

Enhancing the Robustness of GPC via a Simple Choice of the Youla Parameter

P. Ansay¹, M. Gevers² and V. Wertz²

¹Service d'Automatique, Université de Liège, Institut Montefiore, Sart-Tilman, Belgium; ²CESAME, Université Catholique de Louvain, Louvain la Neuve, Belgium

This paper presents a new methodology for enhancing generalised predictive control (GPC) in order to robustify the closed-loop system in the face of neglected dynamics. This methodology consists of two distinct steps. In the first step, a nominal controller is obtained by minimising the GPC tracking performance index; in the second step, the robustness of the controller with respect to model uncertainties is enhanced via a simple choice of a Youla parameter. The overall controller is shown to yield a better compromise between closed-loop performance and robust stability than is obtained with existing methods.

Keywords: Generalised predictive control (GPC); Model uncertainties; Youla parametrisation

1. Introduction

Generalised predictive control (GPC) is model based. It uses an estimated model of the process which has the form of a CARIMA model, $A(q^{-1})\Delta y_t = B(q^{-1})\Delta u_{t-1} + C(q^{-1})e_t$, relating past outputs to past inputs and an estimation of the correlation of the noise via the C polynomial. This polynomial has focused a lot of attention because it represents a trade-off between rapid elimination of disturbances and robustness to measurement noise and unmodelled dynamics.

Estimating C is rarely successful because the contribution of the noise is usually time varying. Therefore, in the GPC derivation, C is used as a

fixed observer for the prediction of the future outputs [7]. In the absence of modelling errors, Clarke and Mohtadi [3] have shown that the C polynomial solely affects disturbance rejection properties of the closed-loop system and has no effects on the tracking response. Later, McInthosh et al. [6], Robinson and Clarke [9], Soeterboek [11] and Yoon and Clarke [12] introduced the problem of robust design to model uncertainties involving the selection of the C filter as its key element and suggested some design guidelines for the selection of C . We shall refer to these methods for robust design of GPC as C -design methods. They all proposed $C = (1 - \mu q^{-1})^n$, $C = A$, or a combination of these two expressions. They showed that usually one of these expressions yields a maximum robustness to model uncertainties for a particular value of μ and n . Despite these efforts and results for the special cases where GPC gives either mean-level or dead-beat closed-loop action [6], no systematic selection of C was proposed [12].

An alternative approach to predictive control robustification is found for the first time in Kouvaritakis et al. [5], where use is made of a Youla parametrisation to robustify the closed loop with respect to unstructured unmodelled dynamics. They obtained a systematic procedure for solving the problem via the minimisation of an H_∞ norm, whose sole purpose is to obtain stability in the presence of plant uncertainties. A similar approach can be found with all details in Hrissagis et al. [4]. The objections to this technique are that it requires an accurate model of the plant uncertainties and that the minimisation of an H_∞ norm can be time con-

Correspondence and offprint requests to: P. Ansay, Service d'Automatique, Université de Liège, Institut Montefiore, Bat. B28, 4000 Sart-Tilman, Belgium. E-mail: ansay@montefiore.ulg.ac.be

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suming in a receding horizon framework, especially in the case of an unstable plant.

The purpose of this paper is to propose a systematic procedure to obtain robustness margins for closed-loop stability in the presence of modelling errors. Our results rely on the relationship between the C -design method and the Youla parametrisation method established in Kouvaritakis et al. [5] and later clarified in Yoon and Clarke [12]. With this relationship GPC design can be split into a two-step procedure. We first design an optimal tracking GPC controller from a deterministic model, i.e. with $C = 1$, and then enhance the controller by a proper and simple choice of a Youla parameter. We compare the performance of the resulting controller with that obtained by the classical C -design method, as well as the $SGPC$ -design method proposed in Kouvaritakis et al. [5], and show that our resulting controller yields a better compromise between robustness and closed-loop performance than was achieved with previous methods, even when only a partial knowledge of the noise is known. This new scheme is not limited to open-loop stable plants. The discussion is illustrated by examples.

2. Two-Step Design of GPC

In GPC, the plant model is chosen to be

$$A(q^{-1})y_t = B(q^{-1})u_{t-1} + \frac{C(q^{-1})}{\Delta}e_t \quad (1)$$

Suppose also that there exists an exact representation of the true plant:

$$A_0(q^{-1})y_t = B_0(q^{-1})u_{t-1} + \frac{C_0(q^{-1})}{\Delta}e_t \quad (2)$$

where A , B , C , A_0 , B_0 and C_0 are polynomials in the backward-shift operator, e_t is white noise and $\Delta = 1 - q^{-1}$. The presence of Δ in the denominator of the noise model is to allow for the rejection of step disturbances, since Δ represents the internal model of a step. For the sake of simplicity, we assume that the system has a unitary dead time. The objective of GPC is to compute the vector of controls by optimising a quadratic cost function such as

$$J_t = \sum_{j=1}^{N_y} (y_{t+j} - r_{t+j})^2 + \lambda \sum_{j=1}^{N_u} \Delta u_{t+j-1}^2 \quad (3)$$

with the constraint $\Delta u_{t+N_u+j} = 0$, for $0 \leq j \leq N_y - N_u - 1$. Suppose that it is possible to obtain a stabilising controller for the model (1) by proper choices of the control horizon N_u , the prediction horizon N_y , the control weighting λ , and a fixed C polynomial. This

controller can be represented as a standard two degree of freedom controller:

$$\Delta \bar{R}(q^{-1})u_t = \bar{T}(q^{-1})r_{t+N_y} - \bar{S}(q^{-1})y_t \quad (4)$$

(see for example, Bitmead et al. [2] for details). Let us now choose the model

$$A(q^{-1})y_t = B(q^{-1})u_{t-1} + \frac{1}{\Delta}e_t \quad (5)$$

i.e. by setting $C = 1$ in (1). By minimising the same criterion with the same tuning knobs (N_y, N_u, λ) as in (3) for the model (5), we obtain the following stabilising 2 DOF controller:

$$\Delta R'(q^{-1})u_t = T'(q^{-1})r_{t+N_y} - S'(q^{-1})y_t \quad (6)$$

The following theorem shows the relation between these two GPC controllers.

Theorem 1. [5] The two controllers (4) and (6) are related by

$$\begin{aligned} \Delta \bar{R} &= \Delta R' C - q^{-1} B \Delta M \\ \bar{S} &= S' C + A \Delta M \\ \bar{T} &= T' C \end{aligned} \quad (7)$$

where M is a C -dependent polynomial.

The block diagram for the model (1) and the controller (4) can be represented as in Fig. 1. The outer loop is the optimal GPC controller R' , S' , T' obtained for the deterministic model (i.e. $C = 1$), while the inner loop is a correction that accounts for the rejection of the coloured noise modelled by $C/(A\Delta)$. In the C -design method, a stabilising controller for the model (5) is first computed. Then a C polynomial is designed to enhance robustness to model uncertainties or disturbances. There exist several guidelines for a good choice of C , but no

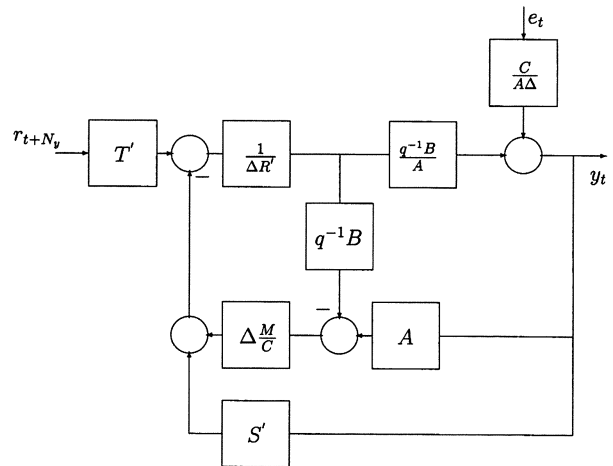


Fig. 1. Equivalent GPC controller structure.

systematic design has been proposed (see, for example, Yoon and Clarke [12]), because polynomial M is C -dependent.

An alternative to this C -design procedure is to use a design based on the Youla parametrisation [5]. This is based on the following observation. Consider the controller $\Delta R'$, S' , T' of (6) and assume, as before, that it stabilises the model (5), and thus also the model (1). Then the set of all controllers yielding the same closed-loop characteristic polynomial $A'_c = A\Delta R' + q^{-1}BS'$ with the plant model (1) is given by

$$\begin{aligned}\Delta R &= \Delta R' - q^{-1}BQ \\ S &= S' + AQ \\ T &= T'\end{aligned}\quad (8)$$

for any stable transfer function Q . The transfer function Q is called the Youla parameter. Observe that the regulator obtained from (4) by dividing all polynomials by C is a special case of (8) in which $Q = (\Delta M)/C$, with M a function of C . In the Youla parametrisation-based design method, however, Q is now entirely free, save that it has to be a stable transfer function. We show in the sequel how to select a simple choice of Q for robustness enhancement.

In order to calculate bounds for robust stability, we consider that the input to output model of (1) and the true plant (2) are linked by the following identity:

$$\frac{B_0}{A_0} = \frac{B}{A} + E$$

An important tool to examine the stability of a system in the presence of model-plant mismatch is the small-gain theorem based on the Nyquist stability criterion.

Theorem 2. [1] Consider the stable closed-loop system obtained by the controller (8) acting on the model (5), and let $A_c = A\Delta R + q^{-1}BS$ be the characteristic polynomial of that closed loop system. Suppose that the input-output transfer functions of the model (1) and the plant (2) have the same number of poles outside the unit circle and the same poles on the unit circle. Then the closed loop system obtained by the controller (8) acting on the true plant (2) is stable if the following inequality is fulfilled:

$$|E| < \left| \frac{A_c}{AS} \right| \quad \forall \omega \in [0, \pi] \quad (9)$$

3. Selection of the Youla Parameter for Stability Robustness

In this section we propose a systematic way of selecting the transfer function Q in (8) for robustness enhancement in the presence of neglected dynamics. With the definition of S as in (8), equation (9) may be rewritten as

$$|E| < \left| \frac{A_c}{AS'} \frac{S'}{S' + AQ} \right| \quad \forall \omega \in [0, \pi] \quad (10)$$

Observe also that A_c is fixed entirely by the first part of the design, because

$$A_c = A\Delta R + q^{-1}BS = A\Delta R' + q^{-1}BS' = A'_c$$

i.e. it is independent of Q . The stability bound, i.e. the right-hand side of (10), consists of two terms: the first term, $A_c/(AS')$, is obtained from the first step of the GPC design, while the second term, $S'/(S' + AQ)$, contains the free parameter Q which we expect to tune in order to satisfy the stability criterion.

We may want to impose a constraint on Q for the rejection of step disturbances. Indeed, observe that, with the controller (8) applied to the system (2), the closed-loop system becomes

$$y_t = \frac{B_0 T}{A_{c0}} r_{t+N_y-1} + \left(\frac{\Delta R' - q^{-1}BQ}{A_{c0}} \right) \frac{C_0}{\Delta} e_t$$

where

$$A_{c0} = A_0 \Delta R + q^{-1} B_0 S$$

Therefore, in order to avoid steady-state errors due to step disturbances, we need to impose that Δ is a factor of Q . In the sequel we shall impose this constraint on Q .

3.1 Stable Model and Plant

We assume in this section that the model and the true plant are stable and we pose

$$Q = \frac{S' \Delta M^*}{A C^*} \quad (11)$$

where M^* is any polynomial and C^* is any Hurwitz polynomial. This choice of Q , i.e. with S'/A as factor of Q , has the property of making the second term in the robustness bound (10) totally independent of the first term, and hence of the first design step:

$$|E| < \left| \frac{A_c}{AS'} \frac{1}{1 + \frac{\Delta M^*}{C^*}} \right| \quad \forall \omega \in [0, \pi] \quad (12)$$

The first term, $A_c/(AS')$, is the robustness bound obtained after the first design step (see (9) with S replaced by S'). It depends on the model (5) and the tuning knobs (N_y, N_u, λ) . The condition (9) with S replaced by S' may or may not be fulfilled because the objective of this first step is nominal performance only.

The second term, $1/[1 + (\Delta M^*)/C^*]$, is entirely free for tuning, i.e. M^* and C^* are free with the only constraint that C^* must be Hurwitz. In order to satisfy the inequality (12), this term must have a high-pass characteristic because the uncertainties on B_0/A_0 are typically dominant in the high frequencies.

We propose the following first-order filter with unit DC gain:

$$\frac{1}{1 + \frac{\Delta M^*}{C^*}} = \frac{1 - \mu_1 q^{-1}}{(1 - \mu_2) + (\mu_2 - \mu_1)q^{-1}} \quad (13)$$

with $\mu_1 < 1$ and $0 < \mu_2 \leq \mu_1$. This implies that

$$\frac{\Delta M^*}{C^*} = \frac{-\mu_2(1 - q^{-1})}{1 - \mu_1 q^{-1}} \quad (14)$$

and (12) becomes

$$|E| < \left| \frac{A_c}{AS'} \frac{1 - \mu_1 q^{-1}}{(1 - \mu_2) + (\mu_2 - \mu_1)q^{-1}} \right| \quad \forall \omega \in [0, \pi] \quad (15)$$

The second step in the Q -design method can provide robustness against high-frequency modelling errors. The parameters μ_1, μ_2 in the filter (13) are used as new tuning knobs to adjust correction in the high-frequency band where the modelling errors are expected to be large. Note that the high-pass characteristic is guaranteed if the inequality $0 < \mu_2 \leq \mu_1$ holds: see Fig. 2. This two degree of freedom filter permits the design of a wide variety of frequency band corrections. We propose the following design procedure:

- First consider $\mu_1 = \mu_2$; the filter in (13) is rewritten as

$$\frac{1}{1 + \frac{\Delta M^*}{C^*}} = \frac{1 - \mu_1 q^{-1}}{1 - \mu_1}$$

Select μ_1 between $0.6 \leq \mu_1 \leq 0.9$, as recommended, for example, in Soeterboek [11], such that the inequality in (15) is satisfied. This step fixes the filter bandwidth.

- Gradually reduce the value of the parameter μ_2 , leaving μ_1 fixed. This has the effect of reducing the high-pass correction, without affecting the system bandwidth: see Fig. 2.

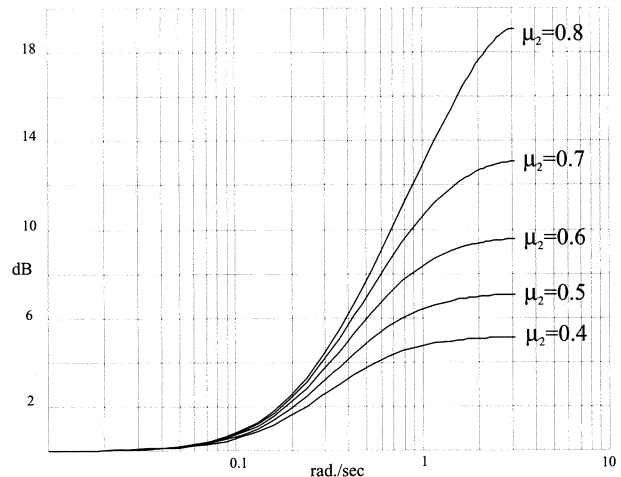


Fig. 2. Amplitude Bode plot of the first-order filter in (13) with fixed parameter μ_1 (here $\mu_1 = 0.8$) and variable parameter μ_2 .

Note that the choice of the first-order filter (13) is such that $(1 - q^{-1})$ is a factor of Q ; this is required for asymptotic step disturbance rejection (see above). Note also the similarity with the C -design method in GPC, where C is often chosen as $C = AC^*$, i.e. equal to the denominator of Q in (11), with $C^* = 1 - \mu_1 q^{-1}$, and $\mu_1 = \mu_2$ between $0.6 \leq \mu_1 \leq 0.9$.

Another interesting interpretation of our two-stage design method with the design choice (13) is obtained by comparing (15) and (9). The effect of the second design step is to replace the feedback path S' by

$$S' \frac{(1 - \mu_2) + (\mu_2 - \mu_1)q^{-1}}{1 - \mu_1 q^{-1}}$$

which represents the nominal filter S' multiplied by a low-pass filter. The physical interpretation is that the feedback path attenuates the loop gain at frequencies where the uncertainties are dominant.

We now compare with the C -design method, as in Yoon and Clarke [12]. With the regulator (7), the small gain condition (9) becomes

$$|E| < \left| \frac{\bar{A}_c}{A\bar{S}} \right| = \left| \frac{A_c C}{A \bar{S}} \right| \quad \forall \omega \in [0, \pi] \quad (16)$$

The right-hand side of (16) consists of two terms: A_c/A , which is independent of C , and a C -dependent term, C/\bar{S} . For the reason described above, if we want to select C such that C/\bar{S} has a high-pass characteristic, then, as observed in Robinson and Clarke [9], one method is to choose $C = AC^*$. Indeed, the definition of \bar{S} is (see (7))

$$\bar{S} = S'C + A\Delta M \quad (17)$$

If $C = AC^*$ in (17), then \bar{S} must contain A as factor. By assuming that $\deg(\bar{S}) = n_c - 1$ (see, for example,

Morari and Zafiriou [8]), the feedback polynomial \bar{S} is then reduced to

$$\bar{S} = KA$$

where K is a polynomial with $\deg(K) = n_{c^*} - 1$. Hence, if $C^* = 1 - \mu_1 q^{-1}$, then (16) becomes

$$|E| < \left| \frac{A_c}{Ak^*} \frac{1 - \mu_1 q^{-1}}{1 - \mu_1} \right| \quad \forall \omega \in [0, \pi] \quad (18)$$

where $k^*(1 - \mu_1) = K$ is a scalar.

Compare the relative degrees in the stability bounds (15) and (18). Because $\deg(S') = n_a$, the right-hand side in (18) typically dominates the right-hand side of (15) in high frequencies. The C -design method thus provides higher bounds than our method. However, these high bounds will provide for a deterioration in performance. This will be illustrated in Section 4.

3.2. Unstable Model and Plant

In the case of an unstable plant and model, the parameter Q cannot be chosen as in (11) because the model is unstable. Let then the polynomial A be split into its stable part A^s and unstable part A^u , $A = A^s A^u$, where $\deg(A^u) = n$ and $\deg(A^s) = n_a - n$. Define

$$Q = \frac{S' \Delta M^*}{A^s C^*}$$

Then the robust condition (10) becomes

$$|E| < \left| \frac{A_c}{AS'} \frac{1}{1 + \frac{A^u \Delta M^*}{C^*}} \right| \quad \forall \omega \in [0, \pi] \quad (19)$$

where the two terms of the right-hand side depend, as before, on the first and the second design step, respectively. In the same way as is done for the stable case, one can select a high-pass filter in the second design step in order to fulfil the robust stability condition of (19). We propose the simplest low-order high-pass filter compatible with the degree of A^u :

$$\frac{1}{1 + \frac{A^u \Delta M^*}{C^*}} = \frac{\prod_{i=1}^{n+1} (1 - \mu_i q^{-1})}{\prod_{i=1}^n k(1 - \alpha_i q^{-1})} \quad (20)$$

where k is selected such that the filter has unit DC gain. Equation (20) implies that

$$C^* = \prod_{i=1}^{n+1} (1 - \mu_i q^{-1})$$

where C^* is any Hurwitz polynomial of degree $n + 1$ and

$$C^* + A^u \Delta M^* = \prod_{i=1}^n k(1 - \alpha_i q^{-1})$$

This filter contains n poles which depend on the unstable poles of the model. Note also that $\deg(M^*)$ is zero as in the stable case. Similar remarks as in Section 3.1 are in order here.

4. Illustrative Examples

In this section we present two examples to illustrate the new Q-design method, and we compare its performance with the more classical C -design method and the $SGPC$ -design method, proposed in Kouvaritakis et al. [5].

4.1. Example 1

Our first example is the well-known Rohrs example (see Rohrs et al. [10]). The discrete-time description of the true system with a sampling period of $T = 0.04$ seconds is given by

$$y_t = \frac{0.00361(1 + 0.196q^{-1})(1 + 2.76q^{-1})q^{-1}}{(1 - 0.961q^{-1})[(1 - 0.55q^{-1})^2 + (0.04q^{-1})^2]} u_t \quad (21)$$

A first-order model is chosen that has the same natural frequency as the true plant

$$y_t = \frac{0.13q^{-1}}{1 - 0.88q^{-1}} u_t + \frac{C}{(1 - 0.88q^{-1})\Delta} e_t. \quad (22)$$

The GPC criterion (3) is used in combination with model (22). The reference signal r_t is a step change. We now compare four different control designs.

1. Nominal design. We choose standard tuning knobs ($N_y = 6, N_u = 1, \lambda = 0.1$) and $C = 1$ providing a good nominal tracking step response for the closed-loop model. The right-hand side of (9) with $S = S'$ is shown in Fig. 3; it intersects the amplitude of the model error $|E|$. The small gain condition (9) is not fulfilled and the achieved closed loop is actually unstable.

2. SGPC-design. We consider the optimal $SGPC$ -design as proposed in Kouvaritakis et al. [5], which is also a two-step procedure. The resulting controller has the same structure as in (8). The nominal controller, $\Delta R', S', T'$, is obtained using the following tuning knobs: $N_y = 6, N_u = 1, N_y - N_u$ terminal constraints and $\lambda = 0.1$ (see Kouvaritakis et al. [5] for details). This controller does not stabilise the true plant. An optimal H_∞ -

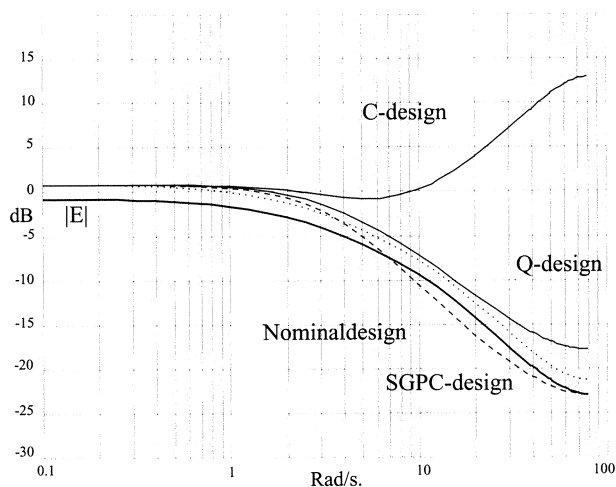


Fig. 3. Amplitude of the model error, $|E|$, and of the right-hand side of (9) in designs 1-4, as a function of frequency.

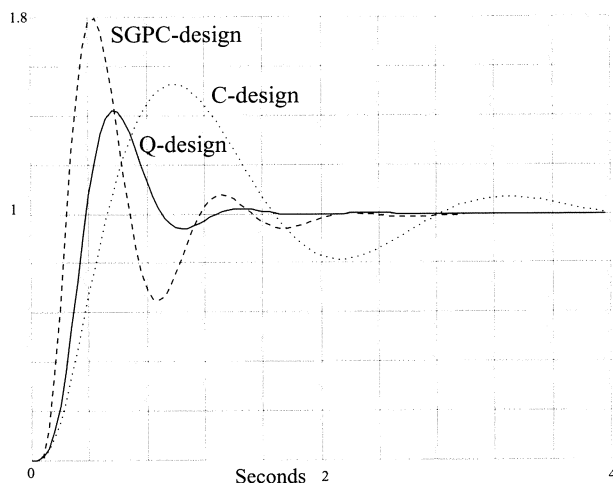


Fig. 4. Step response of the closed-loop system with designs 2-4 (noise-free simulations): Q -design (full), C -design (dotted), $SGPC$ design (dashed).

design is therefore performed as in Kouvaritakis et al. [5]. Results of this design are presented in Figs 3 and 4. In Fig. 3, one can see the right-hand side of (9), and the achieved closed-loop step response on the true plant is shown in Fig. 4. The controller is robust, i.e. the curve noted $SGPC$ -design in Fig. 3 has no intersection with the amplitude of the error model $|E|$.

3. Q -design. The first design step is the nominal design described above. Recall that this controller destabilises the true system. We then perform a second design step to enhance robustness with $Q = S'/A(\Delta M^*/C^*)$ and $\Delta M^*/C^*$ selected as in (14) with $\mu_1 = 0.8$ and $\mu_2 = 0.4$. The small gain condition (9) is now fulfilled (see Fig. 3); the curve noted Q -design is very close to that obtained in the

optimal $SGPC$ -design. Moreover, the step response achieved on the true plant is excellent (see Fig. 4), in fact much better than that obtained with the $SGPC$ -design.

4. C -design. We now compare with classical GPC design using the C -design method. The GPC controller is obtained in one design step with the same tuning knobs ($N_y = 6, N_u = 1, \lambda = 0.1$) as in the nominal design but with $C = A(1 - \mu_1 q^{-1})$, with the same parameter $\mu_1 = 0.8$ as in the Q -design. The curves C -design in Figs 3 and 4 show the right-hand side of (16), and the closed-loop tracking step response on the true plant, respectively. The controller is robust but the closed-loop tracking step response is significantly worse than with the Q -design.

4.1.1. Interpretations

It follows clearly from Fig. 3 that the robustness margin of GPC obtained in the C -design is higher than that obtained in the Q -design with equivalent settings. Moreover, it is possible to find a Q parameter such that the robustness margin in the Q -design is close to that obtained in the $SGPC$ -design, without an accurate model of $|E|$. The advantage of the Q parameter is that it provides a better compromise between performance and robust stability: compare Q -design with C -design and $SGPC$ -design in Figs 3 and 4. Moreover, as we shall see below, the disturbance rejection performance of the Q -designed controller is also significantly better than that of the C -designed controller and is similar to that of the $SGPC$ -designed controller.

4.1.2. Disturbance Rejection

If the noise contribution of the plant is known, i.e. C_0 in (2), the resulting GPC controller (7) is optimal for tracking and disturbance rejection. In practice, noise models are invariably wrong and at most a partial knowledge of the noise can be assumed. In the C -design method, this knowledge of the noise is incorporated in the C polynomial, see (7), while the Q -design method incorporates it via the C^* polynomial, see (11). Due to the fact that the Q -design method provides lower high-frequency uncertainty margins than the C -design method, high-frequency noise rejection will then be better and can be similar to that of the optimal $SGPC$ -design method. This is illustrated by the following example.

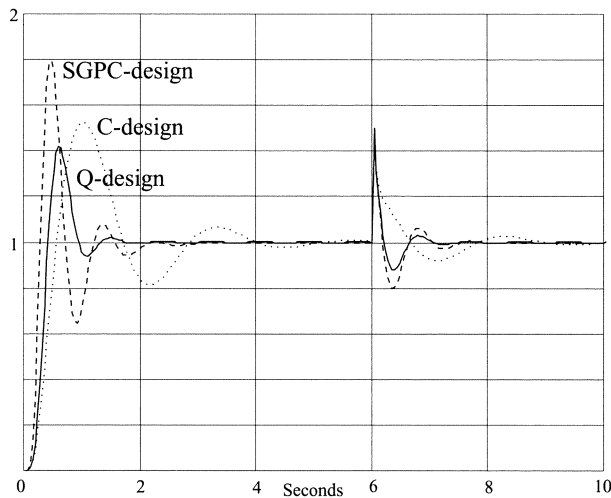


Fig. 5. Comparison of closed-loop step response and step disturbance rejection for example 2, with three different regulators: Q -design (full), C -design (dotted), $SGPC$ -design (dashed).

4.2. Example 2

In this example, the same plant model (21) is used as in Example 1, but now a noise contribution, $C_0/(A_0\Delta)$, is assumed and incorporated in the transfer function as in (2), with

$$C_0 = (1 - 0.7q^{-1})(1 - 0.8q^{-1})(1 - 0.9q^{-1})$$

We suppose that a partial knowledge of the noise is available: we assume $C = A(1 - 0.8q^{-1})$ in the C -design method, and we take $C^* = (1 - 0.8q^{-1})$ in the Q -design method. The model (22) is used. We compare the performances of the C -design, the Q -design and the $SGPC$ -design schemes in rejecting step disturbances. Figure 5 presents the closed-loop step response as well as the response to a step disturbance of magnitude 0.5 applied at time $t = 6$ seconds, for the controller obtained with the C -design, the Q -design and the $SGPC$ -design using the same tuning knobs as in Example 1. The results obtained with the Q -design method are significantly better than those obtained with the C -design method (see Fig. 5), both in terms of tracking and in terms of disturbance rejection. The performance in terms of disturbance rejection is similar to that obtained with the optimal $SGPC$ -design.

5. Conclusions

We have presented a two-step procedure for GPC control design. In the first step, a nominal controller is designed in order to achieve some tracking be-

haviour. The second design step, based on a Youla parametrisation, modifies the initial controller to take account of perturbations and/or unmodelled dynamics. Our procedure leads to very intuitive design guidelines for the controller adjustment in the second step, particularly with respect to the robustness to unmodelled dynamics. Our scheme is not limited to open-loop stable plants.

We have compared our two-step procedure with the prevailing C -design method and the optimal $SGPC$ -design method on the 'Rohrs example'. It shows that our design method is much better adapted to handle model errors, in that it leads to less conservative design than the C -design method, and it shows that it is possible to obtain similar robustness margins to those of the $SGPC$ -design method, with a suboptimal but much simpler – and computationally faster – choice of the Youla parameter.

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