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Model-free subspace-based LQG-design

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Abstract

When only input/output data of an unknown system are available, the classical way to design a linear quadratic Gaussian controller for that system mainly consists of three separate parts. First a system identification step is performed to find the system parameters. With these parameters a Kalman filter is designed to find an estimate of the state of the system. Finally, this state is then used in an LQ-controller. In the literature these three steps are hardly ever considered as one joint problem. Based on techniques from the field of subspace system identification the present paper gives a new, much more direct method to calculate a finite-horizon LQG-controller. The three steps of the LQG-controller design, i.e. system identification, Kalman filter and LQ-control design are replaced by a QR- and a SV-decomposition. The equivalence between the new subspace-based approach and the classical approach is proven.

1 Introduction

A lot of the modern control methods, such as LQG control, use state space models to design a controller. When such a model is not available, the LQG-design first requires a system identification from the available input/output data. In addition, once a state space model is available, the state of the system is estimated with a Kalman filter. Finally, an LQ-controller has to be designed to feed back the estimated state. In this paper we show that, based on techniques from the field of subspace identification, one can bypass these three steps and find an LQG-controller directly from input/output data of the unknown system.

We will be interested in the LQG-controller design for linear time-invariant systems that can be described by the following state space equations:

$$x_{k+1} = Ax_k + Bu_k + Ke_k, \quad (1)$$
$$y_k = Cx_k + Du_k + e_k \quad (2)$$

where the input $u_k \in \mathbb{R}^m$, the output $y_k \in \mathbb{R}^l$, the state $x_k \in \mathbb{R}^n$ and the stationary, ergodic, white noise $e_k \in \mathbb{R}^l$ with the following covariance matrix:

$$\mathbf{E}[e_p e_q^T] = S\delta_{pq}.$$

The available input/output data set is supposed to be measured in open-loop. The main problem we are interested in is the following:

Model-free LQG-control problem:

Given *a set of measurements of the inputs* u_k *and the outputs* y_k, $k \leq 0$, *of the unknown system (1)-(2),* **find** *the first input* u_1 *of the sequence* $u_f = (u_1, \ldots u_N)$ *that minimizes the following quadratic cost function* J *over the horizon* N*:*

$$J = \sum_{k=1}^{N} \widehat{y}_k^T Q_k \widehat{y}_k + u_k^T R_k u_k \quad (3)$$

where $\widehat{y}_k$ *is the k-step-ahead optimal predicted output given past inputs and outputs and future inputs up to time* k*. The matrices* $Q_k \in \mathbb{R}^{l\times l}$ *and* $R_k \in \mathbb{R}^{m\times m}$ *are user-defined non-negative definite weightings of the outputs and the inputs.*

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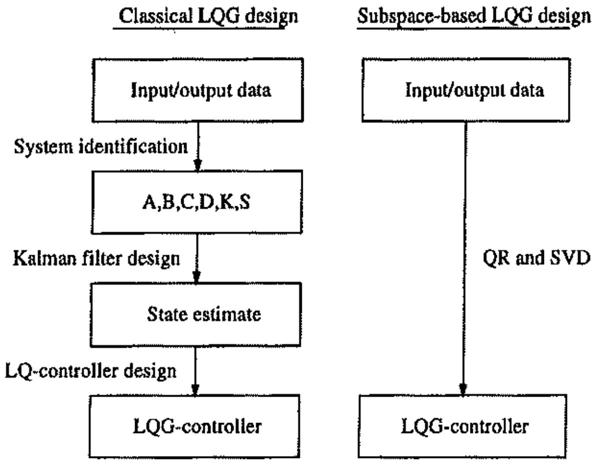

Figure 1: In the classical LQG-framework, first a system identification step is performed to find the state space matrices A, B, C, D, K and S. In a second step, these matrices are used to design a Kalman filter which gives an estimate of the state. Finally, this state estimate is used in a linear quadratic controller. The main result of the paper is that these three steps can be short-circuited and replaced by a QR and a SVD of matrices directly constructed out of input and output data of the system.

We will call the *forward horizon* (N) the number of time steps over which the system output is predicted and the control input is computed. The *backward horizon* (M) is the number of past input/output data points that are used to predict the outputs over the forward horizon.

In the literature the above model-free LQG-problem is hardly ever considered as one identification/control problem. Most authors concentrate on either the identification of the unknown system given a set of input and output measurements or on the LQG-design given the system parameters A, B, C, D, K and S. In the present paper a new method is derived to design LQG-controllers. The method is much more direct than the existing techniques in the sense that the three typical steps in calculating an LQG-controller, i.e. system identification, Kalman filter design and LQ-controller design, are replaced by numerically fast and robust operations of linear algebra, i.e. QR and SV decomposition. The model parameters A, B, C, D, K and S are never calculated in the present approach. This idea is represented graphically in Figure 1.

The results in this paper ground on results from the field of subspace system identification [1], [2], [3]. One of the largely underestimated features that make subspace identification so different from other system identification methods is the fact that *first* a Kalman filter estimate of the states can be found even *before* the system parameters are determined [3]. It is clear that this interesting feature could be exploited for control purposes. Up to now no research has been done on this specific topic. In [4] a method was proposed for the joint design and control by the way of subspace identification. The link with the Kalman filter states is not made however. The work of Bitmead et al. [5] was very inspiring to the present research. But even in their adaptive framework, the system parameters are identified explicitly. The idea of computing an optimal LQG-controller directly from data, without use of a model, has been developed by Hjalmarsson and collaborators (see [6]) using a technique called Iterative Feedback Tuning (IFT) which is also entirely different from the subspace-based technique developed here. The result presented here is an extension of [7] where it was shown that, based on techniques from the field of subspace identification, it is possible to design a predictive controller directly from the data, i.e. without any knowledge of the system.

Since the results presented here are inspired by subspace identification we start the paper by giving a short overview of the basics of subspace identification in Section 2. In Section 3, the classical way of solving the LQG-problem (3) is recalled after which the new subspace-based approach of [7] is presented. The main results of the paper can be found in Section 4 where it is shown that both approaches are equivalent.

2 Brief overview of linear subspace system identification

The problem treated in linear subspace identification is the following:

> **Given** *measurements of the inputs u_k and the outputs y_k of the unknown system (1)-(2)* **find** *an estimate of the system matrices A, B, C, D and the noise related matrices S and K.*

The starting point of all subspace identification algorithms is the construction of block-Hankel matrices containing the input/output data of the system:

$$U_p = \begin{pmatrix} u_0 & u_1 & \dots & u_{j-1} \\ u_1 & u_2 & \dots & u_j \\ \dots & \dots & \dots & \dots \\ u_{M-1} & u_M & \dots & u_{M+j-2} \end{pmatrix}, \quad (4)$$

$$U_f = \begin{pmatrix} u_M & u_{M+1} & \dots & u_{M+j-1} \\ u_{M+1} & u_{M+2} & \dots & u_{M+j} \\ \dots & \dots & \dots & \dots \\ u_{M+N-1} & u_{M+N} & \dots & u_{M+N+j-2} \end{pmatrix} \quad (5)$$

where the indices p and f stand for past and future. In a similar way the block-Hankel matrices Y_p and Y_f can be defined for the outputs y_k. It should be noted

4.2 White inputs

Let us now consider the case where the inputs of the available data are white and zero-mean. This is a quite common situation since white signals are often used as inputs for system identification purposes. In this particular case the equivalence between the classical and the subspace-based LQG-controller (14) can also be made. The proof of the following theorem is very similar to the proof of Theorem 1 and can be found in [10].

Theorem 2

If *the available input data* u_k *are white and zero-mean,* **then** *the subspace-based LQG-controller (14) and the classical LQG-controller equations (8)-(13) with initial conditions* $\Sigma_{-N+1} = 0$, $\widehat{x}_{-N+1} = 0$ *and* $P_0 = 0$ *are equivalent.*

5 Conclusions

In the present paper we have studied a recent algorithm for the calculation of a predictive controller for an unknown plant based on input/output data only, i.e. without any knowledge of the plant [7]. Even though the derivation is based on expressions from the field of subspace system identification theory, the algorithm bypasses the identification step altogether. One could say that the method transfers the properties of subspace identification from the realm of identification to the realm of control design. The main result of the present paper is that, in the case the horizon is infinite or in the case the available input data are white zero-mean, the new subspace-based controller is equivalent to the classical LQG-controller equations.

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