inequality (6). To show stability, the authors reason by contradiction as follows: assume that $u \notin L_2$, and take limits on both sides of (7) as $T \to \infty$. In this case, the right-hand side tends toward zero and therefore, the left-hand side also tends toward zero. Hence the authors conclude that there is a contradiction since, by (6), the left-hand side is actually greater than zero. Therefore it must be true that $u \in L_2$.

This reasoning is fallacious, however, unless condition (6) is strengthened by requiring that the following be satisfied

$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] \geq \delta > 0. \tag{8}$$

In other words, there are only two possibilities of interest in (7)

1)
$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] \geq \delta > 0. \tag{9}$$

If this is the case then indeed there is a contradiction in (7). Condition (9), however, implies that the system is strictly passive, and therefore Lemma 1 becomes a restatement of the passivity theorem (see, for example, [1]), i.e., it says nothing about weak SPR functions.

2)
$$\inf_{u \in L_{2n}^n} \left[\lim_{T \to \infty} \frac{\langle u, Hu \rangle_T + \beta}{\langle u, u \rangle_T} \right] = 0. \tag{10}$$

In this case, there is no contradiction in (7) since the left-hand side also tends toward zero for some function u, without violating condition (6) (in the same way $1/n^p \to 0$ as $n \to \infty$, for all $p \in R^+ \geq 1$).

As a final remark we make the following observations, which emphasize the distinction between weak and strong SPR. It is relatively easy to show that the feedback combination of a (possibly nonlinear) passive plant and a strong SPR compensator is stable. The result can be proved by defining the loop transformation shown in Fig. 1 and noting that it does not alter the stability properties of the original system. It is then straightforward to show that, for small enough $\varepsilon > 0$, the system $H_1' = (1 - \varepsilon H_1)^{-1} H_1$ is passive, while $H' = H + \varepsilon 1$ is strictly passive, and therefore stability follows from the passivity theorem.

The case of a weak SPR system is, however, very different as shown in the following example.

Example 1: Consider the linear time-invariant system $H(s) = (s+c)/[(s+a)(s+b)]$, and let $H'(s) = H(s)/[1-\epsilon H(s)]$. We have

$$H'(j\omega) + H'(-j\omega) = 2\frac{(abc - \epsilon c^2) + \omega^2(a + b - c - \epsilon)}{(ab - \epsilon c - \omega^2)^2 + (a + b - \epsilon)^2} > 0 \tag{11}$$

if and only if

$$abc - \epsilon c^2 > 0, \quad a + b - c - \epsilon > 0. \tag{12}$$

If $a + b > c$, we can always find an $\epsilon > 0$ that satisfies (12). If, however, $a + b = c$ (i.e., when $H(s)$ is weak SPR), no such $\epsilon > 0$ exists.

II. Conclusions

The proof of Lemma 1[1] is incorrect. Since this note does not disprove that the feedback interconnection of a passive plant and a weak SPR controller is stable, we conclude that it remains an open question.

References

[1] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties.* New York: Academic, 1975.

On Interval Polynomials with No Zeros in the Unit Disc

V. Blondel

Abstract—**We give a necessary condition for an interval polynomial to have no zeros in the closed unit disc. The condition is expressed in terms of the two first intervals.**

The stability analysis of polynomials subject to structured uncertainty has received considerable attention this last decade (see [2] for an historical overview; references related to this contribution include [1], [3], [5], [8], and [9]).

In this note we give a necessary condition for an interval polynomial

$$P = \{a_0 + a_1 z + \cdots + a_n z^n : \quad \underline{a}_i \leq a_i \leq \overline{a}_i\}$$

to be D-stable, i.e., such that all members of P have no roots in the closed unit disc. Our condition is expressed in terms of the two first intervals only.

In a corollary we show that if $\underline{a}_0 < \overline{a}_0/2$ and $\underline{a}_0 < \overline{a}_1/9$ then P cannot be D-stable.

The results presented here are easy consequences of a little-known theorem on analytic functions.

Landau's Theorem: Assume that the function f is analytic in the open unit disc $|z| < 1$ and that $f(z) \neq 0, 1$ for all $|z| < 1$. Then

$$|f'(0)| \leq 2|f(0)|(|\log|f(0)|| + A)$$

were A is a constant which can be taken equal to 4.4.

For a proof of this theorem (which is sometime referred to as Landau–Carathéodory theorem) see, for example, Hille [4, p. 221]. The best possible bound for A was given in 1981 by Jenkins [6]; it is equal to $4\pi^2/\Gamma^4(\frac{1}{4}) = 4.37\cdots$.

We now prove our theorem.

Theorem: Let $P = \{a_0 + a_1 z + \cdots + a_n z^n : \quad \underline{a}_i \leq a_i \leq \overline{a}_i\}$ be an interval D-stable polynomial and assume that $\overline{a}_0 > \underline{a}_0 > 0$. Then

$$|\overline{a}_1| \leq 2\underline{a}_0 \left(\log^+ \frac{\underline{a}_0}{\overline{a} - \underline{a}_0} + 4.4 \right)$$

where $\log^+ x = \max(0, \log x)$.

Proof: Define $a_0^* \in [\underline{a}_0, \overline{a}_0]$ by $a_0^* := \min(2\underline{a}_0, \overline{a}_0)$ and choose an arbitrary set of coefficients $a_i^* \in [\underline{a}_i, \overline{a}_i]$ $(i = 2, \cdots, n)$. Consider the polynomial $p(z)$ defined by

$$p(z) := \frac{1}{\underline{a}_0 - a_0^*}(\underline{a}_0 + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n).$$

It is easy to see that $p(z)$ never takes the value zero or one in the open unit disc. Indeed

$$p(z) = 0 \Leftrightarrow \underline{a}_0 + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n = 0$$

and

$$p(z) = 1 \Leftrightarrow a_0^* + \overline{a}_1 z + a_2^* z^2 + \cdots + a_n^* z^n = 0.$$

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These two polynomials belong to P, hence $p(z) \neq 0, 1$ when $|z| < 1$. Applying Landau's theorem on $p(z)$ we obtain

$$\left|\frac{\overline{a}_1}{\underline{a}_0 - a_0^*}\right| \leq 2\left|\frac{\underline{a}_0}{\underline{a}_0 - a_0^*}\right|\left(\left|\log\left|\frac{\underline{a}_0}{\underline{a}_0 - a_0^*}\right|\right| + 4.4\right).$$

Since $a_0^* > \underline{a}_0 > 0$ we get

$$|\overline{a}_1| \leq 2\underline{a}_0\left(\left|\log\frac{\underline{a}_0}{a_0^* - \underline{a}_0}\right| + 4.4\right).$$

The coefficient a_0^* is defined by $a_0^* := \min(2\underline{a}_0, \overline{a}_0)$. If $2\underline{a}_0 \leq \overline{a}_0$ then $a_0^* = 2\underline{a}_0$ and

$$|\overline{a}_1| \leq 2\underline{a}_0 4.4 = 2\underline{a}_0\left(\log^+\frac{\underline{a}_0}{\overline{a}_0 - \underline{a}_0} + 4.4\right)$$

whereas if $2\underline{a}_0 > \overline{a}_0$ then $a_0^* = \overline{a}_0$ and

$$|\overline{a}_1| \leq 2\underline{a}_0\left(\left|\log\frac{\underline{a}_0}{a_0^* - \underline{a}_0}\right| + 4.4\right) = 2\underline{a}_0\left(\log^+\frac{\underline{a}_0}{\overline{a}_0 - \underline{a}_0} + 4.4\right).$$

The theorem is thus proved.

Remarks:

1) A corresponding theorem can be derived for other stability regions. For Schur stability (no roots outside the open unit disc) we obtain a necessary condition for the stability of interval polynomials with uncertainty in the highest order coefficient.
2) It is clear from the proof of theorem that, if $p_1(z) = \alpha + a_1 z + a_2 z^2 + \cdots + a_n z^n$ and $p_2(z) = \beta + a_1 z + a_2 z^2 + \cdots + a_n z^n$ are both D-stable polynomials, then

$$|a_1| \leq 2|\alpha|\left(\left|\log\left|\frac{\alpha}{\alpha - \beta}\right|\right| + 4.4\right).$$

This inequality can be used to derive bounds for other structured uncertainties descriptions.

Corollary: Let $P = \{a_0 + a_1 z + \cdots + a_n z^n: \quad \underline{a}_i \leq a_i \leq \overline{a}_i\}$ be an interval polynomial and assume that $0 < 2\underline{a}_0 < \overline{a}_0$ and $9\underline{a}_0 < \overline{a}_1$. Then P cannot be D-stable.

Proof: Assume by contradiction that P is D-stable. Since $2\underline{a}_0 \leq \overline{a}_0$, the theorem gives $|\overline{a}_1| \leq 2\underline{a}_0 4.4 = 8.8\underline{a}_0$. But this is a contradiction since $9\underline{a}_0 < \overline{a}_1$. The result is thus proved.

References

[1] J. E. Ackermann and B. R. Barmish, "Robust Schur stability of a polytope of polynomials," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 984–986, 1988.

[2] B. R. Barmish and H. I. Kang, "A survey of extreme point results for robustness," *Automatica*, vol. 29, pp. 13–35, 1993.

[3] N. K. Bose, E. I. Jury, and E. Zeheb, "On robust Hurwitz and Schur stability," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 1166–1168, 1988.

[4] E. Hille, *Analytic Function Theory*, vol. II. New York: Ginn, 1962.

[5] C. V. Hollot and A. C. Bartlett, "Some discrete-time counterparts to Kharitonov's stability criterion for uncertain systems," *IEEE Trans. Automat. Contr.*, vol. 31, pp. 355–356, 1986.

[6] J. Jenkins, "On explicit bounds in Landau's theorem. II," *Canadian J. Math.*, vol. 33, pp. 550–562, 1981.

[7] V. L. Kharitonov, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differential'nye Uraveniya*, vol. 14, pp. 1483–1485, 1978.

[8] A. Rantzer, "Kharitonov's weak theorem holds if and only if the stability region and its reciprocal are convex," *Int. J. Nonlinear Robust Contr.*, to appear.

[9] C. B. Soh, C. S. Berger, and K. P. Dabke, "A general stability theorem for linear discrete-time systems," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 505–507, 1985.

The Logical Control of an Elevator

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Abstract—**This paper presents a detailed example of the design of a logical feedback controller for finite state machines. In this approach, the control objectives and associated control actions are formulated as a set of axioms each of the form X implies Y, where X assets that i) the current state satisfies a set of conditions and ii) the control action y will steer the current state towards a given target state; Y assets that the next control input will take the value y. An automatic theorem prover establishes which of the assertions X is true, and then the corresponding control y is applied. The main advantages of this system are its flexibility (changing the control law is accomplished through changing only the axioms) and the fact that, by the design of the system, control actions will provably achieve the control objectives. The illustrative design problem presented in this paper is that of the logical specification and logical feedback control of an elevator.**

I. Introduction

The COCOLOG system (from Conditional Observer and COntroller LOGic) [1], [2] is a logical system for the state estimation and control of finite state machines. In this approach, the control objectives are formulated as axioms (i.e., necessarily true logical formulas) which relate the current state to a target state. The axioms are each of the form X implies Y, where X asserts that i) the current state satisfies a set of conditions and ii) the control action y will steer the current state towards a given target state; Y asserts that the next control input will take the value y. An automatic theorem prover establishes which of the assertions X is true, and then the corresponding control y is applied. The main advantages of this system are its flexibility (changing the control law is accomplished through changing only the axioms) and the fact that, by the design of the system, control actions will provably achieve the control objectives.

This paper applies the COCOLOG system to an idealized version of an elevator control problem. Section II describes the state, dynamics and control of the elevator. Sections III–V briefly outline the COCOLOG system and present a new logical framework specific to the control of an elevator. Sections VI and VII present the results of computer simulations using an Automatic Theorem Prover (due to Mackling, see [3]) and the conclusions which can be drawn from these results.

II. The Elevator Control Example

The logical control of an elevator is a good example with which to illustrate the operation of the COCOLOG system because it shares with many other discrete event systems the features of i) simplicity of dynamics, ii) combinatorial complexity of state description, and iii) great variety of possible control strategies and resulting trajectories. This section describes the basic set-up of the elevator control problem.

A. The State

The elevator control problem studied in this paper consists of a single elevator in a building with five floors, numbered zero to four. The elevator can handle up to three demands, with the

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