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# STABLE ADAPTIVE CONTROLLERS FOR WASTE TREATMENT BY ANAEROBIC DIGESTION

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## ABSTRACT

This paper deals with the control of waste treatment plants by anaerobic digestion. Continuous-time adaptive schemes are proposed for single-step (methanization) and two-step (acidification plus methanization) plants. The stability of the algorithms is demonstated by using Lyapunov functions, in both the disturbance-free and the bounded disturbance cases. The effectiveness of the algorithms is illustrated by simulation experiments. An important feature of the proposed algorithms is that they do not require any analytical description of the microbial specific growth-rate.

## INTRODUCTION

The biological treatment of organic wastes by anaerobic digestion with methane production has been described for a long time in the scientific literature [1]. One of the first comprehensive model was developed by Andrews [2] and has evolved through many other contributions, e.g. [3] - [8]. The anaerobic digestion of organic materials is commonly considered as a two-step process : acidification and methanization. In this paper we consider a continuous flow stirred tank reactor fed with soluble organics. In the acidification stage, the soluble organics are fermented into volatile acids by a group of acidogenic bacteria. In the methanization stage, a group of methanogenic bacteria converts the products of the acidification phase into methane ($CH_4$) and carbondioxyde ($CO_2$). The biological behaviour of the two phases is described by the usual state model of the microbial growth but with unspecified growth-rate functions. The aim of this paper is to present and analyse adaptive algorithms for the regulation of the anaerobic digestion process when it is used for waste treatment purpose (in food industries for instance). The control objective is to regulate the output pollution concentration at a prescribed level despite the input pollution fluctuations by acting on the dilution rate. Adaptive algorithms are presented for the methanization stage alone, and for the complete two-step process. The disturbance-free case and the bounded disturbance case are considered. An important feature is that, with these adaptive algorithms, stability and convergence properties of the overall system (process plus controller) have been emphasized under rather mild assumptions : it is worth noting that, for instance, the adaptive control algorithms are valid for any positive and bounded specific growth-rate. The theoritical proofs are not given here but can be found in an extended version of this paper [9].

## ADAPTIVE CONTROL OF THE METHANIZATION STAGE

### Description of the system

We consider first the case of the methane reactor in a two-phase process or an anaerobic digestion plant in which the methanogenic step is rate limiting. We assume that the dynamic biological behaviour of the anaerobic digestion process can be described by the following set of material balance equations (e.g. [6]) :

$$\frac{dX}{dt} = \mu(t)X(t) - D(t)X(t) \quad (1)$$

$$\frac{dS}{dt} = - k_1\mu(t)X(t) + D(t)\,[S_a(t) - S(t)] \quad (2)$$

$$Q(t) = k_2\mu(t)X(t) \quad (3)$$

with $X(t)$ the bacterial concentration, $S(t)$ the volatile acids concentration, $S_a(t)$ the influent volatile acids concentration, $D(t)$ the dilution rate, $Q(t)$ the methane gas flow rate, $\mu(t)$ the specific growth-rate, $k_1$ and $k_2$ the yield parameters.

The specific growth-rate $\mu(t)$ is known to be a complex function of the fermentation variables and many different analytical expressions have been suggested to account for the influence of substrate inhibition, pH and temperature on the bacterial growth in an anaerobic digester (e.g. [2],[8],[10] - [12]).

In this paper, we do not impose a specific analytical expression for the growth-rate $\mu(t)$. On the contrary, the adaptive algorithms we present are valid for any growth-rate, provided that it fulfills the following (quite physical) conditions:

$$H1 : 0 < \mu(t) < \mu^* \quad (4)$$

$$H2 : \mu(t) = 0 \text{ if } S(t) = 0 \quad (5)$$

where $\mu^*$ is the maximum growth rate.

### Statement of the control objective

As we said in the introduction, we consider the problem of regulating the anaerobic digestion process when it is used for waste treatment purpose. $S_a(t)$ is the input pollution level and $D(t)S_a(t)$ is the rate of the pollutant inflow while $S(t)$ is the output pollution level and $D(t)S(t)$ is the rate of the pollutant outflow. The control objective is to regulate the output pollution $S(t)$ at a prescribed level $Z^*$, despite the fluctuations of the input pollution level $S_a(t)$, by acting on the dilution rate.

To achieve this objective, we assume that :
1) the influent and effluent waste concentrations $S_a(t)$ and $S(t)$ are observed on line (through COD measurements for example)
2) the methane gas flow rate $Q(t)$ is an auxiliary measurable output of the process
3) the dilution rate $D(t)$ is the control input.

Furthermore, we assume that the growth-rate $\mu(t)$ and the yield parameters $k_1$, $k_2$ are a priori unknown and cannot be used in the control law.

To solve this regulation problem, the non linear structure of the system is explicitely used for the design of adaptive control schemes. Such an approach has already been used by the authors in the derivation of discrete time adaptive algorithms of both substrate concentration and production rate control of fermentation processes [13]. Here continuous time non linear adaptive controllers are proposed and their stability properties are analysed by using Lyapunov functions, in the disturbance-free case and in the bounded disturbance case.

### The disturbance-free case

From equations (1)-(3), we have :

$$\frac{dS}{dt} = - K\, Q(t) + D(t)\,[\, S_a(t) - S(t)] \qquad (6)$$

$$\text{with } K = \frac{k_1}{k_2} \qquad (7)$$

Equation (6) can be viewed as a first order dynamic model of the methanization phase with a bounded time varying parameter Q(t) ( the boundedness of Q(t) results from lemma 1 in [13]). This equation is the basis for the derivation of the control algorithm.

### The adaptive control algorithm

For a given desired output waste concentration $Z^*$, we define :

$$\overline{D}(t) = \frac{C_1(t)\,[\, Z^* - S(t)] + \hat{K}(t)\, Q(t)}{S_a(t) - S(t)} \qquad (8a)$$

with $C_1(t)$ a strictly positive function $(C_1(t) > 0)$ and $\hat{K}(t)$ an adaptive estimate of K, and we calculate the control input as follows :

$$D(t) = \overline{D}(t) \text{ if } 0 < \overline{D}(t) < D_{max} \qquad (8b)$$

$$D(t) = 0 \quad \text{if } \overline{D}(t) < 0 \qquad (8c)$$

$$D(t) = D_{max} \text{ if } \overline{D}(t) > D_{max} \qquad (8d)$$

The zero lower bound on D(t) is evident since a dilution rate cannot be negative. The upper bound $D_{max}$ is needed to ensure realistic operating conditions if $\overline{D}(t)$ is infinite.

The adaptive estimation of K is as follows :

$$\frac{d\hat{K}}{dt} = 0 \begin{cases} \text{if } \overline{D}(t) > D_{max} \text{ and } S(t) < Z^* \\ \text{if } K = 0 \text{ and } S(t) > Z^* \end{cases} \qquad (8e)$$

$$\frac{d\hat{K}}{dt} = C_2\, Q(t)\,[\, Z^* - S(t)] \quad \text{otherwise} \qquad (8f)$$

$$\hat{K}(0) > 0 \qquad (8h)$$

$C_2$ is a positive constant.

It is interesting to notice that if we define :

$$e_Z = (Z^* - S(t)) \quad \text{and} \quad e_K = (K - \hat{K}(t)) \qquad (9)$$

the closed loop system (6) - (8) can be written, when $D(t) = \overline{D}(t)$ :

$$\frac{d}{dt}\begin{bmatrix} e_Z \\ e_K \end{bmatrix} = \begin{bmatrix} -C_1(t) & Q(t) \\ -C_2(t) & 0 \end{bmatrix}\begin{bmatrix} e_Z \\ e_K \end{bmatrix} \tag{10}$$

i.e. as a linear time-varying unforced system.

## Stability analysis of the closed loop system

The stability of the closed loop system (6) - (8) is analysed under the following assumptions:

H3 : The input pollution level is bounded as follows :

$$S_{min} \leq S_a(t) \leq S_{max} \tag{11}$$

H4 : $0 \leq S(0) \quad 0 \leq X(0) \quad S(0) + k_1 X(0) \leq S_{max}$ (12)

H5 : $Z^* < S_{min}$ (13)

H6 : $D_{max} > \dfrac{\mu^* S_{max}}{S_{min} - Z^*}$ (14)

We have the following stability results :

Theorem 1 : under assumptions H1 to H6, the closed loop system (6) (8) is stable.

Proof : see [14].

Theorem 2 : under assumptions H1 to H6 and if $Q(t) \geq \epsilon > 0$ for all t, with $\epsilon$ an arbitrary constant, then

a) $\lim_{t\to\infty} S(t) = Z^*$

b) there exists $t_1$ such that $D(t) = \bar{D}(t)$ for all $t \geq t_1$

c) if $C_1(t) = \alpha Q(t)$ $(\alpha > 0)$, then the closed loop system is exponentially stable for $t \geq t_1$.

Proof : see [9].

Comments : 1) Theorem 2 is based on the assumption that the methane gas flow rate Q(t) is strictly positive for all t : $Q(t) > 0$. Within the framework of our previous assumptions, it is not possible to prove that $Q(t) > 0$ since Q(t) depends on $\mu(t)$ which is unspecified. However, for simple models of $\mu(S(t))$, it is fairly easy to show that Q(t) is effectively > 0 for all t.

2) It is worth noting that the controller achieves a zero steady-state error even with a varying input disturbance $S_a(t)$, since the algorithm (8) includes a feedforward action (i.e. takes into account the influence of the measured disturbance $S_a(t)$).

## The bounded disturbance case

In this paragraph, we consider the more realistic situation where a bounded disturbance term is added to equation (6) :

$$\frac{dS}{dt} = -K\,Q(t) + D(t)\,[S_a(t) - S(t)] + W(t) \tag{15}$$

with $|W(t)| \leq \Delta$ for all t.

The term W(t) represents model inaccuracies, external non measured disturbances and measurement noise.

In this case, the "error system" (10) is modified as follows :

$$\frac{d}{dt}\begin{bmatrix} e_Z \\ e_K \end{bmatrix} = \begin{bmatrix} -C_1(t) & Q(t) \\ -C_2(t) & 0 \end{bmatrix}\begin{bmatrix} e_Z \\ e_K \end{bmatrix} + \begin{bmatrix} -W(t) \\ 0 \end{bmatrix} \quad (16)$$

We have the following stability result :

Theorem 3 : under assumptions of theorem 2, the system (16) is BIBS stable.

Proof : see [19].

Comments : 1) under more restrictive assumptions, it can also be shown $\lim_{t\to\infty} |S(t) - Z^*| < \beta\,\Delta$, where $\beta$ can be made arbitrarily small by a proper choice of the design parameters $\alpha$ and $C_2$.

2) In case of a constant disturbance, the controller achieves a perfect static accuracy (i.e. $\lim_{t\to\infty} S(t) = Z^*$) since the adaptation of $\hat{K}$ includes an integral action.

## Simulation experiments

We consider a methanization system with :

a) a Haldane specific growth-rate (i.e. with substrate inhibition) :

$$\mu(t) = \frac{\mu^* \, S(t)}{K_m + S(t) + S^2(t)/K_i} \quad (17)$$

b) $K_m$ = 0.4 g/l, $K_i$ = 2.5 g/l, $\mu^*$ = 0.4 day$^{-1}$

c) a time varying input pollution level : $S_{min}$ = 3 g/l, $S_{max}$ = 5 g/l

The desired output is constant : $Z^*$ = 1.66 g/l

This value corresponds, in open loop, to an unstable stationary state ([15],[16]).

The control algorithm is applied with $D_{max}$= 0.4 day$^{-1}$, $C_1 = C_2 = 10$ and a measurement noise is added to Q(t).

Fig. 1 shows that the process is effectively stabilized by the controller. In order to prove the stability, we have assumed that $C_1(t) = \alpha Q(t)$ but in the experiment, stabilization has been obtained with a constant $C_1$.

## ADAPTIVE CONTROL OF THE TWO-STEP ANAEROBIC DIGESTION PROCESS

We consider now the complete anaerobic digestion process where acidification and methanization are present simultaneously.

The process is described by the following equations (e.g. [6]) :

acidification :

$$\frac{dX_1}{dt} = \mu_1(t)X_1(t) - D(t)X_1(t) \quad (18)$$

$$\frac{dS_1}{dt} = -k_3\mu_1(t)X_1(t) + D(t)[S_r(t) - S(t)] \quad (19)$$

methanization :

$$\frac{dX}{dt} = \mu(t)X(t) - D(t)X(t) \quad (20)$$

$$\frac{dS}{dt} = -k_1\mu(t)X(t) - D(t)S(t) + k_4\mu_1(t)X_1(t) \quad (21)$$

$$Q(t) = k_2\mu(t)X(t) \quad (22)$$

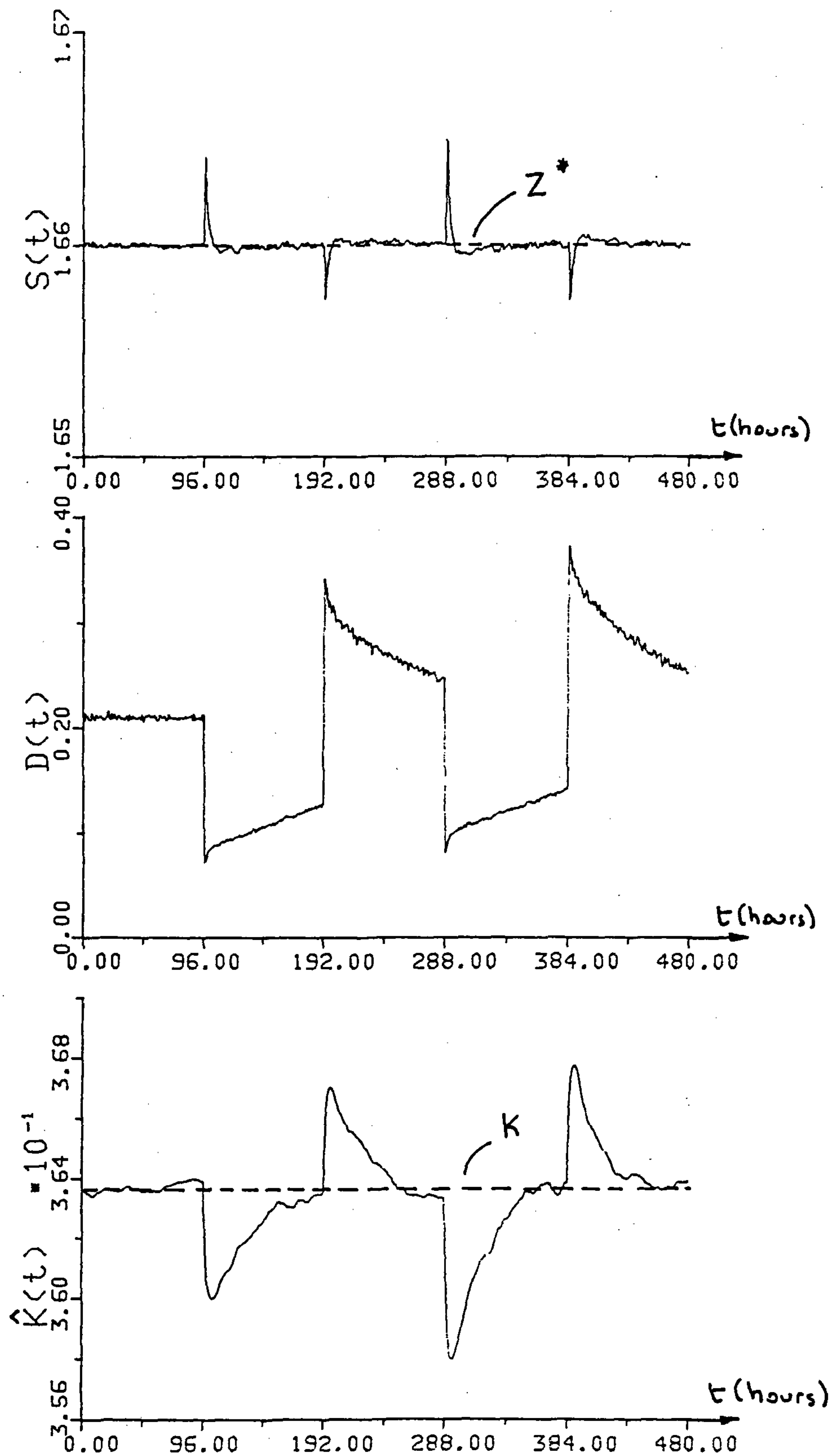


Fig.1. Adaptive control of an (unstable) methane reactor with a Haldane specific growth-rate.

with $X$, $S$, $D$, $Q$, $\mu$, $k_1$, $k_2$ as in section 2 and $X_1(t)$ the acidogenic bacterial concentration, $S_1(t)$ the soluble organics concentration, $S_r(t)$ the influent soluble organics concentration, $\mu_1(t)$ the specific growth-rate of the acidogenic bacteria, $k_3$ and $k_4$ yield parameters. From (18)-(21), the following first order model is readily derived :

$$\frac{dZ}{dt} = -KQ(t) + D(t)[S_r(t) - S(t)] + \varphi(t) \qquad (23)$$

with $Z(t) = S(t) + S_1(t)$

$$\varphi(t) = (k_4 - k_3)\,\mu_1(t)X_1(t)$$

It can be shown that $\varphi(t)$ is bounded as follows [13] :

$$0 < |\varphi(t)| < \frac{|k_4-k_3|}{k_3}\,\mu_1^*\,S_r^{max} = \Delta \qquad (24)$$

This disturbance-free model of the two-step process is, in fact, completely equivalent to the bounded disturbance single phase model (15) with $Z(t)$, $S_r(t)$ and $\varphi(t)$ instead of $S(t)$, $S_a(t)$ and $W(t)$ respectively. Then we can use the same control algorithm (8) and the same stability results will hold (theorem 3). However, the bound $\Delta$ on the "pseudo-disturbance" $\varphi(t)$ can be very large and the conditions for stability are likely to be violated in most cases.

An alternative way that we will now investigate is to estimate $\hat{\varphi}(t)$ on line and to incorporate $\hat{\varphi}(t)$ into the control law. Such an estimation of $\varphi(t)$ is clearly analogous to the on line estimation of OUR in activated sludge processes ([17],[18]).

## The adaptive control algorithm

We define

$$\bar{D}(t) = \frac{C_1(t)\,[Z^* - Z(t)] + \hat{K}(t)Q(t) - \hat{\varphi}(t)}{S_r(t) - Z(t)} \qquad (25)$$

The computation of $D(t)$ and the adaptation of $\hat{K}(t)$ are given by (8b) to (8h) like for the single stage case.

The adaptation of $\hat{\varphi}(t)$ is as follows :

$$\frac{d\hat{\varphi}}{dt} = -C_3\,[Z^* - Z(t)] \qquad (26)$$

Then, if we define $e_\varphi = \varphi(t) - \hat{\varphi}(t)$ and $e_Z = Z^* - Z(t)$, we have the following error system (when $D(t) = \bar{D}(t)$) :

$$\frac{d}{dt}\begin{bmatrix} e_Z \\ e_K \\ e_\varphi \end{bmatrix} = \begin{bmatrix} -C_1(t) & Q(t) & -1 \\ -C_2Q(t) & 0 & 0 \\ C_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} e_Z \\ e_K \\ e_\varphi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ d\varphi/dt \end{bmatrix} \qquad (27)$$

A complete stability analysis of the closed loop system is more involved than in the single stage case and has not yet been achieved. We present the following partial result :

Theorem 4 : under assumptions H1 to H6
if $C_1(t) = \alpha Q(t)$ $(0<\alpha<\infty)$
if $Q(t) > \epsilon > 0$ for all $t$
if there exists $t_1$ such that $D(t)=\bar{D}(t)$ for all $t > t_1$
if $\left|\frac{d\varphi}{dt}\right| < \Delta'$

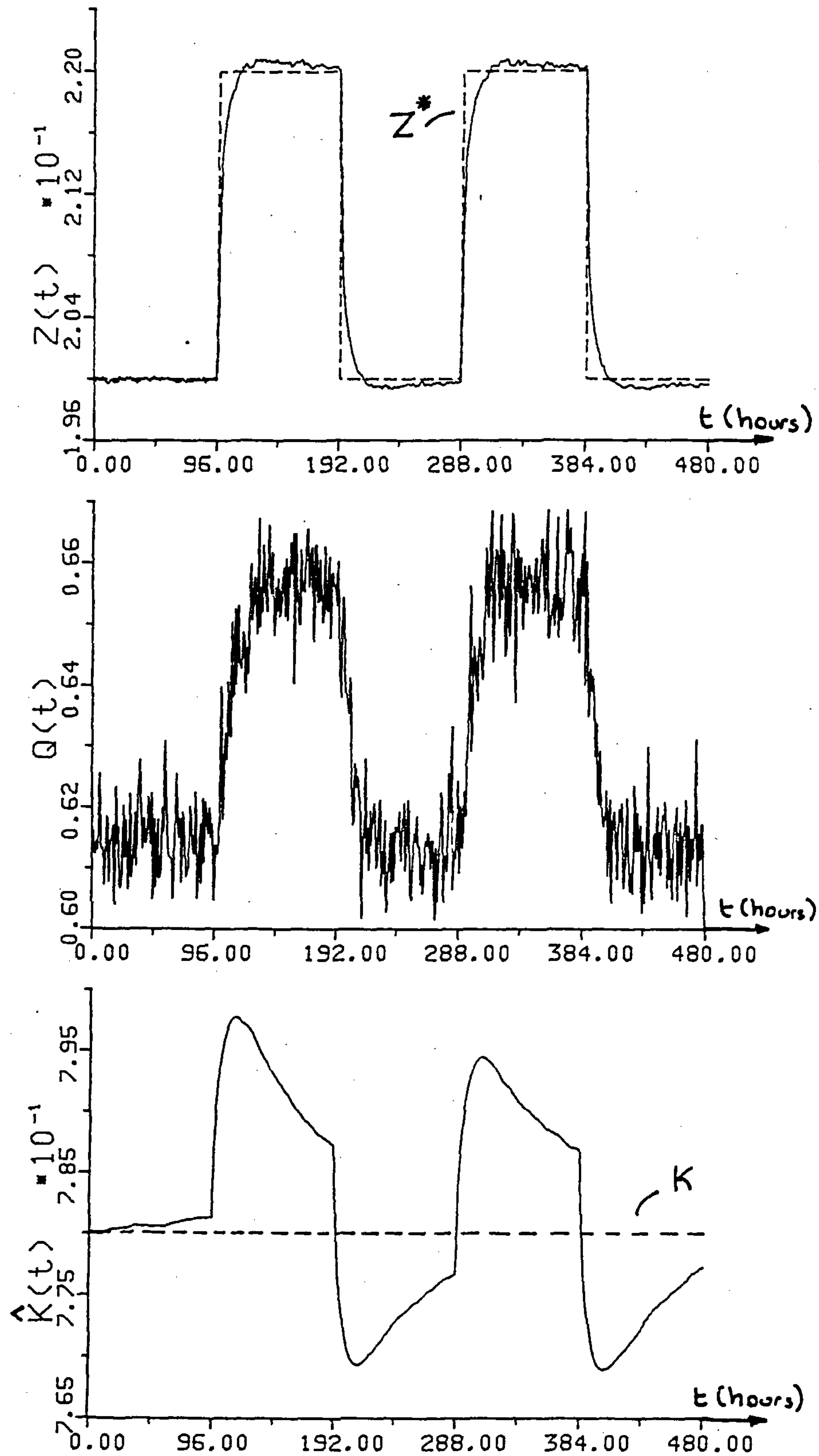


Fig.2. Control without estimation of $\phi$

then the error vector $e^T = (e_Z, e_K, e_\varphi)$ is asymptotically bounded as follows :

$$\lim_{t\to\infty} \|e\| < \gamma\Delta'$$

with $\gamma$ a positive constant.
Proof : see [9].

Comment : clearly, this stability property is much weaker than those derived in section 2. However, it is worth noting that, with theorem 4, the bound on the regulation error depends on the derivative of $\varphi(t)$ rather than on $\varphi(t)$ itself : with algorithm (25)(26), when $D(t) = \bar{D}(t)$, the regulation error $e_Z$ is small provided the fluctuations of $\varphi(t)$ are smooth, even if $\varphi(t)$ is large.

*Simulation experiments*

We consider a plant consisting of equations (18)-(22) with :
a) Michaelis-Menten growth-rates,
b) an additive measurement noise on $Q(t)$.
Fig. 2 shows a succesful simulation result when the control algorithm (8) is applied to the system (i.e. without estimation of $\hat{\varphi}$), in the case of a square wave set point.
Fig. 3 shows a simulation result when the algorithm (25)(26) is used (i.e. with an on-line estimation of $\hat{\varphi}$).
*However, we must say that initial conditions have been tried without success, and therefore that further analysis is needed to improve the algorithms.*

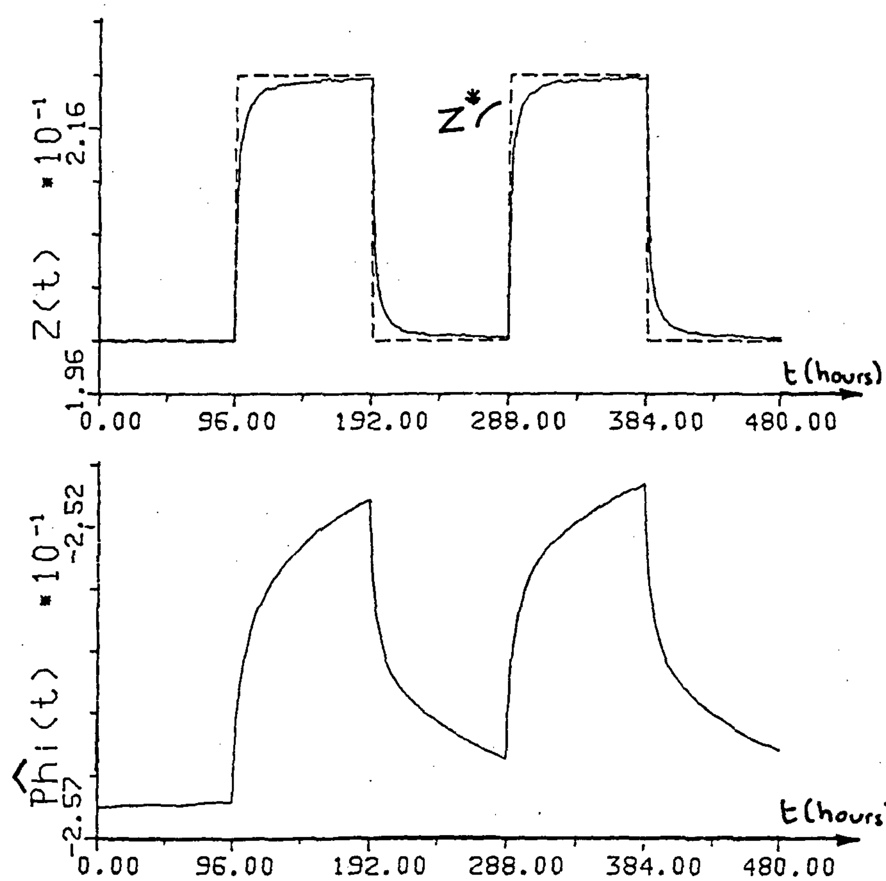


Fig.3. Control with estimation of $\phi$

## CONCLUSIONS

This paper has dealt with the control of waste treatment plants by anaerobic digestion. Various continuous-time adaptive schemes have been proposed for methanization plants and for acidification-methanization plants.

The stability properties of those algorithms have been analysed in both the disturbance-free and the bounded disturbance cases, and their effectiveness has been illustrated by some simulation experiments. It is worth noting that the proposed algorithms does not require any specific analytical description of the microbial growth-rates. Moreover, they can be coupled, if such an information is desired, with on line adaptive estimators of the specific microbial growth-rates [20].

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