite. Indeed, define $M_k(x,e,\eta)$ by

$$M_k(x,e,\eta) = \begin{pmatrix} m_{11}(x,e,\eta) & ..m_{1k}(x,e,\eta) \\ .. & .. \\ m_{k1}(x,e,\eta) & ..m_{kk}(x,e,\eta) \end{pmatrix} \quad 1 \le k \le 2n$$

and let $q_k(x,e,\eta)$ be the leading principal minors of $(M(x,e,\eta) + M^T(x,e,\eta))$, i.e.

$$q_k(x,e,\eta) = \det(M_k(x,e,\eta) + M_k^T(x,e,\eta))$$

The terms involving a (k-n)th power of α (k>n) do not depend on $M_{12}(x,e,\eta)$; since $M_{11}(x,\eta)$ and $M_{22}(x,e,\eta)$ are negative definite, a sufficently high α yields $q_k(x,e,\eta)(-1)^k$ positive, i.e. $M(x,e,\eta)$ negative definite (Sylvester test).

Finally, the origin is exponentially stable for the domain defined by

$$\{(x,e) : V(x,e) \le c\}$$

From this analysis, the size of the convergence domain is clearly limited by nonlinear factors : $W_x(x,\eta)$ depends on the nonlinearity of the ideal closed loop $\dot{x} = f(x,u(x))$, $w_y(\hat{x},e,\eta)$ depends on the nonlinearity of the output equation $y = h(x)$. Note also the dependence of $P_e(\hat{x})$ on $\hat{x}$: in order to reduce the influence of the time derivative of $P_e(\hat{x})$ in $M_{22}(x,e,\eta)$, the factor $1/|P_e(\hat{x})|$ has been introduced in the Lyapunov function. For linear systems, the matrix $M(x,e,\eta)$ is constant and positive definite (for a sufficently high α). Therefore, as is well known, the origin $(x,e) = (0,0)$ is exponentially stable whatever the initial conditions. For nonlinear systems, this property is no longer valid : the previous approach leads, in a conservative way, to a convergence domain in which both the state trajectory x(.) and the state observation error trajectory e(.) are guaranteed to be bounded.

VI. SIMULATION

To illustrate the observer based feedback design and the Lyapunov analysis, we now compare the regulator based on the extended Luenberger observer, with the one based on a linear observer.

Consider the following simple nonlinear system :

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_1 x_2 + u$$

$$y = x_1$$

The feedback law

$$u(x) = -x_1 x_2 - p_1 x_1 - p_2 x_2$$

linearizes and stabilizes the system at the origin for an appropriate choice of p_1 and p_2. The actually implemented law will be

$$u(\hat{x}) = -\hat{x}_1\hat{x}_2 - p_1\hat{x}_1 - p_2\hat{x}_2$$

The extended Luenberger observer design takes the form

$$\dot{\hat{x}}_1 = \hat{x}_2 + k_1(\hat{x})(y-\hat{y})$$

$$\dot{\hat{x}}_2 = -p_1\hat{x}_1 - p_2\hat{x}_2 + k_2(\hat{x})(y-\hat{y}) \qquad (6.1)$$

$$\hat{y} = \hat{x}_1$$

where $k(\hat{x})$ has to be chosen to assign the eigenvalues of the matrix $\Gamma(\hat{x})$ at fixed stable values through the choice of polynomial coefficients p3 and p4 :

$$\det\left(sI_2 - \begin{pmatrix} -k_1(\hat{x}) & 1 \\ -k_2(\hat{x}) + \hat{x}_2 & \hat{x}_1 \end{pmatrix} \right) = s^2 + p_4 s + p_3$$

This implies

$$k_1(\hat{x}) = p_4 + \hat{x}_1$$

$$k_2(\hat{x}) = p_3 + \hat{x}_2 + k_1(\hat{x})\,\hat{x}_1$$

The linear observer takes the form (6.1), but with a constant vector k, obtained as the value of k(x) at the origin, i.e. $k_1=p_4$ and $k_2=p_3$.

The two poles of the state feedback linearized closed loop are chosen equal to -1 and the two eigenvalues of the matrix $\Gamma(x)$ equal to -3, i.e.

$$p_1 = 1,\ p_2 = 2,\ p_3 = 4,\ p_4 = 4$$

The matrix $df/dx(0,0)$ and $\Gamma(x)$ are then given by

$$\frac{df}{dx}(0,0) = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\Gamma(x) = \begin{pmatrix} -4-x_1 & 1 \\ -4-4x_1-x_1^2 & x_1 \end{pmatrix}$$

Let $Q_e = Q_x = I_2$; we obtain

$$P_x = \begin{pmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

and

$$P_e(x) = \begin{pmatrix} \frac{5}{8}+x_1+\frac{25}{32}x_1^2+\frac{x_1^3}{4}+\frac{x_1^4}{32} & -\left(\frac{1}{2}+\frac{21}{32}x_1+\frac{x_1^2}{4}+\frac{x_1^3}{32}\right) \\ -\left(\frac{1}{2}+\frac{21}{32}x_1+\frac{x_1^2}{4}+\frac{x_1^3}{32}\right) & \frac{21}{32}+\frac{x_1}{4}+\frac{x_1^2}{32} \end{pmatrix}$$

The Lyapunov function is

$$V(x,e) = x^T P_x x + \alpha e^T \frac{P_e(x-e)}{\|P_e(x-e)\|} e$$

The following values have been selected :

$$\alpha = 50,\ c = 5.5$$

for which condition (5.1) has been numerically checked to hold. We then have :

$$\frac{d}{dt}V(x,e) < 0\ ,\quad \forall\ (x,e) : 0 < V(x,e) \le c \qquad (6.1)$$

The point $(x,e) = (1,\ 0.5,\ 0,\ 0.5)$ will serve as initial condition in our simulation. Notice this point belongs to the convergence set define by (6.1). Hence, the initial values will be $x = (1,\ 0.5)$, for the nonlinear system, and $\hat{x} = x - e \doteq (1,\ 0)$ for both the linear and the nonlinear observer.

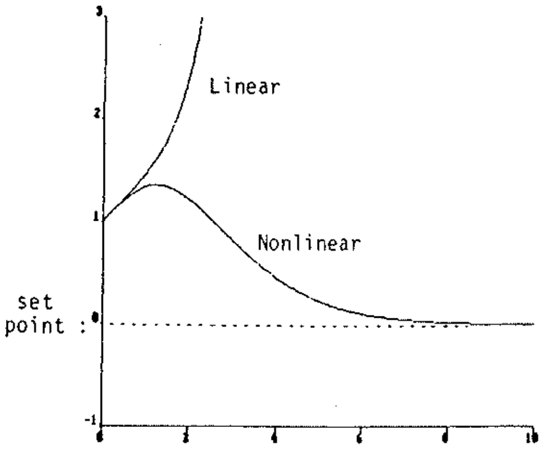

Fig.1 : Output response y(t) using linear and nonlinear observer.

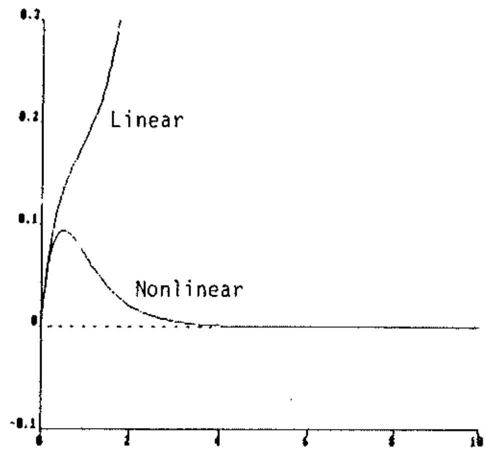

Fig.2 : State observation error e1(t).

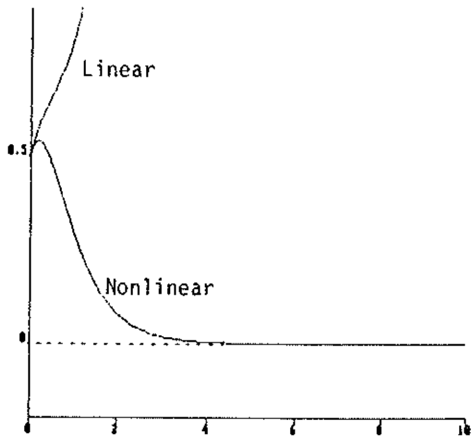

Fig.3 : State observation error e2(t).

The relevance of the nonlinear observer design appears clearly : the linear observer cannot lock on the actual value of x (and causes the divergence of the observation error and the instability of the closed loop) while the nonlinear one converges to x and stabilizes the closed loop.

VII. CONCLUSIONS

An output feedback regulator has been considered, based on an extended Luenberger observer design. It concerns the class of SISO exponentially stabilizable and locally observable nonlinear systems. The state estimate used to implement a suitable regulation law has been proved to lead to exponential stability around the origin : a quadratic Lyapunov function has been obtained and a convergence domain has been characterized.

Note that the stability analysis only deals with a full order observer ; indeed, reduced order extended Luenberger observer design is not trivial since the dependence of the vector $k(\hat{x})$ on $\hat{x}$ prevents from eliminating $\dot{y}$ from the observer equation. However, when the output is a component of the state, e.g. x_1, it may be put in place of $\hat{x}_1$ in the control law $u(\hat{x})$ and in the vector $k(\hat{x})$, leading to improved stabilization. Extension of the Lyapunov analysis to this case causes no particular problems.

Remark finally that the design concerns a broad class of nonlinear systems, as in [1], [14] and as opposed to [5], [6] , [8]. We now briefly compare our extended Luenberger observer design with those proposed in [1] and [14]. According to [A3] (i.e. df/dx(0,0) has no zero eigenvalues and is invertible) there exists a neighborhood of the origin wherein the nonlinear systems considered here admit a one dimensional continuum of operating points ; the corresponding new input is u = u(x) + r where u(x) is the stabilizing control law around the origin, and r parametrizes the operating points. For this law, the extended Luenberger observer design is directly suitable and so the output feedback design is related to the extended linearization method [1] : a fixed dynamical behavior of the nonlinear observation error system is specified locally about the continuum of constant operating points, producing satisfactory closed loop system behavior over a much wider range of operation than the linear observers designed about each single operating point. However, unlike [1], our extended Luenberger observer design requires no differential equation solution and the extension to the MIMO design is straightforward. Similarly, a drawback of the observer design proposed in [14] is its dependence on the derivatives of the input.

APPENDIX

We recall Taylor's theorem for scalar functions of several variables.

Theorem :

Let U be an open set in R^n

$f : U \to R$, a C^k function on U ($k \in N$)

$a \in U$

$h \in R^n : \forall \lambda \in [0,1]\ \ a + \lambda h \in U$

Then there exists $\eta \in]\,0,1[$ such that

$$f(a+h) = f(a) + df(a)(h) + \frac{1}{2}d^2f(a)(h) + \frac{1}{3!}d^3f(a)(h) + \ldots$$

$$\frac{1}{(k-1)!}d^{k-1}f(a)(h) + \frac{1}{k!}d^kf(a+\eta h)(h)$$

with $d^jf(a)(h) = d^jf(a)(h_1, h_2, \ldots h_n)$

$$= \sum_{i_1=1}^{n} \ldots \sum_{i_j=1}^{n} \frac{\partial^j f}{\partial x_{i_1} \ldots \partial x_{i_j}}(a) h_{i_1} \ldots h_{i_j} \qquad \square$$

Remark : the same result may be used for a vector function of several variables ; note that in this case the value η is different for each component of the vector. Throughout this paper, we have used a single notation η in all our Taylor expansion ; it must be understood that η may be different for each component and different in each expansion, but is always in $]0,1[$.

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