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# Model Validation in Closed-Loop†


Michel Gevers(1), Benoît Codrons(1) and Franky De Bruyne(2)

(1) Centre for Systems Engineering and Applied Mechanics (CESAME),
Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium
Tel: +32 10 472590, Fax: +32 10 472180, Email: {Codrons, Gevers}@csam.ucl.ac.be
(2) Research School of Information Sciences and Engineering,
Australian National University, Canberra, ACT 0200, Australia
Tel: +61 2 62798674, Fax: +61 2 62798688, Email: Franky.DeBruyne@syseng.anu.edu.au



## Abstract

In [8] a new model validation method was proposed that is based on the estimation of an unbiased model of the *model error* which relates the inputs to the simulation errors. In this paper, we extend this methodology to closed-loop validation, i.e. the validation of an open-loop model on the basis of closed-loop validation data. We show that, perhaps surprisingly, the same model may fail to be validated with open-loop data, while it is validated by data collected in closed-loop. In addition, we show that the uncertainty sets generated by models validated in closed-loop are much better tuned towards control design, i.e. the controllers that stabilize all members of such a set are less conservative than those that stabilize all members of a set validated in open-loop. Two examples illustrate the comparison between both methods.


## 1 Introduction

The question of '*what is a good model for control design*' has been addressed for years, mainly from the identification point of view (see e.g. [1, 3, 4, 5, 2]). A recurrent message in most of the publications on that topic is that the identification should be performed in closed-loop, and that the controller should be adapted cautiously, leading to some iterative scheme.

Recently however, the fact that the same results could be obtained by open-loop identification with appropriate frequency weightings has been advocated in [7, 8]. An important part of these publications is devoted to the question of model validation. The main idea is that when a model $\hat{P}$ of a system $P_0$ is given, one can use the simulation errors to identify a model for the modeling error, that is for the difference between $P_0$ and $\hat{P}$. This *model error model* is obtained with an uncertainty region that can be used either

(i) to decide whether $\hat{P}$ is validated, or

(ii) as information that one should use when doing control design.

Although this validation method is very appealing, we believe that it is not really adapted to control design as it is, because it does not take the closed-loop aspects into account. More precisely, even if the model is obtained in a way that is relevant for control design, this open-loop validation procedure leads to uncertainty regions that are not tuned for control design, leading to very conservative controllers. Therefore, the validation criterion should be modified to reflect the intended use of the model.

In this paper, a new closed-loop oriented validation method, adapted from the one presented in [8], is proposed, and its relevance for control design is established and illustrated in a numerical example. We believe that our validation scheme will prove to be useful for the control-relevant validation of models in several iterative identification and control schemes; we refer the reader to [3] for further details on such schemes.

The outline of the paper is as follows. In Section 2, the 'standard' validation method of [8] is briefly reviewed. Section 3 shows how it can be adapted to reflect the closed-loop aspects involved in control design. Some engineering aspects are tackled in Section 4, and then


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the two methods are compared on the basis of a numerical example in Sections 5 and 6. Finally some conclusions are drawn in Section 7.

## 2 Standard model validation

In this Section, we review the model validation method presented by Ljung in [8] and we explain why it is not tuned for control design.

Let us assume that the true system is the Single-Input Single-Output (SISO) linear time-invariant system described by

$$\mathcal{S} : \left\{ \begin{array}{rcl} y(t) & = & P_0(q)\, u(t) + v(t), \\ v(t) & = & H_0(q)\, e(t), \end{array} \right. \tag{2.1}$$

where $P_0(q)$ and $H_0(q)$ are unknown rational transfer functions, with $P_0(q)$ strictly proper and $H_0(q)$ stable and stably invertible (the argument $q$ is the shift operator; we shall often omit it to simplify the notations). The restriction to scalar plants is inessential, but notationally convenient. Here $u(t)$ is the control input signal, $y(t)$ is the measured output signal, $e(t)$ is a zero mean white noise signal with variance $\sigma^2$ and $v(t)$ is a process disturbance signal.

We assume here that one has obtained a model

$$\mathcal{M} : y(t) = \hat{P}(q)\, u(t) + \hat{H}(q)\, e(t) \tag{2.2}$$

of the system (2.1) which has to be validated in a way that is relevant for control design, using a set of validation data $Z^N = \{y(1), u(1), ..., y(N), u(N)\}$ collected on the system (2.1), regardless of how this model was obtained.

Most of the model validation tests are based on some statistics over the residuals (which are the difference between the simulated and the measured outputs):

$$\begin{array}{rcl} \varepsilon(t) & = & L(q)\,(y(t) - \hat{y}(t)) \\ & = & L(q)\left(y(t) - \hat{P}(q)\, u(t)\right). \end{array} \tag{2.3}$$

Here $L(q)$ is any frequency weighting prefilter. To facilitate the notations, we shall assume that $L(q) = 1$. Note, however, that a common choice of prefilter is $L(q) = \hat{H}^{-1}(q)$, which makes the residuals $\varepsilon(t)$ equal to the model's prediction errors.

Two sources can be considered for the model residuals $\varepsilon(t)$: one that originates from the input $u(t)$ and which would be zero if the model $\hat{P}$ was a perfect representation of the plant $P_0$, and another that originates from the process noise. According to this separation, we can write

$$\varepsilon(t) = \delta P(q)\, u(t) + v(t) \tag{2.4}$$

where

$$\delta P(q) = P_0(q) - \hat{P}(q) \tag{2.5}$$

is the *model error*. The consideration of two distinct sources for the model residuals is only relevant if the disturbance $v$ has nothing to do with the input $u$. This amounts to say that, in the probabilistic framework we consider here, $u$ and $v$ should be mutually independent sequences of random variables [9]. This requires that the validation data be not collected in closed-loop, which will often be the case if the intention is to use the model for control design. Although this is an important drawback of the standard validation method, we shall see in Section 4 that a proper choice of the prefilter $L$ can help solving that problem.

The validation procedure proposed in [8, 9] consists in identifying an unbiased model $\tilde{P}$ for $\delta P$ using a set of data $\{\varepsilon, u\}$. This *model error model* $\tilde{P}$ is then given within a confidence interval (at a certain level of probability, typically 95% or 99%)

$$\mathcal{I} = \left[\tilde{P} - \Delta\tilde{P},\ \ \tilde{P} + \Delta\tilde{P}\right]. \tag{2.6}$$

Here $\Delta\tilde{P}$ is a multiple (say 2 or 3 times, depending on the probability level considered) of the standard deviation of $\tilde{P}$. The model $\hat{P}$ is said to be *validated* with this uncertainty region $\mathcal{I}$ if $\mathcal{I}$ contains 0 at all frequencies. Indeed, since $\mathcal{I}$ also contains $\delta P$ (with the same probability), this means that the bias error between $P_0$ and $\hat{P}$ is dominated by the variance and that $P_0$ and $\hat{P}$ belong to the uncertainty set $\hat{P} + \mathcal{I}$ at the preset probability level.

According to [8], the model $\hat{P}$ can be used for control design *even if it is falsified*, provided that the model error model $\tilde{P}$ and its confidence interval $\mathcal{I}$ be taken into account. However, as shown in Section 6, such a procedure may lead to control designs that are far too conservative, in the sense that a good control design does not require a good knowledge of the system at all frequencies, but rather around the crossover frequency of $P_0 C_y$, where $C_y$ is the feedback part of the controller present in the loop. As already advocated in [4], a small uncertainty around that frequency makes it possible to cautiously increase the bandwidth of the controller, leading to iterative identification and control design methods.

Therefore, it appears that another way of validating a model should be considered, focusing on the closed-loop relevant dynamics and using closed-loop validation data.

## 3 Closed-loop model validation

In this Section, we present a new model validation method that is directly oriented towards control design.

The motivation for closed-loop validation follows from the fact that, as pointed out in [4], the model $\hat{P}$ should be evaluated by how well it mimics the behaviour of the actual system $P_0$ when both are connected with the same controller; ideally the 'to be designed' controller.

Consider once again the true system $P_0$ and a model $\hat{P}$, and suppose that the input signal is determined by a known two degree of freedom controller

$$\begin{aligned} \mathcal{C} : u(t) &= C(q)\,[r(t) \;\; y(t)]^T \\ &= C_r(q)\,r(t) - C_y(q)\,y(t) \end{aligned} \tag{3.1}$$

where $r$ is an external reference signal which is assumed to be quasi-stationary and uncorrelated with $v$. It is further assumed that the controller $C$ stabilizes $P_0$.

The true and the nominal closed-loop transfer functions are then, respectively,

$$T_0(q) = \frac{P_0(q)C_r(q)}{1+P_0(q)C_y(q)}, \tag{3.2}$$

$$\hat{T}(q) = \frac{\hat{P}(q)C_r(q)}{1+\hat{P}(q)C_y(q)}. \tag{3.3}$$

The following *closed-loop residuals* are considered[1]:

$$\begin{aligned} \varepsilon_{cl}(t) &= L(q)\,(y(t) - \hat{y}_{cl}(t)) \\ &= L(q)\left(y(t) - \hat{T}(q)\,r(t)\right), \end{aligned} \tag{3.4}$$

leading to the definition of the *closed-loop model error* $\delta T$:

$$\varepsilon_{cl}(t) = L(q)\,(\delta T(q)\,r(t) + S_0(q)\,v(t)) \tag{3.5}$$

where

$$S_0(q) = \frac{1}{1+P_0(q)C_y(q)}$$

is the closed-loop sensitivity function and

$$\delta T(q) = T_0(q) - \hat{T}(q) \tag{3.6}$$

describes the mismatch between the actual and nominal closed-loop transfer functions (3.2) and (3.3).

Once again, we shall assume in the sequel that the prefilter $L$ is 1. However, a possible choice would be $L(q) = \hat{H}^{-1}(q)\hat{S}^{-1}(q)$, where

$$\hat{S}(q) = \frac{1}{1+\hat{P}(q)C_y(q)},$$

[1] Note that the computation of these closed-loop residuals requires that $\hat{T}$ be stable. In other words, $\hat{P}$ must be stabilized by $C$, which is obviously a necessary condition for $\hat{P}$ to be closed-loop validated with respect to this controller.

which makes the closed-loop residuals equal to the closed-loop prediction errors.

We now apply the open-loop validation procedure described in Section 2 to the model $\hat{T}$ of the closed-loop transfer function $T_0$. Thus, an estimate $\tilde{T}$ of the closed-loop model error $\delta T$ is identified using a set of data $\{\varepsilon_{cl}, r\}$. $\hat{T}$ is then validated if the confidence interval around $\tilde{T}$,

$$\mathcal{J} = \left[\tilde{T} - \Delta\tilde{T}, \;\; \tilde{T} + \Delta\tilde{T}\right], \tag{3.7}$$

contains zero at all frequencies. Based on the derivations of Section 2, this means that if $\hat{T}$ is validated with an uncertainty set $\mathcal{J}$, then $T_0$ and $\hat{T}$ belong to the uncertainty set $\hat{T} + \mathcal{J}$. The important observation now is that, if $\hat{T}$ is validated with an uncertainty set $\mathcal{J}$, then $\hat{P}$ is validated with an uncertainty set $\mathcal{I}_{cl}$ that can be computed from $\mathcal{J}$. We now show how to compute $\mathcal{I}_{cl}$.

Remark that (3.2), (3.3), (2.5) and (3.6) give the following relation between $\delta T$ and $\delta P$:

$$\begin{aligned} \delta T &= \frac{P_0 C_r}{1+P_0 C_y} - \frac{\hat{P} C_r}{1+\hat{P} C_y} \\ &= \frac{(\hat{P}+\delta P)C_r}{1+(\hat{P}+\delta P)C_y} - \frac{\hat{P} C_r}{1+\hat{P} C_y}. \end{aligned} \tag{3.8}$$

It is easy to compute a new estimate $\tilde{P}_{cl}$ of $\delta P$ once an estimate $\tilde{T}$ of $\delta T$ has been obtained:

$$\hat{P} + \tilde{P}_{cl} = \frac{\hat{T}+\tilde{T}}{C_r - C_y(\hat{T}+\tilde{T})}. \tag{3.9}$$

$\tilde{P}_{cl}$ is a model error model of $\hat{P}$ obtained from closed-loop data. The derivation of a confidence interval $\mathcal{I}_{cl}$ around $\tilde{P}_{cl}$ from $\mathcal{J}$ around $\tilde{T}$ is straightforward, since the covariance of the closed-loop model error models are related to each other (first order approximation) by

$$\mathrm{cov}\left(\tilde{P}_{cl}\right) = \left[\frac{\partial \tilde{P}_{cl}}{\partial \tilde{T}}\right] \mathrm{cov}\left(\tilde{T}\right) \left[\frac{\partial \tilde{P}_{cl}}{\partial \tilde{T}}\right]^*. \tag{3.10}$$

More precisely, if $\tilde{T}$ is parametrized by a vector of parameters $\Theta = [\theta_1 \cdots \theta_n]^T$, the uncertainty around that model will often be given under the form of the covariance matrix of $\Theta$. If this uncertainty has to be represented by ellipses on the Nyquist plot of $\tilde{P}_{cl}$, the formula to apply is

$$\mathrm{cov}\left\{\Re\left(\tilde{P}_{cl}\right), \Im\left(\tilde{P}_{cl}\right)\right\} =$$

$$\begin{bmatrix} \frac{\partial \Re(\tilde{P}_{cl})}{\partial \theta_1} & \cdots & \frac{\partial \Re(\tilde{P}_{cl})}{\partial \theta_n} \\ \frac{\partial \Im(\tilde{P}_{cl})}{\partial \theta_1} & \cdots & \frac{\partial \Im(\tilde{P}_{cl})}{\partial \theta_n} \end{bmatrix} \mathrm{cov}(\Theta) \begin{bmatrix} \ddots & \\ & \ddots \end{bmatrix}^*.$$

It must be evaluated at each desired frequency.

It follows that if $\hat{T}$ given by (3.3) is validated with an uncertainty set $\mathcal{J}$ defined by (3.7) then $\hat{P}$ is validated with an uncertainty set

$$\mathcal{I}_{cl} = \left[\tilde{P}_{cl} - \Delta\tilde{P}_{cl},\ \ \tilde{P}_{cl} + \Delta\tilde{P}_{cl}\right], \qquad (3.11)$$

where $\tilde{P}_{cl}$ and $\Delta\tilde{P}_{cl}$ are defined via (3.9) and (3.10).

Note that the fact that a model is validated only means that the variance error supplants the bias error. Therefore, a model $\hat{P}$ may be open-loop falsified for some data set $\{\varepsilon, u\}$ because of its high bias at some frequencies, but closed-loop validated for some data set $\{\varepsilon_{cl}, r\}$ if the bias is reduced by the feedback at those frequencies. This will be illustrated in a numerical example in Section 5. Section 6 gives some insight into the relevance of the closed-loop validation procedure for control design through another example.

## 4 Engineering aspects

In this Section, we present the intuitive rules that we have adopted for the choice of structure and order of the model error model.

### 4.1 The structure of the model error model

In open-loop as well as in closed-loop validation, the choice of a structure for the model error model should always be done with great care. The main requirement is that $\tilde{P}$ (resp. $\tilde{T}$) must be an *unbiased* estimate of $\delta P$ (resp. $\delta T$). Since the computation of these model error models is done using standard open-loop prediction error identification tools, we refer the reader to [6] for guidelines.

### 4.2 The order of the model error model

Let $\eta_{P_0}$, $\eta_{\hat{P}}$, $\eta_{C_r}$ and $\eta_{C_y}$ denote the McMillan degrees of $P_0$, $\hat{P}$, $C_r$ and $C_y$. It is straightforward to see from (3.8) that the order of the model error $\delta T$ is

$$\eta_{\delta T} = \eta_{P_0} + \eta_{\hat{P}} + 2(\eta_{C_r} + \eta_{C_y}). \qquad (4.1)$$

As the McMillan degree of $P_0$ is unknown, we propose to choose the order of the model error model as

$$\eta_{\tilde{T}} = 2(\eta_{\hat{P}} + \eta_{C_r} + \eta_{C_y}), \qquad (4.2)$$

while its delay should be chosen identical to that of $\hat{T}$.

If $C_y(q) = 0$, our validation procedure reduces to the one outlined in Section 2. The order of the model error model $\tilde{P}$ is then taken as twice the McMillan degree of $\hat{P}$, and its delay should be that of $\hat{P}$.

Note that a higher order for the model error model increases the variance of its estimate, such that more models are validated as this order increases, but with a larger uncertainty region.

### 4.3 Standard validation with closed-loop data

As stated in Section 2, the standard 'open-loop' validation procedure can only be used if the input signal is uncorrelated with the noise. Actually, since we identify a strictly causal model error model, this requirement can be relaxed, and all that is needed is that $u(t)$ be uncorrelated with the future noise $v(t+1)$, $v(t+2)$, ...

Note that, in closed-loop, $u(t)$ can be written as

$$u(t) = \frac{C_r(q)r(t) - C_y(q)v(t)}{1 + C_y(q)P_0(q)}. \qquad (4.3)$$

We can formulate the following observations from (4.3) for the standard validation of a model $\hat{P}$directly from closed-loop data:

- If $v(t)$ is white, then $u(t)$ is only correlated with $v(t)$, $v(t-1)$, ... Otherwise, $v(t)$ is correlated with $v(t+k)$ for some $k \geq 0$, and $u(t)$ is then correlated with $v(t+k)$, $v(t+k-1)$, ... To avoid this problem, one can use a prefilter $L = \hat{H}^{-1}$ which will make $L(q)v(t)$ (almost) white, if $\hat{H}$ is a good model of the noise dynamics $H_0$.
- Assume now that $v(t)$ is white, possibly after filtering through $L$. If $C_y$ does not contain a delay, $u(t)$ is still correlated with $v(t)$. In this case, $P_0$ and $\hat{P}$ must contain a delay (for well- posedness of the closed-loop), and consequently $\delta P$ as well, which ensures by (2.4) that $\varepsilon(t)$ only depends on past inputs $u(t-1)$, $u(t-2)$, ... (which are uncorrelated with $v(t)$) and on the current noise sample $v(t)$. Thus, if the feedback controller does not contain a delay, a model structure with delay should be chosen for $\tilde{P}$.

To summarize these observations, one can use the standard open-loop validation method of $\hat{P}$ with closed-loop data provided an adequate prefilter is used. Recall however that the closed-loop validation procedure presented in Section 3, which first validates $\hat{T}$, allows the input signal $u$ to be correlated with the noise $v$ (the only requirement being that $r$ and $v$ be uncorrelated).

## 5 Numerical illustration I: closed-loop validation of an open-loop unvalidated model

In this Section, we show that a model that is not validated in open-loop may nevertheless be validated in closed- loop if its bias is reduced by the presence of the controller. This example was first proposed by Schrama in [10].

The true system is of order 8, while its model is of order 5. Their expressions, and that of the controller used for validation, are given in equations (5.1) to (5.3) (see below). In this particular case, the controller (5.3) was designed from the model (5.2). For simulation purpose, all these transfer functions are discretized with a sampling time of $0.05s$. $H_0$ is assumed to be 1. The Bode diagrams of the system and the model, both in open-loop and in closed-loop, are given in Figure 5.2. Note that the important low-frequency open-loop bias disappears in closed-loop.

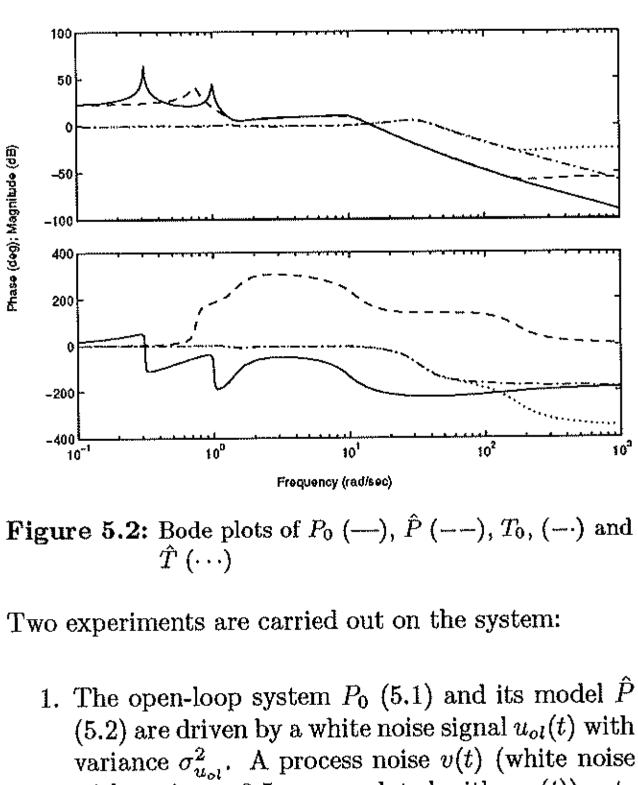



**Figure 5.2:** Bode plots of $P_0$ (—), $\hat{P}$ (– –), $T_0$, (–·) and $\hat{T}$ (···)

Two experiments are carried out on the system:

1. The open-loop system $P_0$ (5.1) and its model $\hat{P}$ (5.2) are driven by a white noise signal $u_{ol}(t)$ with variance $\sigma^2_{u_{ol}}$. A process noise $v(t)$ (white noise with variance 0.5, uncorrelated with $u_{ol}(t)$) acts on the output of $P_0$.
2. The real closed-loop system, defined by (5.1), (5.3), (2.1), (3.1) and (3.2), and the closed-loop model, defined by (5.2), (5.3) and (3.3), are driven with a unit variance white noise reference signal $r(t)$. A process noise $v(t)$ (white noise with variance 0.5, uncorrelated with $r(t)$) acts on $T_0$. The input signal of the true system is denoted $u_{cl}(t)$ and its variance $\sigma^2_{u_{cl}}$.

The variance of the input signal in the first experiment is chosen equal to that in the second experiment: $\sigma^2_{u_{ol}} = \sigma^2_{u_{cl}}$. A data set of 1000 samples is collected during each experiment and used to identify a model error model $\tilde{P}$ of order 10 (experiment 1) or $\tilde{T}$ of order 18 (experiment 2). The latter is then used to compute $\tilde{P}_{cl}$ as in (3.9). In each case, an ARX structure is chosen to reduce computational burden. The orders were chosen in accordance with the rules given in Section 4.

Thus, we end up with two model error models $\tilde{P}$ and $\tilde{P}_{cl}$, which are given with their respective 99% confidence regions $\mathcal{I}$ and $\mathcal{I}_{cl}$. Their amplitude Bode diagrams are depicted in the lower parts of Figures 5.1 (a) and (b). According to these plots, the model is validated with closed-loop data using the closed-loop validation procedure proposed in Section 3, since the uncertainty region around $\tilde{P}_{cl}$ contains 0 at all frequencies: see Figure 5.1 (b). However, it is not validated with open-loop data using the standard open-loop validation procedure. A very natural explanation to this is that for the closed-loop model, the variance error exceeds the bias error at all frequencies, which is clearly not true for the open-loop case. This is an important observation, which actually means that a biased model (typically, a model of order less that the system's) can be validated in closed-loop (the validation has been done here with the 'to be designed' controller, which is the ideal case when control design is of concern).

The upper parts of Figures 5.1 (a) and (b) show the amplitude Bode diagrams of $P_0$, $\hat{P}$ and $\hat{P} + \tilde{P}_{(cl)}$, the latter being a refined approximation of $P_0$. Since $\mathcal{I}_{(cl)}$ is the confidence region of $\tilde{P}_{(cl)}$, that of $\hat{P}+\tilde{P}_{(cl)}$ is given by $\hat{P}+\mathcal{I}_{(cl)}$; it is also shown on the picture. Recall that validation, i.e. $0 \in \mathcal{I}_{(cl)}$, implies that $\hat{P} \in (\hat{P} + \mathcal{I}_{(cl)})$, which is only true in closed-loop (Fig. 5.1 (b)). In that case, $P_0$ is also contained in this uncertainty region at almost all frequencies.

Note that in the absence of noise ($v(t) = 0$), the model would not be validated in open-loop, nor in closed-loop, since the variance of $\tilde{P}_{(cl)}$ would then be zero.

## 6 Numerical illustration II: a glimpse on closed-loop validation for control design

We have shown that a model may be validated according to our new closed-loop validation procedure even though it is not validated in the classical open-loop way. Now, we explain why this new validation test has relevance for control design.

### 6.1 Problem setting

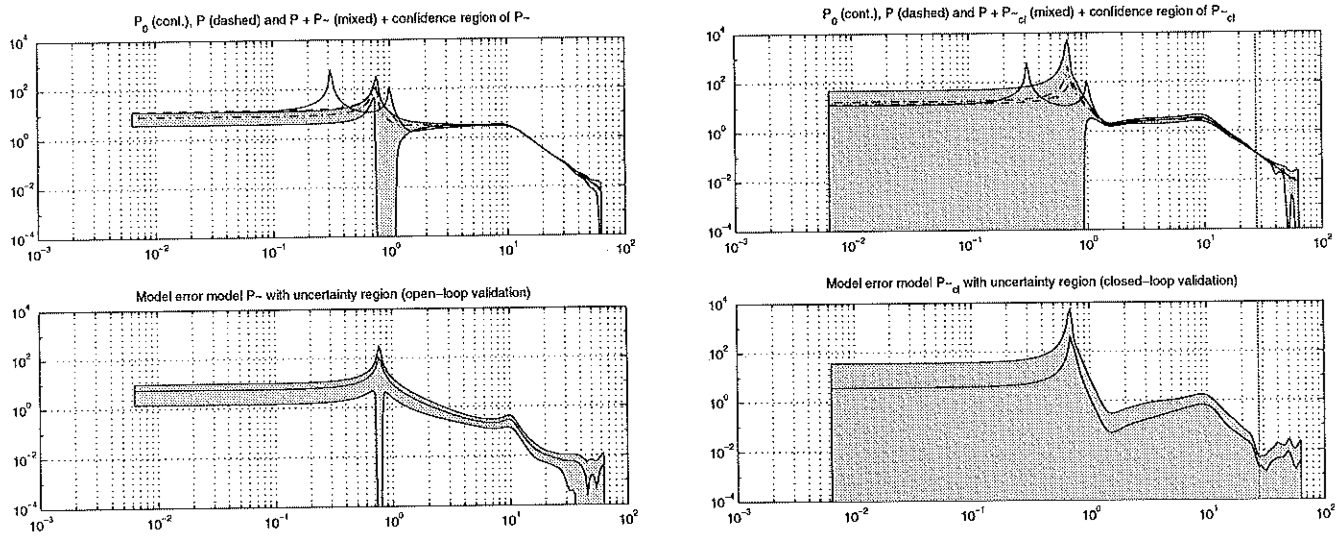
We have taken a 'true system' that has the following ARX structure:

$$(1-1.4q^{-1}+0.45q^{-2})\,y(t) = q^{-1}(1+0.25q^{-1})\,u(t)+e(t),$$

$$P_0 = \frac{30\,s^6 + 3020\,s^5 + 30538\,s^4 + 40373\,s^2 + 41972\,s + 12467}{s^8 + 26.023\,s^7 + 321.7\,s^6 + 2635.9\,s^5 + 10412\,s^4 + 3091.4\,s^3 + 11032\,s^2 + 306.81\,s + 986.86} \tag{5.1}$$

$$\hat{P} = \frac{0.001638\,s^5 - 0.13261\,s^4 + 38.769\,s^3 + 2250.9\,s^2 + 1447.4\,s + 4753.2}{s^5 + 11.397\,s^4 + 156.58\,s^3 + 604.42\,s^2 + 42.466\,s + 337.84} \tag{5.2}$$

$$C_r = C_y = \frac{37.375\,s^2 + 528.48\,s + 5625}{s^2 + 110.01\,s + 1741.6} \tag{5.3}$$




(a) Open-loop validation

(b) Closed-loop validation

**Figure 5.1:** Amplitude Bode diagrams. Upper plots: $P_0$ (—), $\hat{P}$ (– –), $\hat{P} + \tilde{P}_{(cl)}$ (–·) and $\hat{P} + \mathcal{I}_{(cl)}$ (filled). Lower plots: $\tilde{P}_{(cl)}$ (—) and $\mathcal{I}_{(cl)}$ (filled)

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with $e(t)$ white noise of unit variance.

The controller is a simple unit gain output feedback controller:

$$u(t) = r(t) - y(t)$$

with $r(t)$ white noise of unit variance, uncorrelated with $e(t)$.

The model under test is the following:

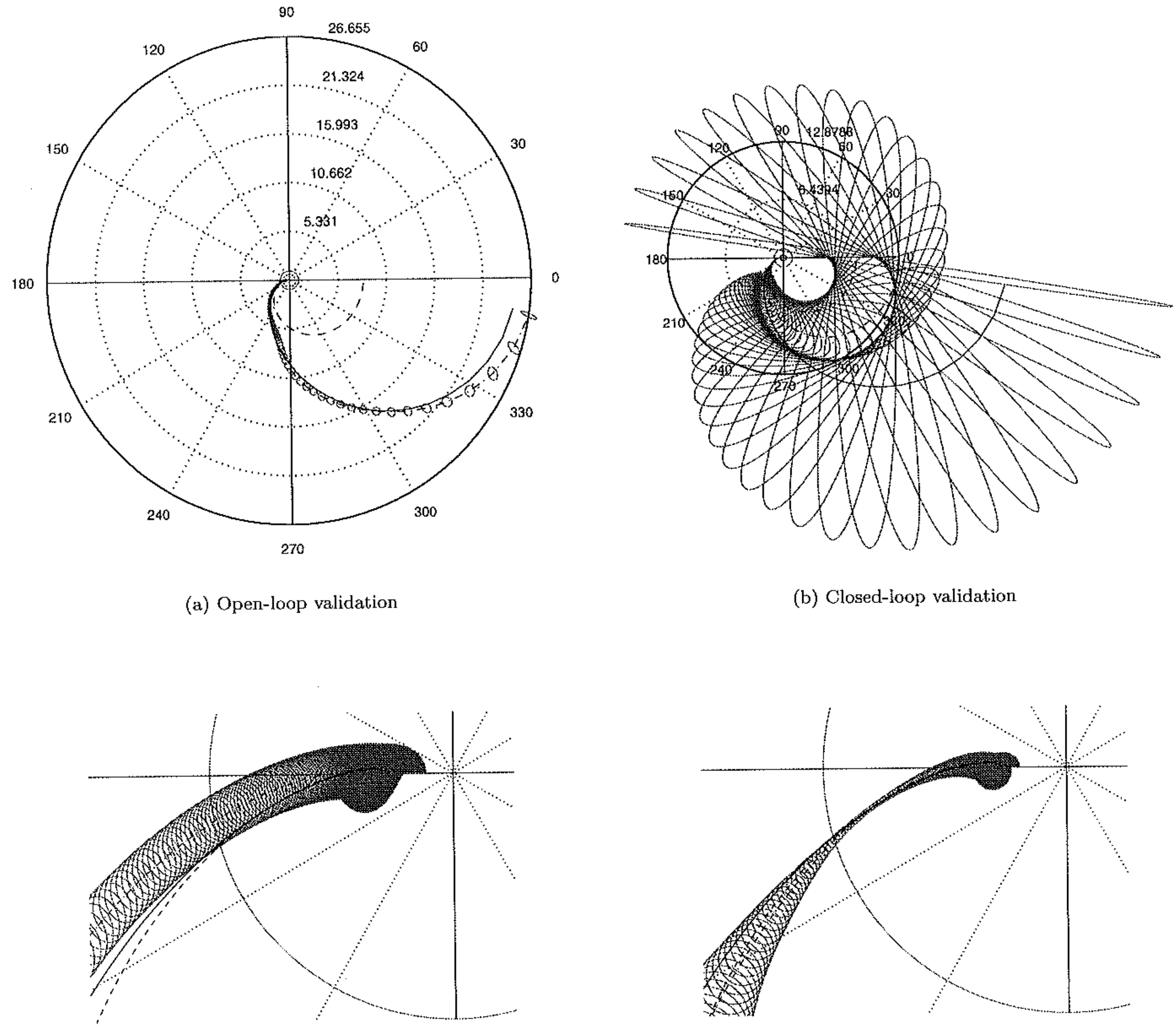
$$\hat{P} = \frac{0.9288 + 0.2604q^{-1}}{1 - 1.3068q^{-1} + 0.4534q^{-2}}q^{-1}.$$

Two data sets are collected on the system. The first one, $Z_{cl}^{1000} = \{r(t), u_{cl}(t), y_{cl}(t)\}$, contains 1000 samples of data collected on the system in closed-loop, $r(t)$ being white noise of unit variance uncorrelated with $e(t)$. It will be used to check if the model is closed-loop validated. The second one, $Z_{ol}^{1000} = \{u_{ol}(t), y_{ol}(t)\}$, contains 1000 samples of open-loop data. Here, $u_{ol}(t)$ is a white noise sequence uncorrelated with $e(t)$, and with a variance equal to that of $u_{cl}(t)$. This data set will be used for the open-loop validation test.

### 6.2 Relevance for control design

The signals of the open-loop data set and $\hat{P}$ are used to identify a model error model $\tilde{P}$ of order 4. An output-error structure is chosen for $\tilde{P}$, since we are not interested in the noise dynamics. The signals $r(t)$ and $y_{cl}(t)$ of the closed-loop data set are used to identify a model error model $\tilde{T}$. An output-error structure of order 4 (since $\eta_{C_y} = \eta_{C_r} = 0$) is also chosen. $\tilde{P}_{cl}$ is computed from $\tilde{T}$. All model error models are obtained with their 99% confidence regions.

Figure 6.1 (a) shows the Nyquist diagrams of $P_0$, $\hat{P}$ and $\hat{P} + \tilde{P}$ with its uncertainty region $\hat{P} + \mathcal{I}$ (ellipses). Recall that $\hat{P}$ is validated if its Nyquist diagram passes through the ellipses. Clearly, it is not the case.




(a) Open-loop validation

(b) Closed-loop validation

(c) Enlargement of (a)

(d) Enlargement of (b)

**Figure 6.1:** Nyquist diagrams of $P_0$ (—), $\hat{P}$ (– –) and $\hat{P} + \tilde{P}_{(cl)}$ (–·) with confidence ellipses

According to [8], one can nevertheless use $\hat{P}$ for control design, provided that $\tilde{P}$ and $\mathcal{I}$ be taken into account. Figure 6.1 (c), which zooms on the central part of Figure 6.1 (a), shows that $\tilde{P}$ and $\mathcal{I}$ do not give a very precise knowledge of the system $P_0$ around the crossover frequency (the circle of modulus 1 is depicted), although $\hat{P}$ is locally validated at that frequency. Therefore it seems obvious that the standard validation criterion is not suitable for control design, even though it informs about the quality of the model at the frequencies that are the most important for control design[2].

Let us now consider the same Nyquist plots when the model error model is built according to our new validation procedure. Figure 6.1 (b) shows the Nyquist diagrams of $P_0$, $\hat{P}$ and $\hat{P} + \tilde{P}_{cl}$ with its uncertainty region $\hat{P} + \mathcal{I}_{cl}$ (ellipses). Figure 6.1 (d) is an enlargement around the crossover frequency. Although the uncertainty ellipses are much larger at most frequencies, they are tight around the crossover frequency, and they capture the real $P_0$. Consequently, a larger gain margin and a finer (less conservative) controller tuning will be allowed.

## 7 Conclusions

A new method has been proposed for model validation, which takes account of the closed-loop properties of the model rather than of its open-loop quality. The validation criterion is an expression of the matching between two closed-loop transfer functions: on the one hand, the true transfer function $T_0$, built by the interconnection of the plant $P_0$ with the controller $C$; on the other hand, the nominal one $\hat{T}$, made up by the model $\hat{P}$ and the same controller. This criterion can then be reformulated in terms of $\hat{P}$ rather than $\hat{T}$, giving an insight into the quality of $\hat{P}$ as a representation of $P_0$ in a closed- loop framework.

A numerical example has shown that the classical approach to model validation may lead to very conservative uncertainty regions, and that it is therefore not suitable for control design. On the contrary, the new one leads to uncertainty regions which are shaped for that purpose. This remains true when the model is biased. This is a typical case often encountered when the purpose is to design a low-order controller (from a low-order model) for a more complex system.

The fact that a biased model can be validated clearly shows that the notion of validation cannot be separated from the intent of the model, nor from the experimental conditions. It is very natural to say that the quality of a model should always be evaluated with regard to its application. The closed-loop validation method is nothing but a formal application of that point.

[2] Indeed, this test – contrary to usual tests based on correlation plots of the residuals – gives more than a yes/no answer to the question of validation. Actually, one can see that the model *is* locally validated in a limited range near the crossover frequency. This validation test allows thus to assess the quality of a model at each frequency, which may prove useful if the model is intended to be used in a certain frequency band outside which no precision is required. The same property holds of course for the closed-loop validation test.

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