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A NEW ROBUST CONTROL DESIGN PROCEDURE BASED ON A PE IDENTIFICATION UNCERTAINTY SET

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Abstract: This paper proposes a new robust control design procedure based on a model and an uncertainty region deduced from classical PE identification. The key step in the procedure is a quality assessment procedure for the pair "model-uncertainty region" taking into account the prescribed performance level.

Keywords: prediction error identification, identification for robust control

1. INTRODUCTION

This paper is part of the wide-spread effort to connect time-domain prediction error (PE) identification and robustness theory. This paper builds specifically on some of our earlier works. In (Bombois *et al.*, 1999; Bombois, 2000), we have shown that PE identification with full-order model structures delivers both a model G_{mod} for control design and an uncertainty region $\mathcal{D}$ containing the true system at a certain probability level. This uncertainty region is a set of parametrized transfer functions whose (real) parameter vector is constrained to lie in an ellipsoid. In (Bombois *et al.*, 2001*a*), we have then developed robustness analysis tools that are adapted to the uncertainty set $\mathcal{D}$ and that verifies whether a given controller (typically designed from G_{mod}) stabilizes and achieves sufficient performance with all systems in the uncertainty set $\mathcal{D}$.

The results of (Bombois *et al.*, 2001*a*) pertain to the validation of a specific controller with respect to the systems in the identified uncertainty region. A controller (designed from the identified model) may be validated with respect to an uncertainty region $\mathcal{D}_1$ delivered by a PE identification under certain experimental conditions and not validated with respect to another uncertainty region $\mathcal{D}_2$ obtained under other experimental conditions. Consequently, it is important to be able to assess the quality of the pair $\{G_{mod}\ \mathcal{D}\}$ delivered by a PE identification experiment with respect to robust control design.

First steps in this direction have been achieved in (Bombois *et al.*, 2000*b*) (see also (Bombois, 2000)). Indeed, we proposed in that paper a quality assessment procedure that was based on a robust stability test. A pair $\{G_{mod}\ \mathcal{D}\}$ was termed tuned for robust control design if the set of G_{mod}-based controllers stabilizing all systems in $\mathcal{D}$ was large enough. The approach in (Bombois *et al.*, 2000*b*) has some drawbacks. It is indeed only based on a ***robust stability*** test, without taking into account the performance specifications that we want to achieve. In the considered set of G_{mod}-based controllers, there are indeed controllers that are not relevant because they do not achieve the desired performance with G_{mod} and will therefore never result from a control design step based on the identified model G_{mod}.

In (Bombois, 2000; Gevers *et al.*, 2001*b*), we have gathered these results into a robust control design procedure where the objective was to design a controller that achieved a level of performance Λ_0 with the unknown true system G_0. In that procedure, a PE identification experiment was first performed on G_0 yielding a pair $\{G_{mod}\ \mathcal{D}\}$. The quality of this pair was then assessed for robust stability using the results of (Bombois *et al.*, 2000*b*); and, if the quality was judged satisfactory, a controller was designed from G_{mod} using a performance criterion Λ_{mod} slightly better (i.e. slightly more demanding) than Λ_0. Using the results of (Bombois *et al.*, 2001*a*), we then verified if the designed controller achieved the prescribed level of performance (i.e. Λ_0) with all systems in $\mathcal{D}$ (and therefore also with the true system G_0).

In the present paper, we propose a much simpler and uniform robust control design procedure whose key step is an improved quality assessment procedure of the identified pair $\{G_{mod}\ \mathcal{D}\}$. The improvement is in the fact that the prescribed performance criterion (and not just the robust stability) is now taken into account in the quality assessment, in contrast to the results of (Bombois *et al.*, 2000*b*).

The new robust control design procedure. As stated earlier, the key step of our new robust control design procedure is an improved method to check whether the identified pair $\{G_{mod}\ \mathcal{D}\}$ is tuned for robust control design. This verification is based on the analysis of the behaviour of a set of controllers $\mathcal{C}(G_{mod})$ over all systems in the uncertainty region $\mathcal{D}$. This set $\mathcal{C}(G_{mod})$ is defined as the set of all controllers achieving a performance level Λ_{mod} with the identified model G_{mod}. This performance level Λ_{mod} (used for control design with the nominal model G_{mod}) is, as usually done in model-based control, chosen slightly better than the prescribed performance level Λ_0. By definition, the controllers in $\mathcal{C}(G_{mod})$ are therefore those that can result from a controller design step based on the model G_{mod}; they are thus the only ones that are relevant in order to establish the quality of the pair $\{G_{mod}\ \mathcal{D}\}$. We then state that an identified pair $\{G_{mod}\ \mathcal{D}\}$ is tuned for "robust control design" if all controllers in the set $\mathcal{C}(G_{mod})$ which achieve the performance level Λ_{mod} with G_{mod}, achieve the prescribed performance Λ_0 with all systems in $\mathcal{D}$.

Determination of the robust controller C. In the case where the identified pair has been termed tuned for robust control design, all controllers in $\mathcal{C}(G_{mod})$ are appropriate robust controllers for the true system G_0 since they are guaranteed to achieve the prescribed performance level (i.e. Λ_0) with all systems in $\mathcal{D}$, and thus in particular with G_0. The choice of a particular controller within that class can then be made on the basis of additional considerations such as lowest complexity.

New experiment design. Conversely, in the case where the quality of $\{G_{mod}\ \mathcal{D}\}$ is not judged satisfactory (the robustness test fails), we propose some guidelines (based on the results of the robustness test) in order to perform a new PE identification experiment providing a new pair "model-uncertainty region" that is likely to be better tuned for robust control design.

2. UNCERTAINTY REGION DELIVERED BY PREDICTION ERROR IDENTIFICATION

In this section, we recall our previous results concerning the uncertainty region $\mathcal{D}$ delivered by PE identification assuming that full-order model structures are used (Ljung, 1999). See (Bombois *et al.*, 1999; Bombois, 2000) for more details.

Proposition 2.1. Consider $G_0 = G(z, \delta_0)$, the true system. A PE identification experiment (with a full order model structure) performed on G_0 delivers an identified model $G(z, \hat{\delta})$ and an uncertainty region $\mathcal{D}$ containing the true system G_0 at a prescribed probability level α. This uncertainty region is centered at $G(z, \hat{\delta})$ and can be described by the following generic form:

$$\mathcal{D} = \left\{ G(z,\delta) \mid G(z,\delta) = \frac{e + Z_N\delta}{1 + Z_D\delta} \; and \; \delta \in U \right\} (1)$$

where $U = \{\delta \mid (\delta - \hat{\delta})^T R(\delta - \hat{\delta}) < \chi^2\}$; $\delta \in \mathbf{R}^{k\times 1}$ is a real parameter vector, $\hat{\delta}$ is the estimated parameter vector defining the identified model, R is a symmetric positive definite matrix $\in \mathbf{R}^{k\times k}$ that is equal to the inverse of the covariance matrix of $\hat{\delta}$, χ^2 is determined by the desired probability level α, $Z_N(z)$ and $Z_D(z)$ are row vectors of size k of known transfer functions and $e(z)$ is a known transfer function.

3. PROBLEM STATEMENT

In this section, we give a precise meaning to the concept that the pair $\{G_{mod}\ \mathcal{D}\}$ is tuned for robust control design. As stated in the introduction, this involves the determination of a class $\mathcal{C}(G_{mod})$ of G_{mod}-based controllers that are relevant for the control design problem.

3.1 *Performance criterion*

Throughout this paper we restrict attention to performance specifications that can be expressed in the following very general framework (see e.g. (Zames, 1981; Glover and Doyle, 1988)):

$$[C\ G]\ \textit{is stable} \tag{2}$$

and

$$\| F(G,C) \|_\infty < 1 \tag{3}$$

where

$$F(G,C) = \begin{pmatrix} W_{11}\dfrac{1}{1+CG} & W_{12}\dfrac{G}{1+CG} \\ W_{21}\dfrac{C}{1+CG} & W_{22}\dfrac{CG}{1+CG} \end{pmatrix} \tag{4}$$

Here $W_{ij}(z)$ are frequency weighting functions. Note that in practice the performance specifications are more often expressed as follows:

$$\begin{aligned} &\left| \frac{1}{1+C(e^{j\omega})G(e^{j\omega})} \right| < | W_{11}(e^{j\omega}) |^{-1} \\ &\left| \frac{G(e^{j\omega})}{1+C(e^{j\omega})G(e^{j\omega})} \right| < | W_{12}(e^{j\omega}) |^{-1} \\ &\left| \frac{C(e^{j\omega})}{1+C(e^{j\omega})G(e^{j\omega})} \right| < | W_{21}(e^{j\omega}) |^{-1} \\ &\left| \frac{C(e^{j\omega})G(e^{j\omega})}{1+C(e^{j\omega})G(e^{j\omega})} \right| < | W_{22}(e^{j\omega}) |^{-1} \end{aligned} \tag{5}$$

However the performance criterion (3) is generally used for control design purposes instead of the four conditions (5) (see e.g. (Zhou *et al.*, 1995; Glover and Doyle, 1988)) since it leads to an attractive computational algorithm. Conditions (5) will nevertheless be used for validation purposes. Note that specifications (3) imply (5), but the converse does not hold.

We consider the situation where the designer is faced with the task of constructing a controller C that achieves a performance level Λ_0 defined by the specifications (2) and (3) (or in fact (2) and (5)) on the *unknown* system G_0 (with some a-priori specified probability level α). For this, we propose the following procedure which combines PE identification and validation theory with robust control design and analysis theory.

3.2 *Identification, control design and validation procedure*

The procedure consists of four steps.

Step 1. Perform an identification experiment on the unknown system G_0. According to Proposition 2.1, this delivers an uncertainty region $\mathcal{D}$ containing the true system at the chosen probability level α and a model $G(z,\hat{\delta})$ at the center of $\mathcal{D}$. Select a model $G_{mod} \in \mathcal{D}$ for control design; for simplicity we shall here take $G_{mod} = G(z,\hat{\delta})$.

Step 2. Determine the set $\mathcal{C}(G_{mod})$ of G_{mod}-based controllers that are relevant for our control design problem. These are the controllers that achieve with the nominal model G_{mod} a performance level Λ_{mod} that is slightly better than the prescribed performance Λ_0 on the true system G_0. The relevant controller class is typically defined as follows:

$$\mathcal{C}(G_{mod}) = \{ C \mid [C\ G_{mod}]\ \textit{stable and}\ \| \frac{1}{\gamma} F(G_{mod}, C) \|_\infty < 1 \} \tag{6}$$

with $\gamma \leq 1$ (e.g. $\gamma = 0.9$). This set $\mathcal{C}(G_{mod})$ can be precisely parametrized as a function of a free parameter $Q(z) \in H_\infty$ such that $\| Q(z) \|_\infty < 1$ (see Section 4).

Step 3. Check whether all controllers in $\mathcal{C}(G_{mod})$ achieve the prescribed performance with all systems in $\mathcal{D}$, i.e. whether $\forall C \in \mathcal{C}(G_{mod})$ and $\forall G \in \mathcal{D}$, the conditions (2) and (5) hold.

If this is the case, the controller class $\mathcal{C}(G_{mod})$ is called validated for robust stability and robust performance, and we say that the pair $\{G_{mod}\ \mathcal{D}\}$ is **tuned for robust control design.**

Step 4. If the controller $\mathcal{C}(G_{mod})$ is validated, then any controller $C \in \mathcal{C}(G_{mod})$ is guaranteed, with probability α, to achieve the prescribed performance Λ_0 with the true system G_0 and the design procedure is finished. If the controller class is not validated (i.e. the pair $\{G_{mod}\ \mathcal{D}\}$ is not sufficiently tuned for control design), we propose guidelines for the design of a new identification experiment providing a new pair $\{G_{mod}\ \mathcal{D}\}$ that is likely to be better tuned for robust control design (see Section 7).

Important comments. Our 4-step identification and control design procedure follows an entirely logical thread from an engineering point of view. Given that the true system is unknown, one first identifies a nominal model G_{mod} and a model uncertainty set $\mathcal{D}$ around it. One then determines the class $\mathcal{C}(G_{mod})$ of controllers that achieves a certain level of performance that has been chosen slightly better than is required. One then checks whether these controllers achieved the required performance with all systems in $\mathcal{D}$, and hence with the true system. The choice of a particular controller within that class can then be made on the basis of additional considerations, such as lowest complexity: see Section 7. Finally, we observe that if the identification experiment has delivered a pair $\{G_{mod}\ \mathcal{D}\}$ that is tuned for robust control, then this is a *one-shot* design procedure. If not, then our method offers guidelines for an iterative design, i.e. the results of the validation analysis of step 3 indicate how to design a new identification experiment that is better tuned for robust control design.

4. PARAMETRIZATION OF THE SET OF RELEVANT CONTROLLERS

The key of the whole procedure is Step 3, which contains two validation tests, one for robust stability and one for robust performance. These tests will be performed by solving a robustness analysis problem with two sources of uncertainty: a plant uncertainty region (i.e. $\mathcal{D}$) and a controller uncertainty region (i.e. $\mathcal{C}(G_{mod})$). We show in the next sections that the set of closed-loop connections made up of the controllers in $\mathcal{C}(G_{mod})$ and the systems in $\mathcal{D}$ can be recast into a LFT framework and that particular robustness analysis tools can be developed for that particular LFT representation. A first step in this direction is achieved in the present section: we show that the systems in $\mathcal{D}$ and the controllers in $\mathcal{C}(G_{mod})$ can both be expressed as linear fractional transformations (LFT's) of some parameter

Proposition 4.1. ((Bombois *et al.*, 2001*a*)). The uncertainty region $\mathcal{D}$ defined by (1) can be rewritten in the LFT framework using a real vector $\phi \in \mathbf{R}^{k\times 1}$:

$$\mathcal{D} = \{G(z) \mid G(z) = \mathcal{F}_u(\mathcal{G}, \phi) \ with \ \phi \in \mathbf{R}^{k\times 1} \ and\ such\ that\ \|\phi\|_2 < 1\} \quad (7)$$

where $\mathcal{F}_u(.,.)$ denotes the classical upper LFT (see (Zhou *et al.*, 1995)) and $\mathcal{G}$ is a known matrix of adequate dimension which depends on e, Z_N, Z_D, $\hat{\delta}$, R and χ^2.

As for the controllers in $\mathcal{C}(G_{mod})$ defined in (6), it was shown in (Glover and Doyle, 1988) that they can also be expressed in an LFT framework.

Proposition 4.2. ((Glover and Doyle, 1988)). The relevant controller set $\mathcal{C}(G_{mod})$ defined in (6) can be rewritten in an LFT framework using a stable transfer function $Q(z)$:

$$\mathcal{C}(G_{mod}) = \{C(z) \mid C(z) = \mathcal{F}_l(\mathcal{K}, Q) \ with \ Q(z) \in H_\infty \ and\ such\ that\ \| Q \|_\infty < 1\} \quad (8)$$

where $\mathcal{F}_l(.,.)$ denotes the classical lower LFT (see (Zhou *et al.*, 1995)) and $\mathcal{K}$ is a matrix of adequate dimension which depend on G_{mod}, $F(.,.)$, and γ.

5. LFT FRAMEWORK OF THE CLOSED-LOOPS

In the previous section, we have given expressions for $\mathcal{D}$ and for $\mathcal{C}(G_{mod})$, both in an LFT framework. We will now consider the loops made up of one controller in $\mathcal{C}(G_{mod})$ and one plant in $\mathcal{D}$ and show that it is straightforward to express these loops in the same LFT framework. The closed-loop connection is entirely described by the four closed-loop transfer functions between the two inputs and the two outputs of $[C\ G]$:

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \overbrace{\begin{pmatrix} \frac{1}{1+CG} & \frac{G}{1+CG} \\ \frac{C}{1+CG} & \frac{GC}{1+CG} \end{pmatrix}}^{\triangleq \begin{pmatrix} T_{11}(G,C) & T_{12}(G,C) \\ T_{21}(G,C) & T_{22}(G,C) \end{pmatrix}} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \quad (9)$$

Expression (9) gives the closed-loop relations of the particular loop $[C\ G]$. The following proposition gives us the LFT representation of the set of all closed-loop connections (9) made up of any controller $C \in \mathcal{C}(G_{mod})$ and any system $G \in \mathcal{D}$.

Proposition 5.1. Consider the uncertainty region $\mathcal{D}$ defined in (1) and (7) and the set $\mathcal{C}(G_{mod})$ of relevant controllers defined in (6) and (8). The set of all closed-loop connections (9) made up of the controllers $C \in \mathcal{C}(G_{mod})$ and the systems $G \in \mathcal{D}$ can be rewritten in the following set of relations constrained by $\| \Delta \|_\infty < 1$:

$$\begin{cases} q = \Delta(z) p \\ \begin{pmatrix} p \\ \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \end{pmatrix} = \overbrace{\begin{pmatrix} M & H_{12} \\ H_{21} & H_{22} \end{pmatrix}}^{H(z)} \begin{pmatrix} q \\ \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \end{pmatrix} \end{cases} \quad (10)$$

where the uncertainty part $\Delta(z)$ is a matrix of size $(k+1) \times 2$ given by

$$\Delta(z) \triangleq \begin{pmatrix} \phi & 0 \\ 0 & Q(z) \end{pmatrix} \quad (11)$$

with ϕ as in (7) and $Q(z)$ as in (8); and the fixed part $H(z)$ of the LFT is a partitioned matrix made up of four stable elements M, H_{12}, H_{21} and H_{22} which depend on $\mathcal{G}$ and $\mathcal{K}$.

6. ROBUSTNESS ANALYSIS PROBLEM CORRESPONDING TO THE VALIDATION TESTS

Based on the previous results, we show in this section how we can perform the two tests involved in the third step of our robust control design procedure (see Section 3). Recall that these tests consist of verifying that the loops $[C\ G]$ made up of $C \in \mathcal{C}(G_{mod})$ and $G \in \mathcal{D}$ are all stable and that they all achieve the prescribed performance level (5).

The LFT representation of these loops given in Proposition 5.1 allows one to perform the first test (the robust stability test) by using the following classical robust stability result (see e.g. (Zhou *et al.*, 1995))

Proposition 6.1. Consider the uncertainty region $\mathcal{D}$ defined in (1) and (7) and the set $\mathcal{C}(G_{mod})$ of relevant controllers defined in (6) and (8). All controllers in $\mathcal{C}(G_{mod})$ stabilize all systems in $\mathcal{D}$ if and only if

$$\mu_\Delta(M(e^{j\omega})) \leq 1 \quad \forall\omega \tag{12}$$

where $M(z)$ is defined in (10) and where μ_Δ is the structured singular value of the structured parameter Δ (see (Zhou *et al.*, 1995)).

According to the previous proposition, we can thus verify whether all controllers in $\mathcal{C}(G_{mod})$ stabilize all systems in $\mathcal{D}$ by computing the structured singular value $\mu_\Delta(M(e^{j\omega}))$ at each frequency and verifying that these values are all smaller than one. In the full version of this paper (Bombois *et al.*, 2001*b*), we show that the computation of μ_Δ at each frequency boils down to an LMI-based optimization problem.

The second test of Step 3 (the robust performance test) can be performed using the methodology presented in the following proposition whose proof is straightforward.

Proposition 6.2. Consider the uncertainty region $\mathcal{D}$ defined in (1) and (7) and the set $\mathcal{C}(G_{mod})$ of relevant controllers defined in (6) and (8). Define the worst case performance related to the closed-loop transfer function $T_{ij}(G,C)$ $(i,j = 1..2)$ (see (9)) at the frequency ω as:

$$J_{WC}(\omega,\mathcal{D},\mathcal{C}(G_{mod}),T_{ij}) \triangleq \max_{\substack{G\in\mathcal{D}\\ C\in\mathcal{C}(G_{mod})}} \mid T_{ij}(G(e^{j\omega}),C(e^{j\omega})) \mid . \tag{13}$$

Then all controllers in $\mathcal{C}(G_{mod})$ achieve the prescribed performance constraints (5) with all systems in $\mathcal{D}$ if and only if:

$$J_{WC}(\omega,\mathcal{D},\mathcal{C}(G_{mod}),T_{ij}) <\mid W_{ij}(e^{j\omega}) \mid^{-1} \quad \forall\omega \tag{14}$$

for the four closed-loop transfer functions T_{ij} $(i,j = 1..2)$ (see (9)).

According to the previous proposition, the robust performance test involves the computation at each ω of the worst case performance $J_{WC}(\omega,\mathcal{D},\mathcal{C}(G_{mod}),T_{ij})$ related to each closed-loop transfer function. The LFT representation of the loops $[C\ G]$ given in Proposition 5.1 allows one to rewrite this worst case performance in a such a way that an LMI-based optimization problem can be developed for its computation. This is shown in the the full version of this paper (Bombois *et al.*, 2001*b*).

7. DETERMINATION OF A ROBUST CONTROLLER OR NEW EXPERIMENT DESIGN

In the previous sections, we have shown how to perform the validation tests involved in the third step of our robust control design procedure (see Section 3). Now, in order to illustrate the fourth step of this procedure, let us distinguish two cases. The first case is where the pair $\{G_{mod}\ \mathcal{D}\}$ passes both the robust stability and the robust performance validation tests. The second case is where the pair does not satisfy one (or both) validation test(s).

In the first case, the pair $\{G_{mod}\ \mathcal{D}\}$ is termed validated for robust control design. We then know that any controller in the set $\mathcal{C}(G_{mod})$ of relevant G_{mod}-based controllers defined in (6) will achieve the prescribed performance specifications (2) and (5) with all systems in $\mathcal{D}$ and thus in particular with the unknown true system G_0. In the second case, the pair $\{G_{mod}\ \mathcal{D}\}$ is not tuned for robust control design and a new PE identification experiment has to be performed on the true system in order to get a better identified pair model-uncertainty region. A nice property of our procedure is that guidelines can be drawn for the design of this new PE identification experiment. Indeed, the frequency regions where the frequency functions μ_Δ and J_{WC} exceed the admissible constraints are in fact the frequency regions where the uncertainty distribution is too large with respect to the desired control objective. The new experiment should be designed in order to reduce the uncertainty distribution in those frequency regions. It should therefore be designed such that the input signal has a larger power spectrum in those particular regions. Indeed, the uncertainty distribution in a particular frequency range is asymptotically inversely proportional to the spectrum of the input signal in open-loop identification (Ljung, 1999) and inversely proportional to the spectrum of the portion of the input signal that is due to the reference signal in closed-loop identification (Gevers *et al.*, 2001*a*).

Following the idea presented in the previous paragraph, our new procedure paves the way for a new research subject. Given a set of performance specifications, it consists of determining the experimental conditions (e.g. the input signal) such that a PE identification experiment under these experimental conditions delivers an uncertainty region $\mathcal{D}$ and a model G_{mod} that are tuned for robust control design.

8. CONCLUSIONS

In this paper, we have proposed a new robust control design procedure based on an uncertainty region and a model delivered by PE identification. The key step of this procedure is the quality assessment of the pair "model-uncertainty region". This pair is termed tuned for robust control design if the controllers in the relevant controller set $\mathcal{C}(G_{mod})$ stabilize and achieve the prescribed performance with all systems in $\mathcal{D}$. If it is the case, then any controller in $\mathcal{C}(G_{mod})$ is an appropriate robust controller for the true system. Conversely, if the pair "model-uncertainty region" is not judged satisfactory, guidelines can be drawn in order to design a new PE identification experiment delivering a pair that is better tuned for robust control design.

In this paper, we assess the quality of the pair $\{G_{mod}\ \mathcal{D}\}$ with respect to the relevant controller set $\mathcal{C}(G_{mod})$. This could seem very demanding since, in fact, we only need one controller in that set to achieve the prescribed performance. However, it is in our opinion that we could not term a pair $\{G_{mod}\ \mathcal{D}\}$ tuned for robust control design if the prescribed performance level is achieved by one controller C_1 in $\mathcal{C}(G_{mod})$ and not by the other controllers that achieve the same level of performance with G_{mod} and that can be obtained from C_1 via a modification of the parameter Q.

Finally, the results developed in this paper for the uncertainty region $\mathcal{D}$ deduced from an identification step with a full-order model structure can be easily extended to the case of an uncertainty region $\mathcal{L}$ deduced from an identification step using a restricted complexity model structure (see (Bombois *et al.*, 2000*a*)). Note also that a simulation example of our method can be found in the full version of this paper (Bombois *et al.*, 2001*b*).

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