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STABLE ADAPTIVE OBSERVERS FOR NONLINEAR TIME-VARYING SYSTEMS (Lund 1986)

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ABSTRACT

We describe an adaptive observer/identifier for single input single output observable nonlinear systems that can be transformed to a certain observable canonical form. We provide sufficient conditions for stability of this observer. These conditions are in terms of the structure of the system and its canonical form, the boundedness of the parameter variations and the sufficient richness of some signals. We motivate the scope of our canonical form and the use of our observer/identifier by presenting applications to a number of nonlinear systems. In each case we present the specific stability conditions.

1. INTRODUCTION

A goal in many practical applications is to combine a priori knowledge about a physical system with experimental data to provide on-line estimation of states or parameters of that system. A common situation is where one has a single input single output (SISO) nonlinear time varying deterministic system described as follows:

$$\dot{z} = f(z,u,p)$$
$$y = z_1 \qquad (1.1)$$

where $u(t) \varepsilon D_u \subseteq R$ is a measurable input, possibly constrained to a subspace D_u of R, $y(t) \varepsilon R$ is a measurable output, $z(t) \varepsilon R^n$ is a state-vector, and $p(t) \varepsilon D_p \subseteq R^q$ is a vector of unknown bounded possibly time-varying parameters. The parameters p(t) can be (possibly unknown) functions of z(t), as in the example of Section 6, and a priori knowledge may constrain p(t) to be in a subspace D_p of R^q. The structure of the system (i.e. the function $f(\cdot)$) is known from physical laws or from the user's experience, i.e. from a priori knowledge. Most often also, the states $z_i(t)$ and some of the unknown parameters $p_i(t)$ in (1.1) have a clear physical significance. Therefore, throughout the paper, we shall call (1.1) the Given Physical System, abbreviated GPS.

Now the user may want to solve one of the following three problems.

Problem 1: the on-line estimation of the non-measured states $z_i(t)$ of the GPS from input-output (I/O) data. This is called adaptive state estimation.

Problem 2: The on-line estimation of some of the physical parameters $p_i(t)$ of the GPS form I/O data. This is called adaptive parameter identification.

Problem 3: The design of an adaptive observer for the on-line estimation of the states, possibly in an equivalent state-space model. This is called adaptive observer design. It is to be distinguished from Problem 1 in that the states here need not be the physical z_i of the GPS; their estimates might be needed for a state-feedback controller, say.

Problem 2 makes sense only if the GPS is parameter identifiable, while Problems 1 and 3 require that, in addition, for all $u(t) \varepsilon D_u$ and all $p(t) \varepsilon D_p$, the GPS be locally observable: see Isidori (1981). We shall therefore make these assumptions throughout the paper.

One commonly used method to solve these three problems is to augment the state z(t) with the parameter vector p(t) and to implement an extended Kalman filter (EKF). However, the EKF is very expensive in computations, it requires a model for the time-variations of p(t) and, most importantly, its stability when applied to parameter estimation of nonlinear systems would be extremely hard to prove.

There is therefore a clear incentive to search for simpler adaptive observers/identifiers that can be guaranteed stable. For linear time-invariant systems, stable adaptive observers have been proposed by e.g. Lüders and Narendra (1973, 1974a, 1974b), Narendra and Kudva (1974), Narendra (1976) and Kreisselmeier (1977, 1979). The robustness of these observers in the case of unmodelled fast parasitic modes has been analysed by Ioannou and Kokotovic (1983).

The purpose of this paper is to show that, for many nonlinear systems of the form (1.1), Problem 3, and to a lesser extent Problems 1 and 2, can be solved using a special adaptive observer/identifier, presented in Section 3, which alleviates some of the disadvantages of the brute force EKF approach. This adaptive observer/identifier is an extenxion to nonlinear time varying systems of the observer of Lüders and Narendra (1974a), which is known to be exponentially asymptotically stable (EAS) when applied to linear time-invariant systems (Morgan and Narendra, 1977). The main advantages of our observer over the EKF are that its stability can be proved under reasonable conditions on the GPS, that it is computationally much simpler than the EKF, and that it does not need any dynamical model of the parameter variations.

A major feature of our approach is to transform the nonlinear GPS into a time-varying observable canonical form (called AOCF) which has the property that it is linear in the unknown quantities. These can include states, parameters or combinations thereof. An adaptive observer is then derived for this canonical form and the main issue is to prove its global stability. The proofs use mostly standard arguments on adaptive systems analysis and persistence of excitation, and extension of these. They have for the most part been deleted from this conference paper and can be found in Bastin and Gevers (1986).

The outline of the paper is as follows. In Section 2, we describe the canonical form (AOCF) mentioned above and motivate its use, while in Section 3 we show how an adaptive observer/identifier can be derived from this form. In Section 4, we give a precise and complete set of sufficient conditions on the GPS and on the signals for the global stability of the observer/

identifier. Our adaptive observer can be applied to all GPS for which a transformation to the AOCF exists. This includes a very large number of observable and parameter identifiable nonlinear systems. Here we illustrate this with two examples:

- the class of time-invariant observable bilinear systems in Section 5.
- a nonlinear biotechnological process in Section 6.

Additional examples can be found in Bastin and Gevers (1986) and Gevers and Bastin (1986).

2. TRANSFORMATION TO A CANONICAL REPRESENTATION

2.1 The adaptive observer canonical form

From now on we consider the nonlinear systems of the form (1.1) which can be transformed, by a time-invariant possibly nonlinear smooth transformation

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = T(z,p,c_2,\ldots,c_n) \qquad (2.1)$$

into the following equivalent form, which we shall call for convenience the adaptive observer canonical form (AOCF):

$$\dot{x}(t) = Rx(t) + \Omega(\omega(t))\theta(t) + g(t)$$
$$y(t) = x_1(t) \qquad (2.2)$$

In (2.1) and (2.2)

- $x(t) \in R^n$ is a state-vector of same dimension as $z(t)$;
- $\theta(t) \in R^m$ is a vector of unknown time-varying parameters, which will be estimated on line;
- $\omega(t) \in R^s$ is a vector of known functions of $u(t)$ and $y(t)$, e.g. $\omega(t) = [u(t)\ y(t)\ y^2(t)\ \sin y(t)]$;
- $\Omega(\omega(t))$ is a nxm matrix whose elements are all of the form $\Omega_{ij}(\omega(t)) = \alpha_{ij}^T\omega(t)$ for known constant, possibly zero, vectors $\alpha_{ij} \in R^s$;
- R is a known constant nxn matrix of the following form

$$R = \begin{bmatrix} 0 & k^T \\ \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & F(c_2,c_3,\ldots,c_n) \end{bmatrix}, \quad k^T \triangleq [k_2,\ldots,k_n] \qquad (2.3)$$

where $k_2,\ldots,k_n$ are known constants and $F(c_2,\ldots,c_n)$ is a (n-1)x(n-1) constant matrix whose eigenvalues can be freely assigned by a proper choice of the constant parameters $c_2,\ldots,c_n$. Typically $F = \mathrm{diag}(-c_2,\ldots,-c_n)$ with $c_i>0$ and all different;
- $g(t) \in R^n$ is a vector of known functions of time;
- $T(\cdot) \in R^{n+m}$ is a continuous smooth transformation from (z,p) to (x,θ) parametrized by n-1 parameters $c_2,\ldots,c_n$.

We shall describe an adaptive observer for the system (2.2) and we shall provide sufficient conditions on the GPS (1.1) to guarantee its global stability. This will provide a solution to Problem 3. If the transformation T in (2.1) is such that the inverse transformation

$$z = H_1(x,\theta,c_2,\ldots,c_n) \qquad (2.4)$$

exists, is unique and is continuous for all $u \in D_u$, then this will simultaneously solve Problem 1. If the inverse transformation

$$p = H_2(x,\theta,c_2,\ldots,c_n) \qquad (2.5)$$

exists, is unique and is continuous for all $u \in D_u$, this will also provide a solution to Problem 2. The application in Section 6 will illustrate these points.

2.2 Motivation

The reader might be bewildered by the strange structure of the AOCF (2.2). Our main reason for using this form is that it leads to an adaptive observer for which global stability conditions can easily be derived, as we shall see in Sections 3 and 4. As it turns out, large numbers of SISO nonlinear systems of practical interest can be transformed into AOCF, even though some effort may be needed to find the transformation T: this will be illustrated in Sections 5 and 6. At this stage we would like to offer some initial motivation by considering the following classes of systems.

Suppose that the GPS can be written in the following "observer form":

$$\dot{z} = \begin{bmatrix} -a_1(z,t) & \\ \vdots & I_{n-1} \\ -a_n(z,t) & 0 \cdots 0 \end{bmatrix} z + \begin{bmatrix} b_1(z,t) \\ \vdots \\ b_n(z,t) \end{bmatrix} u$$
$$y = z_1 \qquad (2.6)$$

It can then be transformed by a constant transformation

$$(x,\theta) = T(z,a,b,c_2,\ldots c_n) \qquad (2.7)$$

into the AOCF (2.2) with

$$\omega^T = [u,y] \qquad g(t) = 0 \qquad (2.8a)$$

$$R = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 0 & -c_2 & & \bigcirc \\ \vdots & & \ddots & \\ 0 & \bigcirc & & -c_n \end{bmatrix}, \quad c_i \neq c_j \quad i \neq j \qquad (2.8b)$$

$$\Omega(\omega) = \begin{bmatrix} y & 0 & \cdots & 0 & u & 0 & \cdots & 0 \\ 0 & y & & \vdots & 0 & u & & \vdots \\ \vdots & & \ddots & 0 & \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & y & 0 & \cdots & 0 & u \end{bmatrix} \qquad (2.8c)$$

This is demonstrated in Bastin and Gevers (1986). Note that all observable linear systems can be represented in the form (2.6), with the $a_i(t)$ and $b_i(t)$ independent of z. If the a_i and b_i are constant, this is the canonical observer form (see, e.g. Kailath (1980)). But in addition, all time-invariant observable bilinear systems can also be represented in the form (2.6) with a special structure for the a_i and b_i and can therefore be transformed to AOCF: see Section 5. The same is true for all nonlinear systems in phase variable form: see Bastin and Gevers (1986).

It follows from this discussion that (2.6) covers several classes of systems that can be transformed to AOCF with the special structure (2.8). However, our AOCF covers a wider class of nonlinear systems, of which (2.8) is just a special case: this will be illustrated in Section 6. The crucial feature of the AOCF (2.2) is its linearity in the unknown quantities $x(t)$ and $\theta(t)$. This allows us to derive the adaptive state and parameter estimator described in the next Section.

3. THE ADAPTIVE OBSERVER

For the system described by (2.2) we propose the following adaptive observer.

State estimation:

$$\dot{\hat{x}}(t) = R\hat{x}(t) + \Omega(\omega(t))\hat{\theta}(t) + g(t) + \begin{bmatrix} c_1\tilde{y}(t) \\ V(t)\dot{\hat{\theta}}(t) \end{bmatrix} \qquad (3.1a)$$

$$\hat{y}(t) = \hat{x}_1(t), \qquad \tilde{y}(t) \triangleq y(t) - \hat{y}(t) \qquad (3.1b)$$

where c_1 is an arbitrary positive constant.

Parameter adaptation:

$$\dot{\hat{\theta}}(t) = \Gamma\phi(t)\tilde{y}(t) \qquad (3.1c)$$

where Γ is an arbitrary positive definite matrix, normally chosen as $\Gamma = \mathrm{diag}(\gamma_1,\ldots,\gamma_m)$, $\gamma_i>0$.

Auxiliary filter:

$V(t)$ is a $(n-1)\times m$ matrix and $\phi(t)$ is a m-vector; they are the solution of the following auxiliary filter:

$$\dot{V}(t) = FV(t) + \bar{\Omega}(\omega(t)), \quad V(0) = 0 \qquad (3.1d)$$

$$\phi(t) = V^T(t)k + \Omega_1^T(\omega(t)) \qquad (3.1e)$$

where Ω_1 is the first row of $\Omega(\omega(t))$ and $\bar{\Omega}$ are the remaining rows, i.e.

$$\Omega \triangleq \begin{bmatrix} \Omega_1 \\ \bar{\Omega} \end{bmatrix} \begin{matrix} \}\ 1 \\ \}\ n-1 \end{matrix} \qquad (3.2)$$

Recall that F and k are submatrices of R defined by (2.3), that $\Omega(\omega(t))$ and $g(t)$ are known functions and that $y=x_1$ is measured. It is worth noting that, most often, $\bar{\Omega}(\omega(t))$ contains a number of zero elements: see, e.g. (2.8c). In such case, the corresponding elements in the matrix $V(t)$ in (3.1d) and (3.1e) will be identically zero. In general, therefore, the solution of the matrix equation (3.1d) requires much less than $(n-1)\times(m-1)$ differential equations. In addition, in many cases F will be diagonal, and the solution of (3.1d,e) simplifies considerably.

4. STABILITY CONDITIONS FOR THE ADAPTIVE OBSERVER

4.1 The error system

We define $\tilde{x} \triangleq x - \hat{x}$, $\tilde{\theta} \triangleq \theta - \hat{\theta}$ and we introduce the following auxiliary error vector:

$$\tilde{x}^* \triangleq \tilde{x} - \begin{bmatrix} 0 \\ V\tilde{\theta} \end{bmatrix} \qquad (4.1)$$

Using (2.2) and (3.1) we can then write, after some lengthy but straightforward manipulations, the following error system.

$$\begin{bmatrix} \dot{\tilde{x}}^* \\ \dot{\tilde{\theta}} \end{bmatrix} = \left[\begin{array}{c|c} R^* & \begin{matrix}\phi^T \\ O\end{matrix} \\ \hline -\Gamma\phi\ \ O & O \end{array}\right] \begin{bmatrix} \tilde{x}^* \\ \tilde{\theta} \end{bmatrix} + \left[\begin{array}{c} 0 \ \ldots\ 0 \\ -V \\ \hline I_m \end{array}\right] \dot{\theta} \qquad (4.2)$$

where

$$R^* \triangleq \left[\begin{array}{c|c} -c_1 & k^T \\ \hline \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & F(c_2,\ldots,c_n) \end{array}\right] \qquad (4.3)$$

Note that $\dim V(t) = (n-1)\times m$. Recall also that F is a constant matrix whose eigenvalues are completely determined by the parameters $c_2,\ldots,c_n$, which are at the designer's disposal in the transformation (2.1) that leads to the AOCF (2.2). As mentioned before, F will often be $\mathrm{diag}(-c_2,\ldots,-c_n)$ with $c_i>0$ and all different. It is then immediately clear from the error system that, if $\hat{\theta}=\theta$, the error $\tilde{x}$ is the solution of a linear time-invariant equation whose poles are entirely determined by the design parameters $c_1,\ldots,c_n$.

4.2 Stability conditions on the error system

We describe a set of sufficient conditions that guarantee:

i) that the homogeneous part of the error system is exponentially asymptotically stable (EAS);

ii) that the error system is therefore BIBS stable;

iii) that $\tilde{x}^*$ and $\tilde{\theta}$ are therefore bounded if $\dot{\theta}$ is bounded.

We denote

$$\varepsilon(t) \triangleq \begin{bmatrix} \tilde{x}^*(t) \\ \tilde{\theta}(t) \end{bmatrix} \qquad (4.4)$$

Theorem 4.1: If

i) $c_1>0$ and $c_2,\ldots,c_n$ are chosen such that $\mathrm{Re}\,\lambda_i(F) < 0$

ii) $\phi(t)$ is bounded $\forall t\geq 0$

iii) $\dot{\phi}(t)$ is bounded $\forall t\geq 0$ except possibly at a countable number of points $\{t_i\}$ such that $\min_{i,j}|t_i-t_j|\geq\Delta>0$ for some arbitrary fixed Δ;

iv) there exist positive constants α and T such that $\forall t\geq 0$

$$0 < \alpha I \leq \int_t^{t+T} \phi(\tau)\phi^T(\tau)d\tau \qquad (4.5)$$

v) there exists a positive constant M_1 such that $\forall t\geq 0$

$$\left|\begin{bmatrix} -V(t) \\ I_m \end{bmatrix} \dot{\theta}(t)\right| \leq M_1 < \infty \qquad (4.6)$$

then there exist finite constants K_1, K_2 and K_3 such that

1) $|\varepsilon(t)| \leq K_1|\varepsilon(0)| + K_2 \quad \forall t \geq 0.$ (4.7a)

2) $\limsup_{t\to\infty} |\varepsilon(t)| \leq K_3M_1$ (4.7b)

Proof: see Bastin and Gevers (1986).

Definition 4.1: We shall call S_Δ the set of signals satisfying conditions ii) and iii) of Theorem 4.1. Recalling (4.1) and denoting

$$e(t) \triangleq \begin{bmatrix} \tilde{x}(t) \\ \tilde{\theta}(t) \end{bmatrix}$$

the following corollary follows immediately.

Corollary 4.1: Under the conditions i) to v) of Theorem 4.1 there exist finite constants K_1', K_2' and K_3' such that

1) $|e(t)| \leq K_1'|e(0)| + K_2' \quad \forall t\geq 0$ (4.10a)

2) $\limsup_{t\to\infty} |e(t)| \leq K_3'M_1$ (4.10b)

The stability conditions of Theorem 4.1 are on the error system. The remainder of this Section is concerned with transferring the conditions of Theorem 4.1 to stability conditions on the GPS and its representation in AOCF. We start by discussing the conditions i) to v) of Theorem 4.1.

Comments 4.1:

i) The c_i's are at the designer's disposal; thus this condition can always be met.

ii),iii) $\phi(t)$ (resp. $\dot{\phi}(t)$) is the output of the BIBO filter (3.1d,e) driven by elements of $\Omega(\omega(t))$ (resp. $\Omega(\dot{\omega}(t))$), which are all of the form $\alpha^T\omega(t)$ (resp. $\alpha^T\dot{\omega}(t)$). Therefore $\phi(t)\epsilon S_\Delta$ if the $\omega_i(t)$ are bounded functions of $u(t)$ and $y(t)$, if $u(t)\epsilon S_\Delta$ and if the GPS is BIBO stable. Hence conditions ii) and iii) are conditions on the given system and on the inputs $u(t)$.

iv) Condition iv) is a persistence of excitation condition that guarantees exponential convergence of the homogeneous part of (4.2). Condition (4.5) will be satisfied if

a) the auxiliary filter (3.1d,e) is output reachable; conditions will be given in Theorem 4.2.

b) the input $\omega(t)$ of that filter is "sufficiently rich", in a sense to be made precise in Theorem 4.3.

v) $|V(t)|$ is bounded if $\omega(t)$ is bounded (see comment ii) above), while the boundedness of $|\dot{\theta}|$ depends on the boundedness of $\dot{p}(t)$ and $\dot{z}(t)$ in the GPS, and on the transformation (2.1). It must be checked case by case.

Persistence of excitation

We now give two results which are important to guarantee the persistence of excitation of $\phi(t)$, i.e. condition iv). First we define a m×m matrix $S(\omega(t))$ as follows:

$$S(\omega(t)) \triangleq \begin{bmatrix} s_1(\omega(t)) \\ \vdots \\ s_m(\omega(t)) \end{bmatrix} \qquad (4.11a)$$

where (see (3.1d,e)):

$$s_1 = \Omega_1, \quad s_j = k^T F^{j-2}\bar{\Omega}, \quad j = 2,\dots,m \qquad (4.11b)$$

Theorem 4.2: The auxiliary filter (3.1d,e) is output reachable from $\omega(t)$ if and only if $S(\omega(t))$ has full column rank over R, i.e. iff there exists no constant m-vector $\beta\neq 0$ such that $S(\omega(t))\beta \equiv 0$.

Proof: See Bastin and Gevers (1986).

Comment 4.2: It is important to note that Theorem 4.2 is a condition on the structure of the canonical form (2.2), since k, F, Ω_1 and $\bar{\Omega}$ are all defined by R and Ω in (2.2). Hence the output reachability of the auxiliary filter of our adaptive observer, which is a necessary condition for the persistence of excitation of $\phi(t)$, can be checked right from the start.

Theorem 4.3: Consider the auxiliary filter (3.1d,e) and assume that:

1) $\Omega(\omega(t))$ in (3.1d) contains q elements that are not identically zero;
2) $S(\omega(t))$ defined by (4.11) has full column rank over R;
3) There exist positive constants γ and T such that

$$\int_t^{t+T} W(\tau)W^T(\tau)d\tau \geq \gamma I > 0 \quad \text{for all } \forall t \geq 0 \qquad (4.12a)$$

where

$$W^T(\tau) = \frac{1}{(s+\delta)^q}\,[\omega^T(\tau)\dot{\omega}^T(\tau)\dots\omega^{(q)T}(\tau)] \qquad (4.12b)$$

with δ arbitrary, but positive. Then $\phi(t)$ obeys (4.5) for some $\alpha>0$.

Proof: See Bastin and Gevers (1986).

4.4 Stability conditions on the GPS

Using Comments 4.1, and Theorems 4.1 to 4.3, we can now spell out conditions on the GPS, the transformation T of (2.1) and some of the signals that will guarantee global stability of our adaptive observer. We shall distinguish between conditions on the system (which can be checked beforehand) and conditions on the signals.

Structural Conditions (on the GPS and the transformation T)

S.1 The GPS (1.1) is BIBO stable

S.2 The GPS and the transformation T are such that

S.2.1 The elements of $\omega(t)$ are bounded functions of u(t) and y(t)

S.2.2 R and Ω in (2.2) make $S(\omega(t))$ in (4.11) of full column rank over R (i.e. the auxiliary filter (3.1d,e) is output reachable).

S.3 The parameter variation $\dot{p}(t)$ and transformation T are such that

$$|\dot{\theta}(t)| \leq M_1 < \infty \text{ for all } t\geq 0.$$

Conditions on the Signals

SI.1 $u(t) \in S_\Delta$ for some $\Delta>0$

SI.2 $\omega(t)$ is sufficiently rich in the sense of (4.12).

Theorem 4.4: Under the conditions S.1 to S.3 and SI.1 to SI.2, there exist finite positive constants K_1^*, K_2^* and K_3^* such that (4.10) is satisfied if c_1 is chosen positive and if $c_2,\dots,c_n$ are chosen such that $\mathrm{Re}\lambda_i(F(c_2,\dots,c_n)) < 0$ for all i.

Proof: Follows from the preceding Theorems and Comments 4.1. □

In Sections 5 and 6 we shall specialise the structural stability conditions to two specific applications. As for the stability conditions on the signals, we shall not discuss them any further. Condition SI.1 can always be met, but no general conditions on the input u(t) of a nonlinear system are presently available that will guarantee SI.2.

5. APPLICATION TO BILINEAR SYSTEMS

Consider that the GPS is a time invariant observable bilinear system described by

$$\dot{z}(t) = M(p_M)z(t) + u(t)N(p_N)z(t) + K(p_K)u(t)$$
$$y(t) = z_1(t) \qquad (5.1)$$

where M and N are constant nxn matrices and K is a constant n-vector, which depend on the constant but unknown vectors of physical parameters p_M, p_N and p_K respectively, and where $u(t)\in S_\Delta$ for some Δ. Then it was shown by Williamson (1977) that there exists a constant nonsingular matrix T_1 such that, with $\xi = T_1 z$, the GPS (5.1) is equivalent with

$$\dot{\xi}(t) = \begin{bmatrix} -a_1 & & \\ \cdot & & \\ \cdot & I_{n-1} & \\ \cdot & & \\ -a_n & 0\ \dots & 0 \end{bmatrix}\xi(t) + \begin{bmatrix} b_1(\xi) \\ \cdot \\ \cdot \\ \cdot \\ b_n(\xi)\end{bmatrix}u(t) = A\xi(t) + b(\xi)u(t) \qquad (5.2)$$

$$y(t) = \xi_1(t)$$

where

$$A = T_1 M T_1^{-1}, \qquad b(\xi) = B\xi + T_1 K \qquad (5.3a)$$

and

$$B = T_1 N T_1^{-1} = \begin{bmatrix} b_{11} & 0 & \dots & 0 \\ b_{21} & b_{22} & \ddots & \cdot \\ \cdot & & \ddots & \cdot \\ \cdot & & & \cdot \\ \cdot & & & 0 \\ b_{n1} & \dots & \dots & b_{nn} \end{bmatrix} \qquad (5.3b)$$

Note that (5.2) is in the form (2.6). It is shown in Bastin and Gevers (1986) that (5.2) can be transformed to the AOCF (2.2) with R and Ω as in (2.8), g(t)=0, and θ and x given by

$$\theta(t) = \bar{T}^{-1}\begin{bmatrix}\begin{bmatrix} t_1 \\ 0 \end{bmatrix} - a \\ b(x)\end{bmatrix}, \quad x(t) = \bar{T}^{-1}\xi(t) \qquad (5.4)$$

where $\bar{T}$ is a known constant nxn matrix and t_1 a known constant (n-1)-vector, $a \triangleq (a_1\dots a_n)^T$ and, using (5.3a),

$$b(x) = B\bar{T}x + T_1 K \qquad (5.5)$$

$\bar{T}$ and t_1 depend only on $c_2,\dots,c_n$.

We can now apply the adaptive observer to this AOCF; the stability conditions are given in the following Theorem.

Theorem 5.1: Let the GPS be given by (5.1). Then (5.1) can be transformed into AOCF by a constant transformation T. The adaptive observer (3.1) for this system is then globally stable (i.e. (4.10) is satisfied) if the following conditions hold:

B.1: The GPS (5.1) is BIBS stable;

B.2: The coefficients $c_1, \ldots, c_n$ are all positive and $c_2, \ldots, c_n$ are all different.

B.3: SI.1 and SI.2 hold.

Proof: See Bastin and Gevers (1986). □

Theorem 5.1 tells us under what conditions the adaptive observer of Section 3 is globally stable for the AOCF obtained from the GPS (5.1). This gives a complete solution to Problem 3: it provides bounded estimates of $\hat{x}$ and $\hat{\theta}$. By (5.4) and (5.5), this yields bounded $\hat{\xi}$, $\hat{a}$ and $\hat{b}$. Whether Problems 1 and 2 can also be solved therefore depends on whether the constant transformation T_1 has a unique inverse for z and/or p: see Section 2.1. This is related to whether the vector of physical parameters $p^T \triangleq (p_M^T, p_N^T, p_K^T)$ is identifiable. See Bastin and Gevers (1986) for details.

6. APPLICATION TO A NONLINEAR BIOTECHNOLOGICAL SYSTEM

The growth of biomass in a continuous stirred tank reactor is most often described by the following second order model (with a unit flow rate):

$$\begin{aligned} \dot{z}_1 &= -\left[\frac{p_1(z_1,z_2,t)}{p_2}\right]z_1z_2 - z_1 - p_3z_2 + u_1 \\ \dot{z}_2 &= p_1(z_1,z_2,t)z_1z_2 - z_2 + u_2 \end{aligned} \quad (6.1)$$

where $z_1, z_2, u_1, u_2, p_2, p_3$ are, respectively, the substrate concentration, the biomass concentration, the substrate feed rate, the biomass feed rate, the yield parameter and the maintenance parameter. The time varying parameter $p_0 = p_1 z_1$ is known as the "specific growth rate". It has been described by many different analytical expressions in the literature; among the most commonly used expressions are

The Monod law: $$p_0(z_1,z_2,t) = \frac{\mu^* z_1}{K_m + z_1} \quad (6.2)$$

The Contois law: $$p_0(z_1,z_2,t) = \frac{\mu^* z_1}{K_c z_2 + z_1} \quad (6.3)$$

where K_m, K_c are positive constants and μ^* is the maximum growth rate (which depends on temperature, pH,...).

One problem of practical interest is to design an adaptive observer/identifier for the on-line estimation of $z_2(t)$, $p_1(t)$, p_2 and p_3 from on-line measurements of $u_1(t), u_2(t)$ and $z_1(t)$; this is Problem 1 and 2 as described in Section 2. We shall now show that our adaptive observer can solve these problems without making any assumptions on a particular structure for $p_0(z_1,z_2,t)$.

On the basis of physical considerations the following assumptions are quite realistic.

Assumptions:

F.1: The specific growth rate is positive and bounded:

$$0 \le p_0(z_1,z_2,t) \le \bar{p}_0 \quad \forall z_1, z_2, t;$$

F.2: p_2, p_3 are constant; $p_2 > 0$ and $p_2 p_3 \ll 1$.

F.3: The derivative of p_1 is bounded:

$$\left|\frac{dp_1}{dt}\right| \le M < \infty \quad \forall t$$

F.4: The biomass and substrate feed rates are positive and bounded as follows:

$$p_2 p_3 u_{max} \le u_1 + \frac{u_2}{p_2} \le u_{max}$$

Transformation to AOCF

We first transform the GPS (6.1) into AOCF as follows

$$x_1 = z_1 \quad (6.4a)$$

$$x_2 = (1-c_2)z_1 - p_3 z_2 \quad (6.4b)$$

$$\theta_1 = -\frac{p_1}{p_2}z_2 + (c_2 - 2) \quad (6.4c)$$

$$\theta_2 = \left[-(1-c_2)\frac{p_1}{p_2} - p_1 p_3\right]z_2 \quad (6.4d)$$

$$\theta_3 = -p_3 \quad (6.4e)$$

This leads to the following AOCF:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -c_2\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}y & 0 & 0 \\ 0 & y & u_2\end{bmatrix}\begin{bmatrix}\theta_1 \\ \theta_2 \\ \theta_3\end{bmatrix} + \begin{bmatrix}u_1 \\ (1-c_2)u_1 - (1-c_2)^2 y\end{bmatrix}$$
$$y = x_1 \quad (6.5)$$

Note that the transformation (6.4) is uniquely invertible as follows:

$$p_3 = -\theta_3 \quad (6.6a)$$

$$z_2 = \frac{-x_2 + (1-c_2)y}{\theta_3} \quad (6.6b)$$

$$p_1 = \frac{(c_2 - 2 - \theta_1)(1-c_2) + \theta_2}{\theta_3 z_2} \quad (6.6c)$$

$$p_2 = \frac{p_1 z_2}{(c_2 - 2) - \theta_1} \quad (6.6d)$$

Therefore, on-line estimates of z_2, p_1, p_2, p_3 can be recovered from on-line estimates of $x_2, \theta_1, \theta_2, \theta_3$, thereby solving Problems 1 and 2.

Stability of the Observer

The stability conditions for the adaptive observer (3.1) applied to (6.5) are given in the next Theorem.

Thereom 6.1: Consider the GPS (6.1) with the assumptions F.1 to F.4 and the associated AOCF (6.5). Then the adaptive observer (3.1) applied to (6.5) is stable if the following conditions hold:

C.1: The initial conditions $z_1(o)$, $z_2(o)$ are such that

$$z_1(o) \text{ and } \frac{z_2(o)}{p_2} \le u_{max}.$$

C.2: $\dot{u}(t)$ is bounded except possibly at a countable number of points of minimum separation Δ.

C.3: $\omega(t)$ satisfies SI.2.

Proof: See Bastin and Gevers (1986).

A more complete overview of adaptive observers applied to a variety of biotechnological problems can be found in Bastin and Dochain (1986), Dochain and Bastin (1985), Bastin, Dochain and Gevers (1986).

7. CONCLUSIONS

We have argued that many observable nonlinear SISO systems can be transformed to a "canonical form" that has the crucial property of being "linear in the unknown quantites". We have then shown how an adaptive observer, inspired by an earlier observer for linear time-invariant sytems, can be applied to this transformed system. Our main contribution, besides this canonical form, has been to establish a precise set of sufficient conditions for global stability of our observer.

In two illustrative examples we have transferred the stability conditions on the AOCF into stability conditions on the GPS and on the regressor vector $\phi(t)$ of our observer. Other examples are studied in Bastin and Gevers (1986). The stability conditions on $\phi(t)$ are persistence of excitation conditions. Clearly, it would be better to translate these conditions into sufficient richness conditions on the input of the GPS. Sufficient richness conditions on u(t) for general nonlinear systems will certainly be very hard to obtain, but conditions can be obtained for systems with slowly time-varying parameters: see Mareels and Gevers (1986).

ACKNOWLEDGEMENTS

G.Bastin acknowledges the scientific support provided by his affiliation with GRECO-SARTA (CNRS, France). Both authors thank R.R. Bitmead for his helpful comments.

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