A Comparison Between Model Reduction and Controller Reduction: Application to a PWR Nuclear Plant†

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Abstract

The most classical way of obtaining a low-order model-based controller for a high-order system is to apply reduction techniques to an accurate high-order model or controller of the plant. An alternative is to use identification for control with a low-order model set. In [1], we compared model reduction, both in open loop and in closed loop, with closed-loop identification of a low-order model. Both methods were applied to the design of a controller for the secondary circuit of a nuclear Pressurized Water Reactor, leading to the conclusions that system identification is a viable alternative to model reduction, and that the key feature for a successful control design is not so much the choice between order reduction or identification, but between open-loop and closed-loop techniques. In the present paper, we go further in this study and we compare model reduction with controller reduction for the same PWR system. We show that closed-loop techniques are more powerful than open-loop ones, and that controller reduction gives better results than model reduction when appropriate frequency weightings are chosen.

1 Introduction

There are several ways of obtaining low-order controllers for high-order systems. One line of thinking is to first obtain a very accurate high-order model and then apply reduction techniques to this model or to a high-order controller computed from that model. There is an extensive literature on this subject. One of the important theoretical messages of this literature is that, if the ultimate objective is the low-order controller (rather than the low-order model), then it is essential that the closed-loop performance objective be incorporated in the reduction technique. This is typically achieved by specific frequency weightings that translate these closed-loop objectives in the model or controller reduction criterion.

The recent research on identification for control has promoted the idea that one can, alternatively, obtain a low-order model directly by closed-loop identification, where the identification criterion takes account of the control performance objective.

The approach based on model reduction (both in open loop and in closed loop) was compared to that based on closed-loop identification in [1]. For that purpose, both methods were applied to the design of a low-order controller for a realistic model of order 42 of a Pressurized Water Reactor (PWR) nuclear power plant. The main contribution of that paper was to add insight to the ongoing debate about identification for control. Two important findings were produced. Firstly, closed-loop identification proved to be a viable alternative to model reduction: we produced a 12-th order controller obtained through closed-loop model reduction techniques that achieved a required level of performance and we showed that the same performance was achieved by a 12-th order controller obtained from a low-order model of the PWR directly identified via a control-oriented identification criterion. In both cases the same LQG[1] criterion was used for the design. Secondly we showed that, whether the route to a low-order controller is via model reduction techniques or via identification of a low-order model, the key to the success of the operation is to inject weightings that reflect the closed-loop performance objectives.

In the present paper, we focus on the approach that was not considered in [1], namely computation of a full-order controller followed by controller reduction. We

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[1]Taking account of LQG weighting filters that are actually part of the designed controllers, their order was in fact 14.

compare this with the model reduction approach considered in [1]. Therefore, the same 42-nd order model of the PWR is used. It is first reduced to a model of order 33, for reasons explained in Section 5. For this 33-rd order model, a full LQG controller of order 40 is computed and then reduced to order 14. We show that its performance is similar to that of the 14-th order controller obtained *via* reduction of the model to order 7 followed by control design with the same LQG criterion. But in addition, we show that if we push down the reduction to the lowest possible order, the results obtained by closed-loop controller reduction can be significantly better than those obtained by closed-loop model reduction or open-loop controller reduction, provided appropriate weightings are used.

The outline of the paper is as follows. In Section 2 we sketch the modelling of the PWR plant, while Section 3 gives a description of the control problem and the control design procedure. Section 4 reviews the coprime factor model order reduction procedure already used in [1], first in open loop, then when closed-loop considerations are taken into account. It shows how the control objective can be used to select an adequate frequency weighting for the reduction. Section 5 describes the procedure used to reduce the controller in an open-loop as well as in a closed-loop sense. The performance on the actual system of the low-order controllers obtained in Sections 4 and 5 are compared in Section 6. Finally, some conclusions are drawn in Section 7.

2 Modelling of the PWR

A realistic nonlinear simulator, based on a first principles' model describing both primary and secondary circuits of the PWR (See Figure 1), has been developed at Électricité de France (EDF). It includes all local controllers involved in both primary and secondary circuits.

In this paper, we focus on the behavior of the plant around a fixed operating point corresponding to 95% of maximum operating power. This results in a high (42-nd) order model P_{42}, which includes the dynamics of the primary and secondary circuits and of all local controllers, except some specific controllers of the secondary circuit that we want to redesign; these are denoted K_{tb} and K_{cd} in Figure 2. They control the electrical power and the condenser water level, respectively, and their structures are very simple: K_{tb} is a PI controller acting on the difference between its two inputs, while K_{cd} is a second order two-input-one-output controller which includes an integrator. For the sake of simplicity, K_{tb} and K_{cd} will both be called "PID" controllers in the sequel.

In Figure 2, W_e is the electrical power produced by

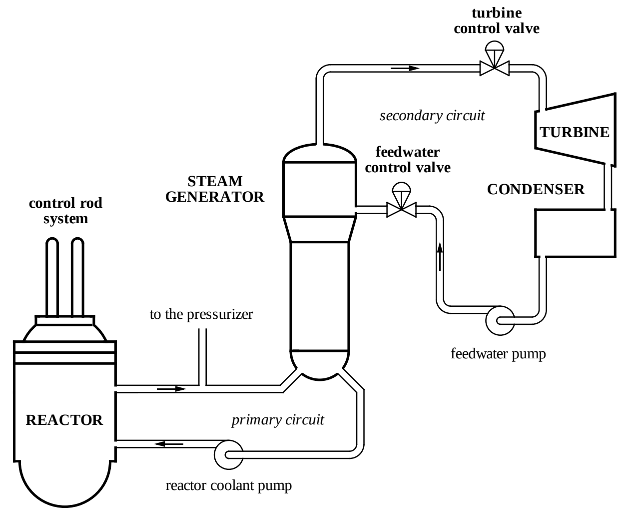

Figure 1: PWR plant description

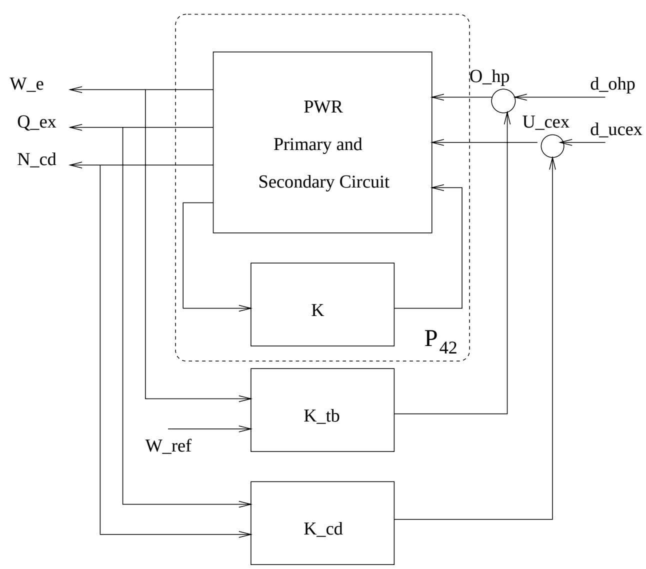

Figure 2: Interconnection of P_{42} with the PID controllers.

the plant, controlled to follow the demand W_{ref} of the network, and directly related to the steam flow in the turbine, which depends on the high pressure turbine control valve aperture O_{hp} - see Figure 1; Q_{ex} is the extraction water flow, and N_{cd} the water level in the condenser (both are related to the locally controlled speed of the feedwater pump and to the extraction valve aperture U_{cex}). d_{Ohp} and d_{Ucex} represent additive terms on the control inputs that can be either disturbances, or excitations for identification purposes. Obviously, there is a strong coupling between W_e and O_{hp} on the one hand, and between N_{cd} and U_{cex} on the other hand, which would explain the structure of the present PID controllers. However, the control performance might be enhanced by taking the cross-couplings into account.

3 Control design strategy

Our goal is to redesign controllers for the electrical power control in the secondary circuit, *i.e.* to replace the present PID controllers K_{tb} and K_{cd} by a single multivariable controller in order to achieve a better performance. The chosen control design is a Linear Quadratic Gaussian (LQG) controller computed from a reduced order model $\hat{P}_r$ of the plant P_{42} (or from that plant itself).

The control objective is to use the feedwater tank, rather than the control rods in the primary circuit, to absorb the fast and medium range variations in the power demands by acting on the valve apertures. The controller will have to ensure that the electrical power supply W_e follows accurately the reference signal W_{ref}, and to regulate the condenser water level N_{cd} around its nominal value: see Figure 2. Also, it will have to reject possible disturbances acting on the system at the inputs d_{Ohp} and d_{Ucex}. The validation will be done with step signals on W_{ref}, d_{Ohp} and d_{Ucex}.

In order to remain consistent in the comparative study, the same LQG criterion is used with each (reduced order) model:

$$J_{LQG}(u) = \int_0^\infty \left(y_{filt}^T Q y_{filt} + u^T R u \right) dt, \qquad (1)$$

where $u = \begin{bmatrix} O_{hp} & U_{cex} \end{bmatrix}^T$ is the control vector and $y_{filt} = \begin{bmatrix} F(s)(W_e - W_{ref}) & F(s) N_{cd} \end{bmatrix}^T$ is the controlled output vector filtered through a filter $F(s) = 1 + 20/s$ to ensure a zero static error. This filter is then connected to the corresponding inputs of the designed controller. Since the main goal is to control W_e (the regulation of N_{cd} being only a secondary requirement), more weight is put on the electrical power tracking error than on the condenser water level in J_{LQG}. On the other hand, since the nominal value of U_{cex} is 0.01 while it is 1 for O_{hp}, 10^4 times more weight is put on U_{cex} to ensure a correct scaling. The chosen weighting matrices are

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 10^{-2} \end{bmatrix}, \quad R = \begin{bmatrix} 10^2 & 0 \\ 0 & 10^6 \end{bmatrix}. \qquad (2)$$

These weightings have proved very satisfactory[2].

For the design of the Kalman filter, the external signals d_{Ohp}, d_{Ucex} and W_{ref}, which are in the low frequency ranges, are modeled as independent Gaussian white noises filtered through a low-pass filter $N(s) = 1/(s + 0.01)$. Since the external signals have a typical amplitude of 1 for d_{Ohp} and W_{ref}, and of 0.01 for d_{Ucex}, their covariance matrix is chosen as $Q_n = diag(1, 0.0001, 1)$. In order to ensure a good roll-off at high frequency, the measurement noise is parametrized as a Gaussian white noise with large covariance $R_n = diag(10, 10, 10)$ (remember that Q_{ex} is measured and used for state estimation, although it is not regulated, which is why R_n is 3×3).

The presence of the filters $F(s)$ (twice) and $N(s)$ (3 times) in the model used for the design will yield a controller with order equal to that of this design model, *i.e.* the order of $\hat{P}_r$ plus 5. Furthermore, $F(s)$ must be explicitely added to the designed controller before it can be used with $\hat{P}_r$ or P_{42}, and the total order of the controller will therefore finally be that of $\hat{P}_r$ plus 7. This will determine the order of $\hat{P}_r$ if a controller of some fixed order is desired.

[2] The weighting matrices and filters used here are different from those used in [1]. However, we have verified that all conclusions and observations made in [1] would remain valid with the weightings used here.

4 First approach to a low-order controller: model reduction followed by control design

Our first approach consists in reducing the order of the 42-nd order plant model P_{42} to a model of order 7 (for the sake of comparison with the model identified in Section 4 of [1]), and to use this resulting low-order model to design a controller. Since the system P_{42} includes unstable modes, a straight balanced truncation is not achievable. Therefore we use a factorization method in which the system transfer function is factored into stable coprime factors, as proposed by Meyer [2]. This section reviews material already presented in [1]; however, Subsection 4.2 has been re-written such that the new results of Subsection 5.2 can be derived from the same expressions, making them much easier to understand.

4.1 Open-loop coprime factor reduction

Since the unstable system under consideration is detectable, we can construct a stable left coprime factorization (LCF) $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$ such that $P_{42} = \tilde{M}_{42}^{-1} \tilde{N}_{42}$. We refer the reader to [1] for more details on how such a factorization can be obtained.

The coprime factors can be *normalized*, meaning that

$$\tilde{N}_{42} \tilde{N}_{42}^\star + \tilde{M}_{42} \tilde{M}_{42}^\star = I$$

where $X^\star(s) = X^T(-s)$. Since $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$ is stable, it can be reduced using standard balanced truncation. One advantage of using normalized coprime factors is that the error between the full- and reduced-order models can then be interpreted in the *graph metric* or *gap metric* [3, 2, 4]: the error is an upper bound on the distance between the graphs of the full and reduced order models. Let $\begin{bmatrix} \tilde{N}_r & \tilde{M}_r \end{bmatrix}$ denote the reduced LCF realization. To make the reduction process useful, we must ensure that the reduced factors $\tilde{N}_r$ and $\tilde{M}_r$ have the same denominator, and that their realizations have the same state, so that the order of the reduced model $\hat{P}_r = \tilde{M}_r^{-1}\tilde{N}_r$ is that of $\tilde{N}_r$ and $\tilde{M}_r$ rather than the sum of these.

We apply the LCF reduction method to P_{42}. We denote by $\hat{P}_r^{ol}$ the reduced order model obtained by truncating all but the first r singular values. Recall that for such model an upper bound on the committed approximation error in the $\mathcal{H}_\infty$ norm is given by twice the sum of the truncated Hankel singular values (HSV): $\|P_{42} - \hat{P}_r^{ol}\|_\infty \leq 2\sum_{i=r+1}^{42}\sigma_i$. We have observed in [1] that the model $\hat{P}_7^{ol}$ produces a destabilizing LQG controller of order 14, $K_{14}^{\hat{P}_7^{ol}}$, when applied to the true system, and that it is necessary to limit the reduction at the order 12 ($\hat{P}_{12}^{ol}$) to obtain a stabilizing controller. The latter is denoted $K_{19}^{\hat{P}_{12}^{ol}}$ and is of order 19.

4.2 Closed-loop coprime factor reduction

Let us rewrite the two PID controllers K_{tb} and K_{cd} of Figure 2 as a single controller K_{PID}. The idea of closed-loop model reduction is to compute the reduced-order model $\hat{P}_r$ that ensures the best possible matching between the closed-loop transfer functions $T(P_{42}, K_{PID})$ and $T\left(\hat{P}_r, K_{PID}\right)$.

The closed-loop system can be redrawn as in Figure 3, where $y = \begin{bmatrix} Q_{ex} & W_e & N_{cd} & \phi \end{bmatrix}^T$, $r = \begin{bmatrix} \phi & \phi & \phi & W_{ref} \end{bmatrix}^T$, $u = \begin{bmatrix} -O_{hp} & -U_{cex} \end{bmatrix}^T$ and $d = \begin{bmatrix} d_{Ohp} & d_{Ucex} \end{bmatrix}^T$. Here, ϕ denotes a fictitious nil signal. Note that a fictitious output has been added to P_{42} to make the dimensions of all vectors compatible.

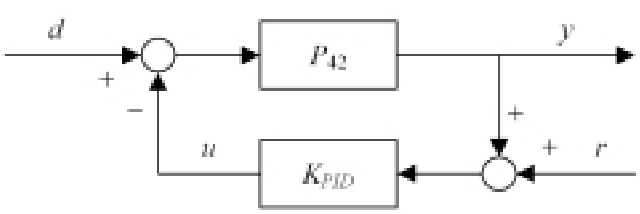

Figure 3: Closed-loop configuration for the model P_{42} in feedback with the PID controller K_{PID}.

Consider a right coprime factorization (RCF) $K_{PID} = UV^{-1}$ of the controller. Some basic calculations show that

$$\begin{aligned}\begin{bmatrix} u \\ y \end{bmatrix} &= \begin{bmatrix} K_{PID} \\ I \end{bmatrix} (I + P_{42}K_{PID})^{-1} \\ &\quad \times \begin{bmatrix} P_{42} & I \end{bmatrix} \begin{bmatrix} d \\ r \end{bmatrix} \\ &= \begin{bmatrix} U \\ V \end{bmatrix} \Phi^{-1} \begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix} \begin{bmatrix} d \\ r \end{bmatrix} \qquad (3)\end{aligned}$$

where $\Phi = \tilde{N}_{42}U + \tilde{M}_{42}V$ can be made equal to I for some choice of U and V (to simplify the notations, we shall assume that in the sequel). Minimizing the approximation error between the two closed-loop transfer functions, in which K_{PID} remains unchanged, is then equivalent with minimizing a frequency weighted difference between the LCF's of P_{42} and $\hat{P}_r$:

$$\begin{array}{c} \min_{\hat{P}_r} \left\| T(P_{42}, K_{PID}) - T\left(\hat{P}_r, K_{PID}\right) \right\| \\ \Updownarrow \\ \min_{\{\tilde{N}_r, \tilde{M}_r\}} \left\| W_{out} \left(\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix} - \begin{bmatrix} \tilde{N}_r & \tilde{M}_r \end{bmatrix} \right) W_{in} \right\|. \end{array} \qquad (4)$$

The weightings W_{in} and W_{out} are chosen in order to take account of the entries of y, r and d that are important for the control design, namely W_e, N_{cd}, d_{Ohp}, d_{Ucex} and W_{ref}. Considering (3) and the definition of y, r and d yields

$$W_{out} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \qquad (5)$$

and

$$W_{in} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (6)$$

This defines an *output frequency weighted (OFW) balanced truncation* problem where the object to approximate is $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$. The resulting $\tilde{N}_r$ and $\tilde{M}_r$ have the same state matrix and define a LCF of $\hat{P}_r$.

We apply the OFW balanced truncation method to the stable normalized LCF of the system P_{42}. The reduced model of order 7 is denoted $\hat{P}_7^{cl}$ and the controller of order 14 computed from this model is denoted $K_{14}^{\hat{P}_7^{cl}}$. For the sake of further discussion and comparison, we have also computed the model $\hat{P}_4^{cl}$ of order 4 obtained by this procedure, and the corresponding controller of order 11, $K_{11}^{\hat{P}_4^{cl}}$. The approximation error has the same property as in open-loop reduction: $\|P_{42} - \hat{P}_r^{cl}\|_\infty \leq 2\sum_{i=r+1}^{42} \sigma_i$ (here, the σ_i's denote HSV's of the OFW LCF). The same interpretation in the gap metric holds, since the same normalized LCF of P_{42} is used.

5 Second approach to a low-order controller: control design followed by controller reduction

Our second approach consists in first designing a high-order LQG controller for the plant P_{42} and then reducing its order. The control design criterion is as explained in Section 3. However the realization P_{42} is nearly non-minimal, which causes some numerical problems in the computation of the Kalman filter part of the controller. Therefore 9 nearly cancelling modes must be dropped before designing the controller, which consequently has order 40 instead of 49. This optimal LQG controller is denoted K_{40}.

5.1 Open-loop coprime factor reduction

K_{40} being unstable, we have to perform the reduction on its normalized coprime factors. All the derivations of Subsection 4.1 are still valid here if we replace $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$ by (normalized) coprime factors $\begin{bmatrix} \tilde{U}_{40} & \tilde{V}_{40} \end{bmatrix}$ of K_{40} such that $K_{40} = \tilde{V}_{40}^{-1}\tilde{U}_{40}$.

In order to establish comparisons with the previoulsy computed low-order controllers, we have reduced K_{40} down to orders 14 and 11, the corresponding controllers being denoted $\hat{K}_{14}^{ol}$ and $\hat{K}_{11}^{ol}$. We have also computed the controller of order 9 $\hat{K}_9^{ol}$ in order to compare it with $\hat{K}_9^{cl}$ which will be obtained in Subsection 5.2.

Note that a right coprime factorization $\begin{bmatrix} U_{40} \\ V_{40} \end{bmatrix}$: $K_{40} = U_{40}V_{40}^{-1}$ can also be used, leading to the same results.

5.2 Closed-loop coprime factor reduction

The basic idea that leads us to reduce the coprime factors of the controller in closed-loop is the same as that used in Subsection 4.2 for closed-loop model reduction, where the closed-loop transfer function $T(P_{42}, K_{PID})$ was approximated by $T\left(\hat{P}_r, K_{PID}\right)$, P_{42} being reduced and K_{PID} remaining unchanged. Here, we approximate $T(P_{42}, K_{40})$ by $T\left(P_{42}, \hat{K}_r\right)$, where the controller K_{40} is the object to be reduced and P_{42} is fixed.

Consider again equation (3). We used it in Subsection 4.2 to transform the problem of reducing P_{42} in closed loop to that of reducing $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$ with frequency weightings depending on $\begin{bmatrix} U \\ V \end{bmatrix}$. Here we do the opposite: the same equation can be used to transform the problem of reducing K_{40} in closed loop to that of reducing $\begin{bmatrix} U \\ V \end{bmatrix}$ with weightings that now depend on $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$. Of course, here, $\begin{bmatrix} U \\ V \end{bmatrix}$ is no longer a RCF of K_{PID}, but a normalized RCF $\begin{bmatrix} U_{40} \\ V_{40} \end{bmatrix}$ of K_{40}. $\begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix}$ does no longer have to be normalized and can be chosen such that $\Phi = I$, which is assumed in the sequel.

Minimizing the approximation error between the two closed-loop transfer functions $T(P_{42}, K_{40})$ and $T\left(P_{42}, \hat{K}_r\right)$ is thus equivalent with minimizing a frequency weighted difference between the RCF's of K_{40} and $\hat{K}_r$:

$$\begin{array}{cc} \min\limits_{\hat{K}_r} & \left\| T(P_{42}, K_{40}) - T\left(P_{42}, \hat{K}_r\right) \right\| \\ \Updownarrow & \\ \min\limits_{\{U_r, V_r\}} & \left\| W_{out} \left(\begin{bmatrix} U_{40} \\ V_{40} \end{bmatrix} - \begin{bmatrix} U_r \\ V_r \end{bmatrix} \right) W_{in} \right\|. \end{array} \tag{7}$$

Similarly to what was done in Subsection 4.2, the weightings are chosen in order to take account of the entries of y, r and d that are concerned by the control objective, namely W_e, N_{cd}, d_{Ohp}, d_{Ucex} and W_{ref}. However, it seems very reasonable to require that the reduced-order controller be as much as possible optimal with respect to the criterion (1) that was used to compute K_{40}. Therefore, contrary to the choice made in Subsection 4.2, W_{out} also penalizes the control signals in u, namely O_{hp} and U_{cex}, and integrates the control criterion *via* weightings put on O_{hp}, U_{cex}, W_e and N_{cd}, and selected to reflect as much as possible the weightings imposed in the design criterion (1). Consequently, the input and output frequency weighting filters are respectively

$$W_{in} = \begin{bmatrix} \tilde{N}_{42} & \tilde{M}_{42} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

and

$$W_{out} = \begin{bmatrix} \beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 F(s) & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 F(s) & 0 \end{bmatrix}. \tag{9}$$

Here the α_i's are the square roots of the entries of Q in (2), the β_i's are those of R in (2), and $F(s)$ is the LQG filter. Since $F(s) = (s+20)/s$ is unstable, we have replaced it by $(s+20)/(s+\varepsilon)$ where $\varepsilon = 10^{-6}$.

This defines an *input-output frequency weighted (IOFW) balanced truncation* problem where the object to approximate is $\begin{bmatrix} U_{40} \\ V_{40} \end{bmatrix}$. The resulting U_r and V_r have the same state matrix and define a RCF of $\hat{K}_r$.

In order to establish comparisons with the previoulsy computed low-order controllers, we have reduced K_{40} down to orders 14 and 11, the corresponding controllers being denoted $\hat{K}_{14}^{cl}$ and $\hat{K}_{11}^{cl}$. We have also computed the controller of order 9, $\hat{K}_9^{cl}$, which is the lowest-order controller that can be obtained by this method and that still has good performance when applied to P_{42}.

Note that we have also tried to perform the reduction with a filter W_{out} that only puts a unit penalty on W_e and N_{cd}, and no penalty at all on the two control signals. This case is thus the dual situation of Subsection 4.2. The results proved much less satisfactory, with important oscillations appearing in the control signals when the reduction was pushed down to order 11 or less.

6 Comparative study of controller performance

In this section we compare the performance on the actual system P_{42} of the controllers computed in the two previous sections.

6.1 Performance of controllers of order 14

We first consider the four controllers of order 14, $K_{14}^{\hat{P}_7^{ol}}$, $K_{14}^{\hat{P}_7^{cl}}$, $\hat{K}_{14}^{ol}$ and $\hat{K}_{14}^{cl}$, obtained respectively *via* open-loop model reduction, closed-loop model reduction, open-loop controller reduction and closed-loop controller reduction. $K_{14}^{\hat{P}_7^{ol}}$ destabilizes P_{42}. The other three have very good performance when applied to P_{42}; the last two are almost undistinguishable (their performance is very close to that of K_{40}, which is not illustrated in order to keep the plots readable) and a bit better than the second one when considering their responses to a step disturbance d_{Ucex}, as shown in Figure 6. Figures 4 and 5 show the closed-loop responses of P_{42} with those controllers, resp. to a step reference W_{ref} and to a step disturbance d_{Ohp}.

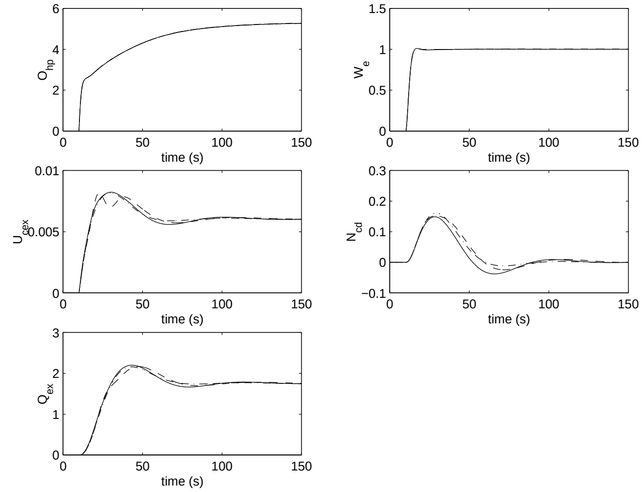

Figure 4: Responses to a step on W_{ref}: P_{42} controlled by $K_{14}^{\hat{P}_7^{cl}}$ (—), $\hat{K}_{14}^{ol}$ (−−) and $\hat{K}_{14}^{cl}$ (−·).

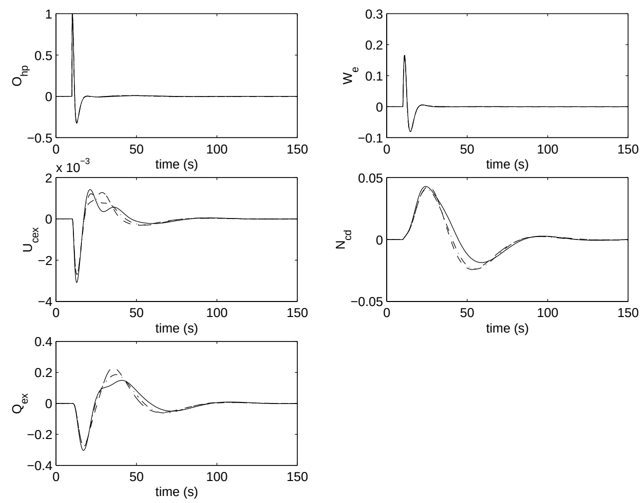

Figure 5: Responses to a step on d_{Ohp}: P_{42} controlled by $K_{14}^{\hat{P}_7^{cl}}$ (—), $\hat{K}_{14}^{ol}$ (−−) and $\hat{K}_{14}^{cl}$ (−·).

6.2 Performance of controllers of order 11

Here we consider the three controllers of order 11, $K_{11}^{\hat{P}_4^{cl}}$, $\hat{K}_{11}^{ol}$ and $\hat{K}_{11}^{cl}$, obtained respectively *via* closed-loop model reduction, open-loop controller reduction and closed-loop controller reduction. Recall that 11 is the lowest order reachable by closed-loop model reduction while ensuring stability. Their performance are depicted in Figures 7 to 9.

Trends observed in the previous subsection are confirmed here: the responses of $K_{11}^{\hat{P}_4^{cl}}$, although still very acceptable, deviate from those of the other two controllers, which remain close to the optimal one K_{40} (not shown on the plots).

6.3 Performance of controllers of order 9

Finally, we consider the two controllers of order 9, $\hat{K}_9^{ol}$ and $\hat{K}_9^{cl}$, obtained respectively *via* open-loop and closed-loop controller reduction. Their performance are depicted in Figures 10 to 12, where they are compared to that of the optimal LQG controller K_{40}.

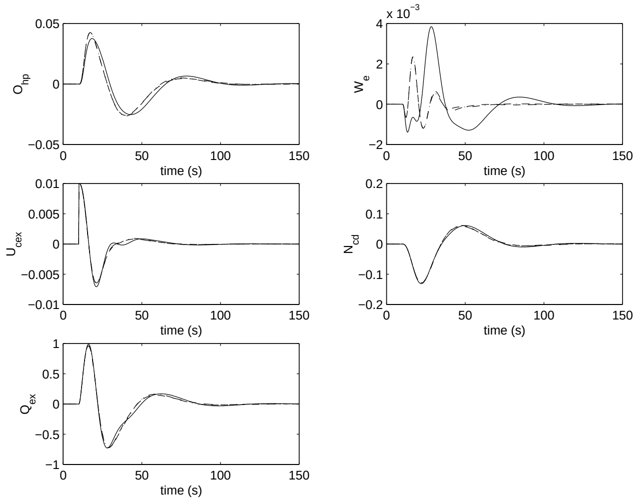

Figure 6: Responses to a step on d_{Ucex}: P_{42} controlled by $K_{14}^{\hat{P}_7^{cl}}$ (—), $\hat{K}_{14}^{ol}$ (– –) and $\hat{K}_{14}^{cl}$ (–·).

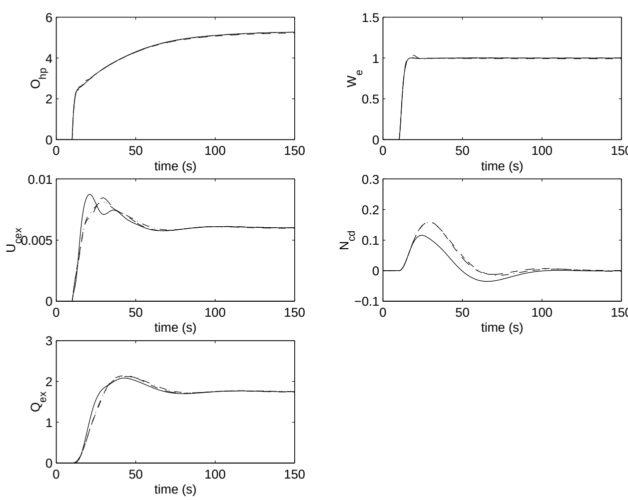

Figure 7: Responses to a step on W_{ref}: P_{42} controlled by $K_{11}^{\hat{P}_4^{cl}}$ (—), $\hat{K}_{11}^{ol}$ (– –) and $\hat{K}_{11}^{cl}$ (–·).

These plots clearly show the superiority of closed-loop controller reduction when the order is reduced as much as possible. While the open-loop reduced controller has unacceptable performance when applied to P_{42}, the closed-loop reduced one remains very close to the optimal high-order controller K_{40}, even at such low order as 9. However, recall that such a good result has been obtained with weightings chosen to reflect the design criterion (see Subsection 5.2). Such weightings might be difficult to choose in some cases, while the weightings used in closed-loop model reduction are obtained in a much more starightforward way.

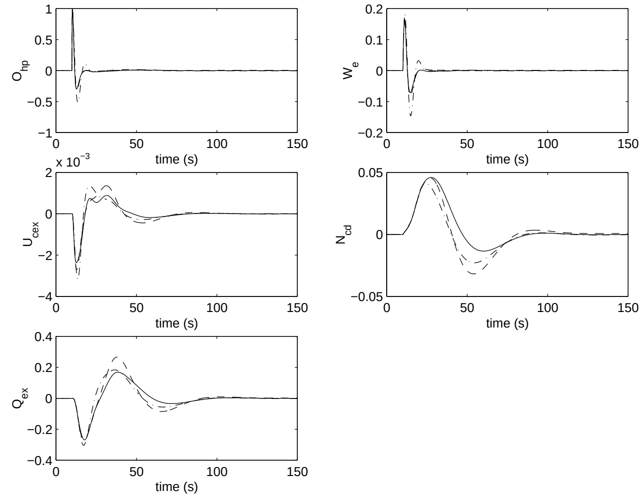

Figure 8: Responses to a step on d_{Ohp}: P_{42} controlled by $K_{11}^{\hat{P}_4^{cl}}$ (—), $\hat{K}_{11}^{ol}$ (– –) and $\hat{K}_{11}^{cl}$ (–·).

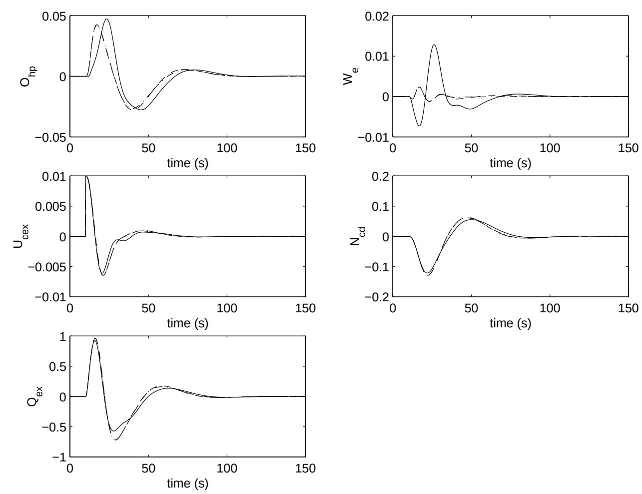

Figure 9: Responses to a step on d_{Ucex}: P_{42} controlled by $K_{11}^{\hat{P}_4^{cl}}$ (—), $\hat{K}_{11}^{ol}$ (– –) and $\hat{K}_{11}^{cl}$ (–·).

7 Conclusions

In this study we have compared four different ways of obtaining a low-order controller for a complex plant: open-loop or closed-loop reduction of the plant model followed by control design, and full-order control design followed by open-loop or closed-loop reduction of the optimal controller. Furthermore, in [1], a fifth method was studied: direct closed-loop identification of a low-order plant model followed by control design.

The methods have been tested on a realistic, high-order linearized model of a PWR nuclear power plant, the goal being the replacement of two PID controllers by a multivariable controller for the electrical power while ensuring an acceptable water level regulation in the condenser.

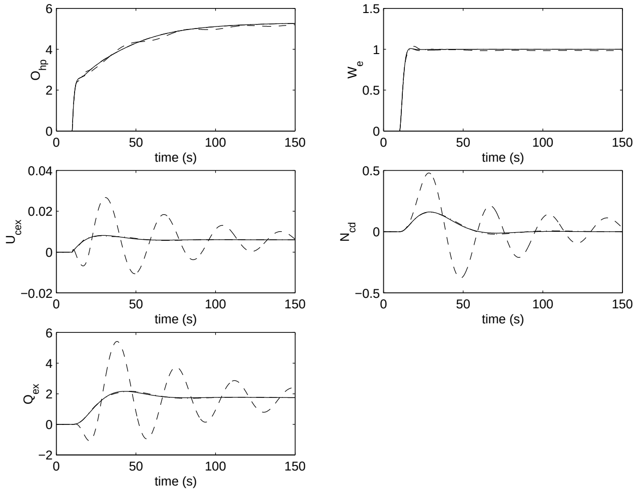

Figure 10: Responses to a step on W_{ref}: P_{42} controlled by K_{40} (—), $\hat{K}_9^{ol}$ (− −) and $\hat{K}_9^{cl}$ (−·).

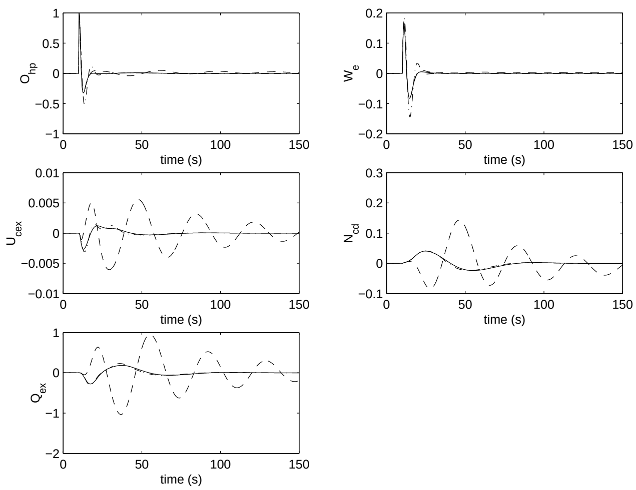

Figure 11: Responses to a step on d_{Ohp}: P_{42} controlled by K_{40} (—), $\hat{K}_9^{ol}$ (− −) and $\hat{K}_9^{cl}$ (−·).

Following the observations made in Section 6 of this paper and in [1], two tables can be drawn up. Table 1 classifies the methods with respect to the performance achieved by controllers of the same order (when this order is sufficiently low to make differences appear). The performance of each controller is evaluated in terms of the discrepancy between its responses and those of the optimal one (K_{40}). On the other hand, Table 2 classifies the methods with respect to the lowest order that can be reached with a pre-specified level of performance.

Methods based on controller reduction appear to be potentially more powerful than those based on model reduction, when they are both considered either in open loop or in closed loop. This result is quite logical since reduction implies a loss of information: it is best to keep all the information (*i.e.* the richness of the model) as long as possible. In other words, it may be difficult

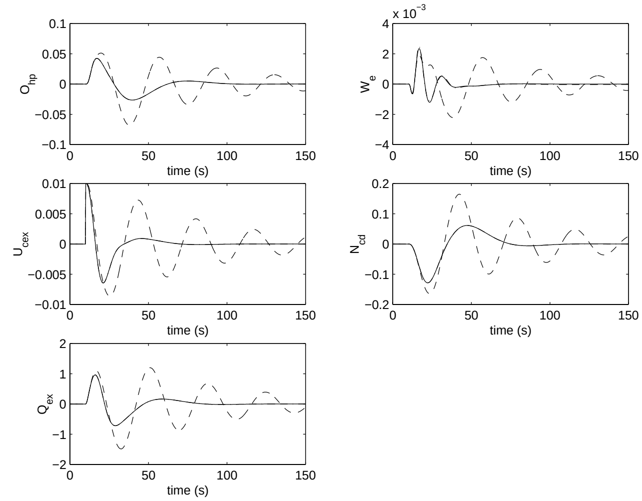

Figure 12: Responses to a step on d_{Ucex}: P_{42} controlled by K_{40} (—), $\hat{K}_9^{ol}$ (− −) and $\hat{K}_9^{cl}$ (−·).

Performance	Method
MAX	CL controller reduction
↓	CL model identification
	CL model reduction
↓↓	OL controller reduction
MIN	OL model reduction

Table 1: Classification of the methods w.r.t. the performance achievable with controllers of the same order

to predict how model reduction will affect the to-be-designed controller while, on the other hand, when doing controller reduction, the starting point is the optimal high-order controller and the loss of performance is directly related to the approximation error committed during the reduction. However, controller reduction has an important drawback: to give the best results, it requires specific weightings that are not completely defined by the objective of maintening the closed-loop transfer function, as it is the case in closed-loop model reduction. The weightings have to be chosen such that the reduction criterion matches as much as possible the design one, and this requirement might be difficult

Order	Method
MIN	CL controller reduction
↑	CL model reduction
	OL controller reduction
↑↑	CL model identification
MAX	OL model reduction

Table 2: Classification of the methods w.r.t. the order reachable with a pre-specified level of performance

to fulfill in some cases . One might also argue that balanced truncation is possibly not the most appropriate method to reduce a controller, since it rests on controllability and observability notions that are more adapted to design models than to controllers.

In order to improve the quality of the results obtained *via* model reduction, further study will investigate the possible advantage of a matching between the control design and the model reduction criteria (here, this matching was only considered in the controller reduction approach, while model reduction was simply based on a closed-loop transfer function preservation criterion). We shall also consider using the full-order controller (instead of the suboptimal PID controller) in the frequency weightings used in model reduction.

This study has also confirmed an important finding previously made in [1], which now clearly extends to controller reduction. This finding is closed-loop techniques are always preferable to open-loop ones, since they allow to achieve a specified level of performance with lower-order controllers; alternatively, for a given order, controllers obtained *via* open-loop techniques generally perform worse than those obtained *via* closed-loop methods.

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