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Semi-adaptive control of convexly parametrized systems with application to temperature regulation of chemical reactors

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SUMMARY

In this paper, we are interested in the problem of adaptive control of non-linearly parametrized systems. We investigate the viability of defining a stabilizing parameter update law for the case when the plant model is *convex* on the uncertain parameters. We show that, when the only prior knowledge is convexity, there *does not exist* an adaptation law—derivable from the standard separable Lyapunov function technique of Parks—applicable *for all* the state space. Therefore, we propose a semi-adaptive state feedback controller where adaptation takes place only in the region of the state space where convexity can be used to reduce parameter uncertainty. In the remaining part of the state space we freeze the adaptation and switch to a robust controller. This scheme ensures *semi-global* stability for convexly parametrized non-linear systems with matched uncertainty. The proposed controller is then applied to the problem of *temperature* regulation of continuous stirred exothermic chemical reactors where reaction heat is convex in the uncertain parameters. Copyright © 2001 John Wiley & Sons, Ltd.

1. INTRODUCTION

It is widely recognized that designing adaptive (identification or control) algorithms for non-linearly parametrized systems is a difficult and poorly understood problem. To make it mathematically tractable the *ad hoc* assumption of linearity in the parameters is often introduced. Since physical parametrizations are almost invariably non-linear, this assumption is quite unnatural. For linear systems, a standard procedure to overcome the problem is to over-parametrize the system in order to obtain a linear parametrization. This suffers from the

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well-known shortcoming of robustness degradation due to the slower convergence intirinsic to a search in a bigger space and the potential loss of identifiability. Furthermore, over-parametrization stymies the incorporation of prior knowledge available from the physical parameters, for instance, when defining a constrained estimation region. When the system is non-linear overparametrization is possible only in some very special cases. It may even be argued that it is somehow superfluous to try to extend the existing theory for linear systems to the non-linear case without addressing this central issue.

Early research in this direction, which motivates our present work, was reported in Reference [1], see also Reference [2]. In these works, it is shown that a sufficient condition for adaptive stabilization of non-linearly parametrized non-linear plants is that the derivative of the standard separable Lyapunov function is *convex* in the unknown parameters. This is unfortunately a state-dependent assumption which is difficult to verify *a priori*. Instead, in this paper,† we assume convexity only of the *plant* model, and explore the implications of this assumption on the limits of adaptation. In other words, we are interested in knowing whether it is possible to derive (with the classical Parks' separable Lyapunov function technique [3]), an adaptation law that will reduce the parameter uncertainty in *all the state space* with only the prior knowledge of convexity. Our answer to this question is, unfortunately, *negative*. Our proposition to overcome this obstacle is a so-called *semi-adaptive* control policy, where adaptation takes place in the region of the state space where convexity can be used to ensure a good gradient search. In the remaining part of the state space we freeze the estimation and switch to a standard robust controller. Our motivation to consider semi-adaptation, instead of robust control in all the state space, is the conventional wisdom that the uncertainty reduction feature of parameter estimation enhances *performance*.

After the publication of Reference [4], which brought to the attention of the western literature the work of Fradkov [1, 2], several authors have tried to exploit the convexity property to design stable adaptive systems. In Reference [5], the property is used, in an identification context for the adaptive pole-placement problem. In Reference [6], the main idea of Reference [1] is combined with a (relay-based) high-gain design to achieve stable adaptive control. This approach is similar in spirit with the algorithm presented in this paper, but instead of using a relay, we freeze the adaptation and switch to a (constant parameter) robust controller. To reduce the deleterious effect of the high gain, the (off-line) computation of convex majorant/concave minorants is proposed in Reference [6]. Recently, in Reference [7], to apply the techniques of Reference [6] for a general non-linearly parametrized system, we have explored the possibility of *convexification via* reparametrization. Other works addressing the problem of adaptive control of non-linearly parametrized systems are References [9–12].

The rest of the paper is organized as follows. In Section 2 we present in detail a motivating example of a simple integrator, that we believe captures the essential features of the problem. Section 3 is devoted to the study of the limits of adaptation. Here again, for pedagogical reasons, we consider the case of the integrator, but as pointed out in Remark 4.2 the analysis applies as well to more general systems. A semi-adaptive state feedback controller which ensures *semiglobal* stability when the uncertainty is matched, is presented in Section 4. The proposed controller is then applied in Section 5 to the problem of *temperature* regulation of continuous stirred exothermic chemical reactors where reaction heat is convex in the uncertain parameters. We wrap up the paper with some concluding remarks.

†A conference version of this work was reported in 1997 in Reference [8].

2. MOTIVATION

To understand the difficulty of the problem, let us consider the simple task of regulating to zero the following scalar plant:

$$\dot{x} = f(\theta, x) + u \tag{1}$$

where $\theta \in \mathscr{R}^q$ is a vector of *unknown* parameters and $f \triangleq f(\theta, x)$ is differentiable in θ. If the parameters are *known* the control $u = -x - f$ achieves the objective. In the case of *unknown* parameters, we might adopt the certainty equivalence approach to get

$$u = -x - \hat{f} \triangleq -x - f(\hat{\theta}, x) \tag{2}$$

where $\hat{\theta}$ is an estimate of the unknown parameters. To define the parameter estimator one is tempted to try a Taylor expansion of f around $\hat{\theta}$ as

$$f = \hat{f} + \nabla_{\hat{\theta}} \hat{f}(\theta - \hat{\theta}) + \text{h.o.t.} \tag{3}$$

where $\nabla_{\hat{\theta}} \triangleq \partial / \partial \hat{\theta}$, and h.o.t. denotes the higher-order terms. Then, we construct a first-order *approximation* of (1) as

$$\dot{x}_a = f(\hat{\theta}, x_a) - \nabla_{\hat{\theta}} f(\hat{\theta}, x_a)(\hat{\theta} - \theta) + u$$

Noting that the first two right-hand terms above are known, we see that (2) is also a certainty equivalent controller for the approximate system, which results in a closed loop

$$\dot{x}_a = -x_a - \nabla_{\hat{\theta}} f(\hat{\theta}, x_a)\tilde{\theta} \tag{4}$$

where $\tilde{\theta} \triangleq \hat{\theta} - \theta$ denotes the parameter error. A globally stabilizing parameter update law for (4) can be easily obtained considering the standard separable Lyapunov function candidate

$$V_a = \tfrac{1}{2}(x_a^2 + |\tilde{\theta}|^2)$$

with $|\cdot|$ the Euclidean norm, whose derivative along the trajectories of (4)

$$\dot{V}_a = -x_a^2 - x_a \nabla_{\hat{\theta}} f(\hat{\theta}, x_a)\tilde{\theta} + \tilde{\theta}^{\mathrm{T}} \dot{\tilde{\theta}}$$

suggests the parameter update law

$$\dot{\tilde{\theta}} = \nabla_{\hat{\theta}}^{\mathrm{T}} f(\hat{\theta}, x_a) x_a$$

The key question is:

- How this first-order approximation approach will work when applied to the actual system? To answer this question we study the stability of the actual closed-loop system

$$\dot{x} = -x + f - \hat{f}$$

with estimation law

$$\dot{\tilde{\theta}} = (\nabla_{\hat{\theta}}^{\mathrm{T}} \hat{f})x \tag{5}$$

To this end, consider the Lyapunov function candidate

$$V = \tfrac{1}{2}(x^2 + |\tilde{\theta}|^2)$$

whose derivative yields

$$\dot{V} = -x^2 + x[f - \hat{f} + (\nabla_{\hat{\theta}} \hat{f})\tilde{\theta}] \tag{6}$$

Noticing that the right-hand terms in square brackets are precisely the higher-order terms of the Taylor series expansion (3), we see that the closed loop will be stable if f is linear in θ. That is, if there exists $\phi(x)$, $f_0(x) \in \mathscr{R}^q$ such that $f = f_0(x) + \phi^{\mathrm{T}}(x)\theta$. Besides this particular case very little can be said without further qualifications on f. For instance, for $f = \theta^2$ we get

$$\dot{V} = -x^2 + x\tilde{\theta}^2$$

It is easy to see that $\dot{V} > 0 \Leftrightarrow 0 < x < \tilde{\theta}^2$. Hence, the equilibrium point $(x, \tilde{\theta}) = (0, 0)$ is unstable. A similar situation arises when $f = \sqrt{|x|}\theta^2$.

It is clear that to get some workable results we have to impose some additional restrictions on f. For instance, if we additionally impose the conditions of $f(0, \theta) = 0$, and f being twice continuously differentiable in x, then a Taylor expansion argument allows us to prove that the closed loop of the certainty equivalent adaptive system based on the first-order approximation is *locally* stable.‡ This is the case considered in Reference [12].

In this paper, we assume no additional prior knowledge on f, except that it is *convex*§ in θ, a natural extension to linearity. That is, we assume

$$f \geqslant \hat{f} + (\nabla_{\hat{\theta}} \hat{f})(\theta - \hat{\theta})$$

for all x, θ and $\hat{\theta}$. Notice that in this case, the higher-order terms of (3) are *positive*. Hence, the sign of the second right-hand term in (6) will coincide with the sign of x. This suggests the following.

Semi-adaptive control policy. Use adaptation in the half line $x \leqslant 0$, and when $x > 0$ freeze the adaptation and switch to a robust e.g. high gain, control law.

Although the procedure may seem to be a little contrived and determined by our choice of Lyapunov function candidate, estimation and control laws, in the next section we show that–without further qualifications on the function f—this is the best we can do from the point of view of adaptation.

‡We should note that in this case local stability is ensured even if the adaptation is *frozen*.

§As will become clear later, similar arguments apply when f is concave.

3. CONVEX PARAMETRIZATION: LIMITS OF ADAPTATION

The proposition below proves that, even restricted to convex parametrizations, we cannot derive—from a standard separable Lyapunov function—an adaptation law that will reduce the parameter uncertainty in *all* the state space. To keep the exposition simple we will still consider throughout this section the simple integrator (1). As pointed out in remark 4.2, this is done without loss of generality. To present our result we need the following preliminary proposition.

Proposition 3.1.

Consider the class $\mathscr{P}$ of scalar functions $f(\theta, x)$, $\alpha(x)$, where $f(\theta, x)$ is differentiable in $\theta \in \mathscr{R}^q$, and $\alpha(x)$ is such that

$$\nabla_x W \alpha(x) \leqslant 0$$

for some differentiable positive-definite function $W(x)$. If $\mathscr{P}$ includes *all* functions $f(\theta, x)$ which are *convex* in θ, then there *does not* exist a vector function $\Theta(\hat{\theta}, x)$: $\mathscr{R}^q \times \mathscr{R} \rightarrow \mathscr{R}^q$ such that

$$S(x, \theta, \hat{\theta}) \triangleq \nabla_x W [\alpha(x) + f(\theta, x) - f(\hat{\theta}, x)] + (\hat{\theta} - \theta)^{\mathrm{T}} \Theta(\hat{\theta}, x) \leqslant 0 \tag{7}$$

for all x, θ, $\hat{\theta}$.

Proof. Adding and substracting $\nabla_x \mathrm{W}(\nabla_{\hat{\theta}} \hat{f})\tilde{\theta}$ to (7) we get

$$S(x, \theta, \hat{\theta}) = \nabla_x W \alpha(x) + \nabla_x W [f - \hat{f} + (\nabla_{\hat{\theta}} \hat{f})\tilde{\theta}] + \tilde{\theta}^{\mathrm{T}} [\Theta(x, \hat{\theta}) - (\nabla_{\hat{\theta}}^{\mathrm{T}} \hat{f}) \nabla_x W]. \tag{8}$$

Recall that $S \leqslant 0$ should hold *for all* f convex in θ. Choose $f = |\theta|^2$. For this particular choice we have

$$S = \nabla_x W \alpha(x) + \nabla_x W |\tilde{\theta}|^2$$
$$+ \tilde{\theta}^{\mathrm{T}} [\Theta(x, \hat{\theta}) - (\nabla_{\hat{\theta}}^{\mathrm{T}} \hat{f}) \nabla_x W]$$

For any fixed x taking $|\tilde{\theta}|$ sufficiently large we conclude that W must satisfy $\nabla_x W \leqslant 0$. But this contradicts the requirement of positive definiteness of W. This completes the proof. □

We are in a position to present the main result of this section, whose proof follows immediately from the above proposition.

Corollary 3.1.

Consider the plant (1) in closed loop with the certainty equivalent adaptive design $u = \alpha - \hat{f}$. Then, there *does not* exist an adaptation law

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = \Theta(\hat{\theta}, x)$$

such that the Lyapunov function candidate

$$V = W(x) + \tfrac{1}{2}|\hat{\theta} - \theta|^2$$

insures $\dot{V} \leqslant 0$ *for all* f which are convex in θ. □

Remark 3.1.

The proof of the proposition above is inspired by Theorem 2 of Reference [13], (see also Reference [2]), where a similar result is established for linear systems with linear parameterizations. It is interesting to note that in Reference [2] it is shown that including *cross-terms* in the Lyapunov function does not allow us to enlarge the class of adaptively stabilizable systems, but only add some additional freedom in the choice of the adaptation law. Some investigations of this issue for non-linear linearly parametrized systems are reported in Reference [14].

Remark 3.2.

It is clear from the proof of the proposition that the condition $\dot{V} \leqslant 0$ cannot be satisfied even on an (arbitrarily small) neighbourhood of the surface $\nabla_x W(x) = 0$.

In the next section, we will present the stability analysis of the semi-adaptation procedure proposed in the previous section as applied to a class of non-linear systems with matched uncertainty.

4. SEMI-ADAPTIVE STABILIZATION

Proposition 4.1.

Consider the non-linear system described by

$$\dot{x} = f_0(\theta, x) + g(\theta, x)[f(\theta, x) + u] \tag{9}$$

where $x \in \mathscr{R}^n$, $u \in \mathscr{R}$, and $\theta \in \mathscr{R}^q$ is a vector of *unknown* parameters and f_0, g, f are functions of suitable dimensions. Assume:

A.1 $f(\theta, x)$ is *convex* in θ.

A.2 The system $\dot{x} = f_0(\theta, x) + g(\theta, x)u$ is asymptotically *stabilizable* without knowledge of θ. That is, we know a function $\alpha(x)$ and a positive-definite function $W(x)$ such that[¶]

$$\nabla_x W[f_0(\theta, x) + g(\theta, x)\alpha(x)] \leqslant -c(x) < 0 \quad \forall x \neq 0$$

for some continuous function $c(\cdot)$.

[¶]Both functions are independent of θ.

Define the *semi-adaptive* control law

$$u = \alpha(x) - f(\hat{\theta}, x) - k\mu(y) \tag{10}$$

where the parameters are updated with the estimator

$$\dot{\hat{\theta}} = \Gamma \nabla_{\hat{\theta}}^{\mathrm{T}} f(\hat{\theta}, x)[y - \mu(y)] \tag{11}$$

with $\Gamma = \Gamma^{\mathrm{T}} > 0$, $\mu(y)$ is a switching function

$$\mu(y) = \begin{cases} 0 & \text{for } y(x) \leqslant 0 \\ y(x) & \text{for } y(x) > 0 \end{cases} \tag{12}$$

with the switching surface $\mathscr{S}$ defined as $\mathscr{S} \triangleq \{x : y(x) = 0\}$ and

$$y(x) \triangleq (\nabla_x W) g(x)$$

Then, for any $\varepsilon > 0$ and any bounded set of initial conditions $\mathscr{D} \triangleq \{(x(0), \hat{\theta}(0))\}$ there exists a constant gain $\bar{k}(\varepsilon, \mathscr{D}) > 0$ such that for any $k > \bar{k}$ all trajectories of the closed-loop system (9)–(12) are bounded. Furthermore, their limit set is contained in the set

$$\mathscr{R}_\varepsilon \triangleq \{x : y(x) \geqslant 0, |x| \leqslant \varepsilon\}$$

Proof. First, using A.1, A.2 and the arguments of Section 2, we can prove that the standard separable Lyapunov function candidate

$$V(x, \hat{\theta}) = W(x) + \tfrac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}$$

satisfies $\dot{V} \leqslant 0$ when $y \leqslant 0$. For $y > 0$ adaptation is frozen and we have

$$\dot{V} \leqslant -c(x) + y[f - f(\hat{\theta}_F, x) - ky]$$

where $\hat{\theta}_F$ are the frozen values of the estimates. Now, specify some bounded region of initial conditions $\mathscr{D}$ and find V_0 such that $\mathscr{D} \subset \mathscr{D}_0 \triangleq \{(x, \hat{\theta}) : V(x, \hat{\theta}) \leqslant V_0\}$. Let $\kappa \triangleq \sup_{\mathscr{D}_0} |f - f(\hat{\theta}_F, x)|$ and

$$\delta(\varepsilon) \triangleq \inf_{\substack{x \in \mathscr{D}_0 \\ |x| \geqslant \varepsilon}} c(x)$$

Then, for $x \in \mathscr{D}_0$, $|x| \geqslant \varepsilon$ we have

$$\begin{aligned} \dot{V} &\leqslant -\delta(\varepsilon) + \kappa y - ky^2 \\ &\leqslant -\delta(\varepsilon) + \frac{\kappa^2}{4k} \end{aligned}$$

and for $k > \kappa^2/4\delta(\varepsilon)$ we get $\dot{V} < 0$ for $(x, \hat{\theta}) \in \mathscr{D}_0$, $|x| \geqslant \varepsilon$. Therefore, all the trajectories of the system are bounded and the limit set is contained in the set $\mathscr{D}_0 \cap \mathscr{R}_\varepsilon$. □

Remark 4.1

The adaptation law (11) for $y \leqslant 0$ is just the *speed gradient* law of References [1, 2], which can be rewritten as

$$\dot{\hat{\theta}} = -\Gamma \nabla_{\hat{\theta}}^{\mathrm{T}} \dot{W}(x, \hat{\theta})$$

with $\dot{W}$ the derivative of $W(x)$ along the trajectories of the system. That is, the parameters are searched in the direction of the negative of the gradient of the *speed* of the Lyapunov function. This feature is the main characteristic of speed gradient methods.

Remark 4.2

It is clear that the statement of Proposition 3.1 applies as well to plants of form (9), by simply taking (1) as a particular case of the latter.

Remark 4.3

For simplicity, we have chosen (9) to illustrate the idea of semi-adaptive control. The result applies *mutatis mutandis* to other classes of plants. Also, we can replace the simple proportional controller with other kind of robust laws, e.g., the one proposed in Reference [9]. Finally, it is easy to see that we do not need to freeze the adaptation, provided we can dominate the sign indefinite terms with a relay action—like in Reference [6].

5. TEMPERATURE REGULATION OF CHEMICAL REACTORS

We will apply now our semi-adaptive controller to the class of continuous stirred tank reactors (CSTR) studied in Reference [15]

$$\dot{x}_0 = Cr(x, \theta) + d(x_0^{in} - x_0) \tag{13}$$

$$\dot{x}_n = h^{\mathrm{T}} r(x, \theta) - q x_n + u \tag{14}$$

where $x_0 \triangleq [x_1, \ldots, x_{n-1}]^{\mathrm{T}} \in \mathscr{R}^{(n-1)}$ is a vector of concentrations, $x_0^{in} \in \mathscr{R}^{(n-1)}$ are the *constant* feed concentrations, $x_n \in \mathscr{R}$ is the reactor temperature to be controlled, $r(x, \theta) \in \mathscr{R}^m$ are the reaction kinetics, $h \triangleq [h_1, \ldots, h_m]^{\mathrm{T}}$ are the reaction heats, $\theta \in \mathscr{R}^q$ is a vector of *unknown* parameters, d, q are positive constants (dilution rate and heat transfer coefficient, respectively), $C \in \mathscr{R}^{(n-1) \times m}$ is the stochiometric matrix, and u, the manipulated heat, is the control input. Eventhough in practice the control signal is limited to $u \geqslant 0$ we will not consider this restriction here. As explained in Remark 5.1, some techniques to handle this hard constraint may be incorporated to our scheme. We will assume the state $x \triangleq [x_0^{\mathrm{T}}, x_n]^{\mathrm{T}}$ is measurable.

The control *objective* is to drive the temperature to a small neighbourhood of some constant reference $x_n^* \geqslant 0$. The latter is such that

$$h^{\mathrm{T}} r(x_0, x_n^* \theta) - q x_n^* < 0 \tag{15}$$

for all $x_i \geqslant 0$, $i = 1, \ldots, (n-1)$. This is a physically motivated feasibility condition which ensures the existence of a non-negative steady-state control corresponding to the temperature set point x_n^* (cf., hypothesis H5 in Reference [15]).

In Reference [15] it is shown that if the matrix C satisfies a (physically reasonable) mass conservation principle then x_0 is bounded,[‖] independently x_n and u.

Similar to Reference [15], we will also assume that the isothermal dynamics

$$\dot{x}_0 = Cr(x_0, x_n^*, \theta) + d(x_0^{in} - x_0)$$

have a unique equilibrium point $\bar{x}_0$ which is globally asymptotically stable.

As explained in Reference [15], the reaction rate functions $r_i(x, \theta)$ are of the form

$$r_i(x, \theta) = K_i(x_n, \theta)\phi_i(x_0, \theta) \tag{16}$$

where $K_i(x_n, \theta)$, $\phi_i(x_0, \theta)$ are positive scalar functions of their arguments. Some standard forms for these functions include the Arrhenius law

$$K_i(x_n, \theta) = \theta_{1i} \mathbf{e}^{-\theta_{2i}/x_n}$$

and polynomial fractions. It is interesting to note that both classes of functions posses some *convexity* properties. For exothermic reactions, the reaction heats h_i are positive. Since positive linear combinations (weighted sums) of convex functions are still convex [16], the problem is a suitable candidate for our semi-adaptive scheme.

We have the following proposition.

Proposition 5.1

Consider the *exothermic* CSTR (13)–(16) in closed loop with

$$u = q x_n - [k_1 + k\mu(y)]y - h^{\mathrm{T}} r(x, \hat{\theta})$$

where $y \triangleq x_n - x_n^*$ is the temperature error, $k_1 > 0$, and the parameters are updated with the estimator

$$\dot{\hat{\theta}} = \Gamma \nabla_{\hat{\theta}} h^{\mathrm{T}} r(x, \hat{\theta})[y - \mu(y)]$$

‖Actually, it is also shown that the evolution of x_0 is restricted to the positive orthant.

with $\Gamma = \Gamma^{\mathrm{T}} > 0$, and

$$\mu(y) = \begin{cases} 0 & \text{for } y \leqslant 0 \\ y & \text{for } y > 0 \end{cases}$$

Assume $r_i(x, \theta)$, $i = 1, \ldots, m$ are convex on θ.

Under these conditions, we have

$$\liminf_{t \to \infty} y(t) \geqslant 0$$

Furthermore, for any $\varepsilon > 0$ and any bounded set of initial conditions $\mathscr{D} \triangleq \{x(0), \hat{\theta}(0)\}$ there exists a constant gain $\bar{k}(\varepsilon, \mathscr{D}) > 0$ such that for any $k > \bar{k}$ all trajectories of the closed-loop system are bounded.

$$\limsup_{t \to \infty} y(t) \leqslant \varepsilon$$

Proof. The proof follows immediately from Proposition 4.1. First, assumption A.1 follows from the condition of convexity of $r_i(x, \theta)$, $i = 1, \ldots, m$. Second, in view of the assumption on C and the stability of the isothermal dynamics mentioned above, we know an α such that assumption A.2 holds. For instance, we can take $\alpha(x) = -qx_n - k_1 y$. We can, finally, invoke a standard converse Lyapunov theorem, to prove that $W(x_0) + \frac{1}{2}y^2$ qualifies as a Lyapunov function for the system without uncertainty. □

Remark 5.1.

A statement on positivity of u, similar to Theorem 3.2 of Reference [15], is possible if we assume some further prior knowledge on the plant. Namely, we can prove the existence of some $\bar{x}_n^* > x_n^*$ such that $u(x_0, x_n, \hat{\theta}) \geqslant 0$ on $\mathscr{R}^{(n-1)} \times (0, x_n^*] \times \Omega$, provided we assume a known compact set Ω for the uncertain parameters such that

$$\Omega \triangleq \{\theta : h^{\mathrm{T}} r(x_0, x_n^*, \theta) - q x_n^* \leqslant 0\}$$

then adding a projection to the estimator that ensures $\hat{\theta}(t) \in \Omega$ for all $t \geqslant 0$. For instance, we can assume the parameters leave in hypercubes, and use the bounding techniques developed in Reference [6] to estimate this set. We refer the reader to this paper for further details on the actual construction of the embedding set and the description of the projections that might be implemented.

6. CONCLUSIONS

In this paper we have investigated the limitations of the tools we have used for linearly parametrized systems, namely the use of separable Lyapunov functions first advocated by Parks [3], as applied to the far more challenging (but still very restrictive) case of convex parametrizations. The outcome of this study is a semi-adaptive scheme, whose main ingredient is the definition of regions in state space where parameter uncertainty can be reduced. Lack of a better proposal, we freeze the adaptation and switch to a robust scheme outside these regions. Eventhough we have restricted ourselves here to a very simple class of non-linear systems the scheme applies, *mutatis mutandis*, to the class of systems currently considered in the literature. An alternative to our proposal, which has been recently proposed in Reference [6], is to leave the adaptation all the time but add a (high-gain) relay action to dominate the uncertain terms. This clearly requires some additional prior knowledge on the systems parameters.

Many open questions remain to be answered, and little clues are available in the literature. The negative results reported here seem to suggest that additional prior knowledge should be incorporate to achieve a fully adaptive design. Further investigations on physical parametrizations that enjoy the convexity property is required. This opens a new line of research for parameter projections techniques, which is more physically motivated. In a recent paper [7] we characterized a class of non-linearly parametrized systems that can be *convexified* via reparametrization (without overparametrization).

We have explored here the classical separable Lyapunov function technique. An interesting question is whether cross-terms could provide some additional flexibility. For a class of linear systems the answer reported in Theorem 3.6 of Reference [2] is, again, negative. It is not clear at this point whether this is the case also for other linear systems, e.g. model reference. Encouraging results along this lines have been reported in Reference [14].

It is the authors' opinion that the non-linear parametrization problem is the main stumbling block in the research of adaptive neural nets, see Reference [4]. The flurry of activity on this area has already provided some interesting partial answers [17, 11].

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