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Identification with the Youla parameterization in identification for control.[1]

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Abstract

This paper highlights the role of the dual Youla parameterization in identification for control, when the Youla parameters are used for control design. The well known Hansen scheme is first modified in order to estimate the Youla parameters separately. The modification naturally arises when the ultimate objective of the identification is to use these estimates directly for control design.

1 Introduction

The typical context in identification for control is where it is desired to estimate a plant model, with the view of designing a new controller that achieves better performance on the true system while providing robustness guarantees: see e.g. [1]. The plant model is estimated in closed-loop, i.e. with data collected on the closed-loop formed by the feedback connection of the unknown system and some stabilizing controller $\hat{C}$. Most closed-loop identification techniques have in common the ability to estimate approximate models of the open-loop system on the basis of closed-loop data. The identification in the Youla framework is one of these, known in the literature as the Hansen scheme [2]. It is based on the parameterization of all plants stabilized by the controller $\hat{C}$. The true system is parameterized as a controller-based perturbation of some plant model, defined by a Youla parameter. The identification of this Youla parameter is an open-loop identification problem. The new identified model is then obtained as a Youla-parameter correction of the previous model. Typically, in iterative identification for control schemes, the open-loop models for both input-output and noise dynamics are reconstructed from the closed-loop estimated parameters, followed by the design of a new controller on the basis of these reconstructed open-loop models: see e.g. [3]. In this paper an alternative methodology is used; we parameterize the controller such that its design is directly a function of the estimated parameters of the closed-loop model in lieu of those of the reconstructed open-loop model.

A quick description of our methodology is as follows. We modify the Hansen scheme such that it allows to estimate separately the dual Youla parameters: the input-output perturbation of the plant model as well as the disturbance dynamics are estimated separately in an open loop framework. Finally, we relate these estimates to a controller perturbation. The controller perturbation is obtained by parameterizing the set of all controllers that stabilize the new plant model (Youla parameters). We shall see that it is the knowledge of the dual Youla parameters that allows us to design an LQG controller with the optimization performed directly on the Youla parameters.

The initial setup to be considered is depicted in Figure 1. In this configuration, the external signals are $r(t)$, the known reference signal, and $e(t)$, a white noise of zero mean and variance σ^2. These signals are independent. $u(t)$ is the control input signal and $y(t)$ is the observed output signal. The vector notation $S : [P\ H]$ is used to describe the true system, which maps the inputs $u(t)$ and $e(t)$ into the output $y(t)$: see Figure 1. $P(q)$ and $H(q)$ are proper scalar rational function operators and $H(q)$ is stable and minimum phase. Here q^{-1} is the backward shift operator ($q^{-1}u(t) = u(t-1)$). Finally $\hat{C} : [C_r\ C_y]$ denotes a two-degree of freedom controller, designed on the basis of a known plant model $\hat{S} : [\hat{P}\ \hat{H}]$, and this controller internally stabilizes the system S.

2 The Youla Parameterization

In this section we briefly recall the Youla parameterization of all controllers that stabilize a given plant model, and the dual Youla parameterization of all systems that are stabilized by a given controller.

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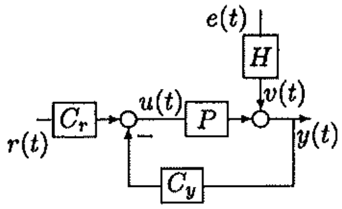

Figure 1: The internally stable feedback loop formed by true system S and the controller $\hat{C}$

Theorem 1 *[4], [2] Let $\hat{S}$ and $\hat{C}$ have coprime factorizations $\hat{S}$: $[N_{\hat{p}} \quad U_{\hat{p}}]D_{\hat{p}}^{-1}$, $\hat{C}$: $[T_{\hat{c}} \quad N_{\hat{c}}]D_{\hat{c}}^{-1}$, where $N_{\hat{p}}$, $D_{\hat{p}}$, $U_{\hat{p}}$, $N_{\hat{c}}$, $D_{\hat{c}}$, $T_{\hat{c}}$ belong to $\mathcal{R}H_\infty$, the set of all stable proper rational transfer functions. Assume that the following Bezout equation holds*

$$N_{\hat{c}}N_{\hat{p}} + D_{\hat{c}}D_{\hat{p}} = 1. \quad (1)$$

This equation expresses both that the factors are coprime and that the feedback loop formed by the plant model and the controller is internally stable. Then, the set of all LTI two degree of freedom controllers that stabilize $\hat{S}$ is given by

$$\mathbf{C}_{\hat{S}} = \{\mathcal{C}_{\hat{S};\hat{C}}(T,Q) : [T \quad (N_{\hat{c}} + D_{\hat{p}}Q)](D_{\hat{c}} - N_{\hat{p}}Q)^{-1} : T, Q \in \mathcal{R}H_\infty \, with \, D_{\hat{c}} - N_{\hat{p}}Q \neq 0\}. \quad (2)$$

Symmetrically, the set of all LTI plants stabilized by the controller $\hat{C}$ is given by

$$\mathbf{S}_{\hat{C}} = \{\mathcal{S}_{\hat{S};\hat{C}}(V,U) : [(N_{\hat{p}} + D_{\hat{c}}V) \; U](D_{\hat{p}} - N_{\hat{c}}V)^{-1} : U, V \in \mathcal{R}H_\infty \, with \, D_{\hat{p}} - N_{\hat{c}}V \neq 0\}, \quad (3)$$

Without loss of generality U can be taken as a unit, i.e. its inverse also belongs to $\in \mathcal{R}H_\infty$.

3 The Hansen Scheme

We consider that the plant S is unknown and that the designer has selected some model $\hat{S}$, typically much simpler than the true system, and has designed a controller $\hat{C}$ on the basis of that model that stabilizes S.

Invoking Theorem 1, there exist $U, V \in \mathcal{R}H_\infty$ with U a unit such that the true plant S can be expressed as

$$\mathcal{S}_{\hat{S};\hat{C}}(V,U) : [(N_{\hat{p}} + D_{\hat{c}}V) \quad U](D_{\hat{p}} - N_{\hat{c}}V)^{-1}. \quad (4)$$

The dual Youla parameters, U and V, are the values that correspond to the true plant and represent the plant perturbation with respect to $\hat{S}$. The pleasing aspect is that the identification of these parameters can be achieved in a standard open loop framework (see e.g. [2]). We present an alternative method for the estimation of these parameters, also in an open loop framework. Consider the feedback system represented in Figure 2, by replacing in Figure 1 both the controller $\hat{C}$ and the plant S by their factorizations in Theorem 1. The equations that describe this closed-loop system are given by

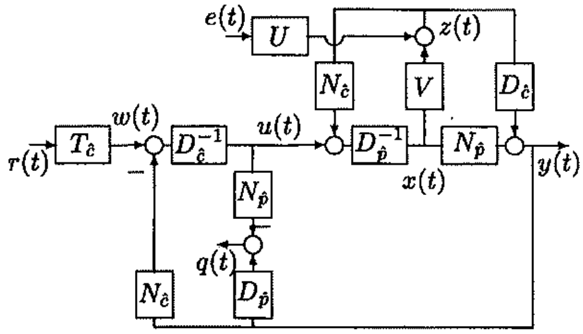

Figure 2: The internally stable feedback loop formed by the true plant $\mathcal{S}_{\hat{S};\hat{C}}(V,U)$ and the controller $\hat{C}$

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} (N_{\hat{p}} + D_{\hat{c}}V)T_{\hat{c}} & D_{\hat{c}}U \\ (D_{\hat{p}} - N_{\hat{c}}V)T_{\hat{c}} & -N_{\hat{c}}U \end{bmatrix} \begin{bmatrix} r(t) \\ e(t) \end{bmatrix}. \quad (5)$$

Let us introduce the auxiliary signal $q(t)$, defined in Figure 2. From closed loop equations (5), with the Bezout equation (1) holding, the transfer functions from signals $w(t) = T_{\hat{c}}r(t)$ and $e(t)$ to $q(t)$ are obtained as follows:

$$\begin{aligned} q(t) &= D_{\hat{p}}y(t) - N_{\hat{p}}u(t) \\ &= D_{\hat{p}}[(N_{\hat{p}} + D_{\hat{c}}V)T_{\hat{c}}r(t) + D_{\hat{c}}Ue_t] \\ &\quad - N_{\hat{p}}[(D_{\hat{p}} - N_{\hat{c}}V)T_{\hat{c}}r(t) - N_{\hat{c}}Ue_t] \\ &= [N_{\hat{c}}N_{\hat{p}} + D_{\hat{c}}D_{\hat{p}}]VT_{\hat{c}}r(t)) + [N_{\hat{c}}N_{\hat{p}} + D_{\hat{c}}D_{\hat{p}}]Ue_t \\ &= Vw(t) + Ue(t). \end{aligned} \quad (6)$$

Since $w(t)$ is known, and since $w(t)$ and $e(t)$ are uncorrelated, the identification of V and U from the signals $w(t)$ and $q(t)$ appears as an open loop identification problem. Classical open loop techniques, as e.g. described in [5], can be used.

This result is somehow dual to that obtained in the Hansen scheme, where the identification of the parameters V and U is obtained from auxiliary signals $z(t)$ and $x(t)$ in Figure 2, with $z(t) = Vx(t) + Ue(t)$. The signals $z(t)$ and $x(t)$ can be constructed from the known signals u, y and r. The difference with the *Hansen* scheme is that here we generate the signals from the controller rather from the plant.

The second step in the Hansen scheme is to reconstruct the open loop model. Given estimates $\hat{U}$, $\hat{V}$, of U and V, respectively, we can compute an estimate of the plant $\mathcal{S}_{\hat{S};\hat{C}}(V,U)$ as

$$\mathcal{S}_{\hat{S};\hat{C}}(\hat{V},\hat{U}) : [(N_{\hat{p}} + D_{\hat{c}}\hat{V}) \quad \hat{U}](D_{\hat{p}} - N_{\hat{c}}\hat{V})^{-1}. \quad (7)$$

4 Estimation and Validation

In prediction error identification one typically considers a general model structure of the form

$$q(t,\theta) = \hat{V}(q,\theta)w(t) + \hat{U}(q,\theta)e(t), \tag{8}$$

parameterized in terms of a parameter vector θ. The parameter estimates are obtained by the minimization of a least-squares criterion function of the form,

$$J_{id}^{N}(\theta) = \frac{1}{N}\sum_{t=1}^{N}\epsilon(t,\theta)^2, \tag{9}$$

with a set of data $Z^N = \{q, w\}$ collected on the closed loop system via (6). Here $\epsilon(t,\theta)$ are the prediction errors, which for the model (8) can be expressed as

$$\epsilon(t,\theta) = L(q,\theta)\{q(t) - \hat{V}(q,\theta)w(t)\}, \tag{10}$$

where $L(q,\theta)$ is any stable filter. Since the two estimated Youla parameters belong to $\mathcal{R}H_\infty$, there is no constraint on the model structure in (8): we are able to apply the standard (open loop) prediction error analysis. Moreover, the fact that $w(t)$ is a user-defined reference signal allows us to estimate the Youla parameters U and V separately. Indeed, consider the following two identification experiment setups with their corresponding prediction errors

- Let $L(q,\theta) = \hat{U}(q,\theta)^{-1}$ and $w(t) = 0 \ \forall t = 1,...,N$. Then (10) becomes

  $$\epsilon_u(t,\theta) = \hat{U}(q,\theta)^{-1}q(t),$$

  where $q(t)$ is generated from the signal $y(t)$, $u(t)$ collected on the closed-loop system with a zero reference signal: $r(t) = 0, t = 1,...,N$.

- Let $L(q,\theta) = L(q)$ be any stable filter and $w(t)$ any sequence of input data. Then (10) becomes

  $$\epsilon_v(t,\theta) = L(q)\{q(t) - \hat{V}(q,\theta)w(t)\}.$$

The minimization of the criterion (9) with the prediction errors $\epsilon_u(t,\theta)$ and a set of data $Z_u^N = \{q, w = 0\}$ provides an estimate $\hat{U} = U(q,\theta^N)$ of U, while the minimization of the criterion (9) with the prediction errors $\epsilon_v(t,\theta)$ and a set of data $Z_u^N = \{q, w\}$ provides an estimate $\hat{V} = V(q,\theta^N)$ of V.

Note the simplicity with which one can generate the informative data about the noise model U, which is estimated in closed loop. Observe that U defines the true noise transfer function H, but we shall see that it is the parameter U and not H that is needed for the derivation of the controller. A particular choice of the filter L can be selected such that the identification criterion

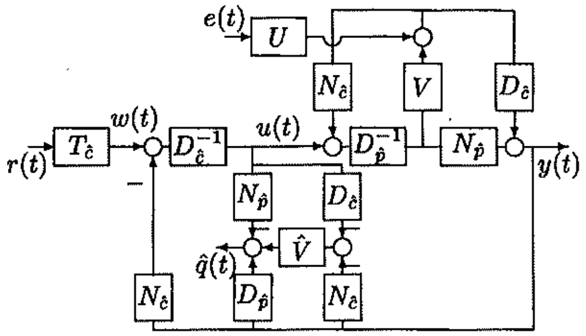

Figure 3: The Modeling error experiment setup

(9) matches asymptotically a performance degradation criterion: see e.g. [6]. This is the main heuristic motivation for performing closed loop identification. With the two estimates $\hat{U}$ and $\hat{V}$, a new plant model is constructed according to the parameterization in (7). Let us introduce the auxiliary signal $\hat{q}(t)$, as defined in Figure 3. Compare Figures 2 and 3; the auxiliary signal is now constructed from the signals $y(t)$ and $u(t)$ using the new model $(N_{\hat{p}} + D_{\hat{c}}\hat{V})(D_{\hat{p}} - N_{\hat{c}}\hat{V})^{-1}$ for P instead of the initial nominal model $N_{\hat{p}}D_{\hat{p}}^{-1}$. The transfer functions from the same signals $w(t)$ and $e(t)$ to $\hat{q}(t)$ are obtained as follows:

$$\begin{aligned}\hat{q}(t) &= [D_{\hat{p}} - N_{\hat{c}}\hat{V}]y(t) - [N_{\hat{p}} + D_{\hat{c}}\hat{V}]u_t \\ &= [D_{\hat{p}}y(t) - N_{\hat{p}}u_t] - \hat{V}[N_{\hat{c}}y_t + D_{\hat{c}}u_t] \\ &= Vw(t) + Ue(t) - \hat{V}w(t) \\ &= \{V - \hat{V}\}w(t) + Ue(t).\end{aligned} \tag{11}$$

The signal $\hat{q}(t)$ contains the modeling error represented by $\{V - \hat{V}\}$ and the noise term $Ue(t)$. The model error $\{V - \hat{V}\}$ can be estimated with e.g. a FIR model or a non-parametric model to reveal whether specific dynamics have been neglected in the model. As we shall see in the next section, it will be used in the control design for guaranteeing stability.

5 Plant and Controller perturbation

In this section, we consider controller perturbation, and we use design schemes that relate the estimated parameters to a controller perturbation: see [7] and [8]. Consider the parameterization of the true system: $\mathcal{S}_{\hat{\mathcal{S}};\hat{\mathcal{C}}}(V,U)$. According to Theorem 1, the set of all LTI controllers that stabilize the true system is given by

$$\begin{aligned}\mathbf{C}_{\mathcal{S}} = \{&\mathcal{C}_{\mathcal{S};\hat{\mathcal{C}}}(T,Q) : [T \quad (N_{\hat{c}} + D_pQ)](D_{\hat{c}} - N_pQ)^{-1} : \\ &T, Q \in \mathcal{R}H_\infty \, with \, D_{\hat{c}} - N_pQ \neq 0\},\end{aligned} \tag{12}$$

where $N_p = N_{\hat{p}} + D_{\hat{c}}V$ and $D_p = D_{\hat{p}} - N_{\hat{c}}V$. The closed loop system composed of the plant model $\mathcal{S}_{\hat{\mathcal{S}};\hat{\mathcal{C}}}(V,U)$ and any member, $\mathcal{C}_{\mathcal{S};\hat{\mathcal{C}}}(T,Q)$, of the set of stabilizing

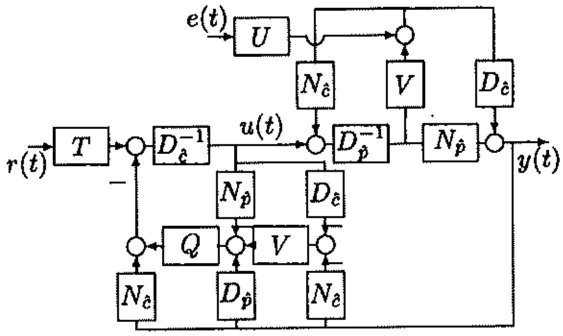

Figure 4: The set of all stable closed-loop systems for the true plant.

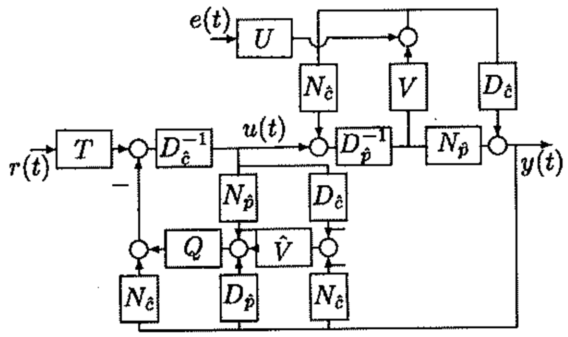

Figure 5: The true plant in feedback with the set of model-based controllers.

controllers is represented in Figure 4. The equations that describe this closed loop system are given by

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} N_pT & (D_{\hat{c}} - N_pQ)U \\ D_pT & -(N_{\hat{c}} + D_pQ)U \end{bmatrix} \begin{bmatrix} r(t) \\ e(t) \end{bmatrix}. \quad (13)$$

Here, the Youla parameter T influences only the tracking part, while Q influences only the disturbance rejection part. The controller belonging to the controller set $\mathbf{C}_S$ of stabilizing controllers for the true plant cannot be implemented because the true V is unknown. The best one can do is to replace V by an estimate $\hat{V}$, leading to the model-based control design of Figure 5. The closed-loop system is no longer guaranteed internally stable for any parameter Q in $\mathcal{RH}_\infty$; it is internally stable if and only if $V - \hat{V}$ and Q together define a stable unity negative feedback loop: see [9].

In [8] the Youla framework was used to design LQG controllers with the optimization performed directly on the Youla parameters T and Q. Consider the following LQG control criterion

$$J_{LQG} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \{y(t+d) - r(t)\}^2 + \lambda u(t)^2 \quad (14)$$

where d is the delay and λ the weighting on the control effort. It is assumed that the true plant $\mathcal{S}$ is known. Consider the closed-loop system composed of the true plant and any member of the set of all stabilizing controllers in (12). As can be seen in [8], using Parseval's Theorem, we get an expression for the disturbance rejection contribution of the LQG index as,

$$J_{dr} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mid D_{\hat{c}} - N_pQ \mid^2 + \lambda \mid N_{\hat{c}} + D_pQ \mid^2 \phi_u, \quad (15)$$

where ϕ_u is the spectrum of the noise model $Ue(t)$. A similar expression is given in [8] for the tracking cost. The stable minimizing Q can be computed analytically, by means of spectral factorizations and projections, as follows:

$$Q = -\mathcal{A}^{-1}[\mathcal{A}^{-*}\mathcal{B}]_{st}, \quad (16)$$

where

$$\mathcal{A}\mathcal{A}^* = [\mid N_p \mid^2 + \lambda \mid D_p \mid^2] \mid U \mid^2 \sigma^2$$
$$\mathcal{B} = [\lambda D_p^* N_{\hat{c}} - N_p^* D_{\hat{c}}] \mid U \mid^2 \sigma^2.$$

Here $\mathcal{A}$ is the minimum phase, stable spectral factor of relative degree zero. The operator * denotes complex conjugate, and the operator $[.]_{st}$ denotes the stable part of the operand.

6 Example

We apply the procedure to a simulation example, using an LQG control criterion. We show that the procedure allows one to derive a controller whose optimal performance cost is close to that obtained with the true plant. The true plant $\mathcal{S}$ has an ARMAX model structure $\mathcal{S} = A^{-1}[B\ C]$, with $\sigma^2 = 0.01$. It has poles at $q = -0.0030 \pm j0.9119$, $0.8082 \pm j0.5605$ and $0.9837 \pm j0.1491$, zeros at $q = 1.1865 \pm j0.7993$, $0.4624 \pm j0.2838$ and 0.4307. The roots of the observer C are at $q = 0.95, 0.80, 0.80, 0.60, 0.60, 0.30$, and the DC gain is 0.4897. The sampling period $T_s = 0.05$ second. The design parameter λ of the LQG control cost (15) is set to 0.1. We first design a full order optimal LQG controller for the true plant and compute the optimal performance cost $J_{dr} = 1.9695$. Now we identify a model for the plant in order to design a model-based LQG controller. We first seek a model that produces a stabilizing controller for the true plant. We perform an open-loop identification experiment with a reduced order ARMAX model structure of order 2 in order to capture the first resonant frequency of the true plant. The input signal is white noise with power spectrum $\phi_u = 10$, and 2000 samples are used. The nominal estimated plant model $\hat{\mathcal{S}} = \hat{A}^{-1}[\hat{B}\ \hat{C}]$, where $\hat{B}(q^{-1}) = 0.0291 * (1 - 0.5395q^{-1})$, $\hat{A}(q^{-1}) = 1 - 1.967q^{-1} + 0.9898q^{-2}$ and $\hat{C} = (1 - 0.8q^{-1})^2$. The

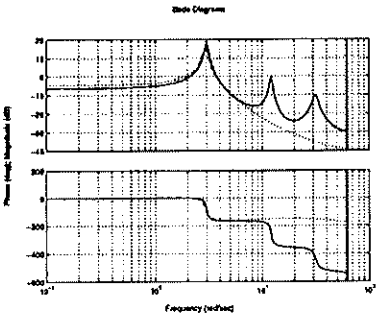

Figure 6: Bode diagrams of the transfer functions P (solid), $\hat{P}$ (dotted) and $\hat{P}_1$ (dashed, but indistinguishable from P).

designed LQG controller, with the same design parameter as the full order controller, stabilizes the true plant. It serves as a nominal stabilizing controller $\hat{C}$. We perform the coprime factorization such that (1) is satisfied and with the following constraint $\mid N_{\hat{c}} \mid^2 + \lambda \mid D_{\hat{c}} \mid^2 = 1$. This constraint is used for performance considerations: see [10] for a complete discussion. Now we estimate

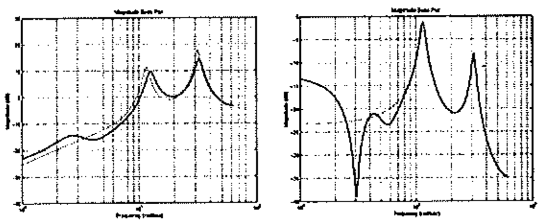

Figure 7: Amplitude Bode plots of the true dual Youla parameter U (left,solid) and the 6^{th} order estimate $\hat{U}$ (left,dotted). Amplitude Bode plots of the true dual Youla parameter V (right, solid) and the 4^{th} order estimate $\hat{V}$ (right, dotted).

a Youla parameter correction for the first model using the closed-loop identification technique described in Section 3. The dual Youla parameter is estimated using a zero reference signal. A 6^{th} order estimate is selected. The amplitude Bode plots of the dual Youla parameter U and its estimate $\hat{U}$ are presented in Figure 7. The true dual Youla parameter V is of order 12, see its plot in Figure 7. It contains the two resonances that are missing in the model $\hat{P}$. We select a 4^{th} order model structure for the estimate $\hat{V}$ in order to capture the two missing resonances. The excitation signal $r(t)$ used in the identification experiment is taken to be white noise with power spectrum $\phi_r = 10$; 2000 samples are computed. The amplitude Bode plots of the Youla parameter V and its estimate $\hat{V}$ are presented in Figure 7. From these estimated Youla parameters, a plant model $\hat{P}_1$ is derived as in (7). As Figure 6 shows, the amplitude Bode plot of the new plant model is almost indistinguishable from that of the true plant. An LQG controller is computed based on the estimated Youla parameters $\hat{V}$ and $\hat{U}$ as in section 5. This controller stabilizes the true plant. The designed performance cost is $J_{des} = 1.9808$, i.e. it is the LQG cost calculated from the closed loop system formed by the model and the LQG controller, while the achieved performance cost is $J_{ach} = 1.7806$, i.e. it is the LQG cost calculated from the closed loop system formed by the true plant and the controller. The achieved performance cost is close to the optimal performance cost.

7 Conclusion

In identification for control, the ultimate objective is to use the estimated model for control design. We have shown how the knowledge of the two dual-Youla parameters U and V allows us to directly design the LQG controller with the optimization performed directly on the Youla parameters T and Q.

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