Green Growth or low growth: Modelling the balanced transition to a sustainable economy

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We present a simple mathematical model for the transition to a sustainable economy in the line proposed by Peter Victor [Victor and Rosenbluth, 2007] and Tim Jackson [Jackson, 2009]. The modelling approach is in the continuation of the "Limits to Growth" of [Meadows et al., 1972, 2004] which have emphasized the unsustainable character of the current economic trend and the necessity of a major change in the economic structure and the consumption behaviour. The "Limits to Growth" projections are confirmed by [Turner, 2008] in his recent comparison with empirical data.

Some authors (e.g. [Spash, 2012]) have expressed their doubts as to the possibility of correctly analysing the sustainable transition with the toolbox of mainstream economics and ask for the development of an epistemological questioning. Although we totally agree with the relevance of the epistemological issue, we believe that the current debate may be clarified by looking more closely into the potential and the limits of the neo-classical formalism for the understanding of the sustainability transition. In this paper we intend to set some preliminary basis for further critical discussion.

The model is build to assess public policies to attain sustainability. Our ultimate objective is to use the model to explore long-run evolution of an economy that achieves limitation of atmospheric greenhouse gas, environment protection and full recycling of material resources with high public investment. However in this communication, we restrict the focus on greenhouse gas limitations, and more precisely on carbon dioxide (CO_2) which is the major contributor to greenhouse gas emissions. The issues of material resources and environment protection are addressed in a companion paper.

The model is a conceptual representation of a "decentralized economy" (see e.g. [Wickens, 2008, Chapter 5]) where the decisions of producers, consumers and government are distinguished. In order to address the objectives mentioned above, the model involves the main economic and environmental variables that are essential for analyzing a sustainable economy. In addition to standard macroeconomic variables (such as production, consumption, investment, capital and labour), we therefore also consider environmental variables (such as CO_2 emissions and atmospheric CO_2 concentration). As it is usual in macroeconomic modeling, the model consists essentially of "flow balance equations" that combine aggregate stock variables and flow functions.

We restrict our attention to balanced economic paths. Our main concern is to investigate how balanced paths are modified under public policies for transition to sustainability. The reason for restricting to balanced paths is to have consistent models that are as simple and flexible as possible. Simple to be easily implemented, even by users who are not familiar with the use of optimal control methods in neo-classical economic theory. Flexible to easily include extensions like subregions, economic subsectors or explicit fiscal policies.

We define a fictional pseudo-world economy with two subregions that are endowed with the CO_2 emissions of OECD and non-OECD countries respectively. Then, for the OECD subregion, we examine two major options towards sustainability: the "Green Growth" option and the "Low Growth" option. In the green growth option, it is believed that the greenhouse gas emissions will be limited by developing public novel technical innovations without changing the final output nor the economic structure. In contrast, the low growth option aims at developing zero or low carbon emission activities without having to rely on major discoveries of new green technologies, which results in structural change and lower growth.

The paper is organized as follows. The baseline system is presented in Section 1. It is a simple single-sector economy with a standard neo-classical production function in capital and labour. The system is supposed to follow a balanced trajectory along which the marginal product of capital and the output-capital ratio are constant. In Section 2, we set up a benchmark numerical model which is initialized with orders of magnitude corresponding to the state of OECD economy during the period 1998-2008 and which is consistent with the empirical data. Section 3 is devoted to modelling of CO_2 intensity and to the quantitative estimation of the relative decoupling between GDP growth and CO_2 emissions. For the simulations, the model equations are solved with Matlab-Simulink. A first "business as usual" simulation experiment is presented in Section 4. In this simulation, the economy continues to follow its current trend and makes the planet reaching unsupportable CO_2 atmospheric concentrations at the end of the century. Section 5 deals with the green growth public policy. The baseline system is extended with a sector producing green technical knowledge. The investment in this sector is assumed to increase proportionally to the excess of CO_2 emissions. The simulation result shows how the investment policy in public green technologies stabilizes the atmospheric CO_2 concentration at the value of 450 ppm (recommended by IPCC) with a public cost in the range 2-8 % of GDP. Finally, in Section 6, we examine how the transition to sustainability may be achieved with a low-growth public policy that consists in fostering the development activities with low or zero carbon intensity, with the results of low productivity growth and structural change. For this purpose, we consider an economy with two sectors: a conventional sector endowed with the economic features of the baseline system and a transition sector of activities having zero carbon intensity and constant labour productivity. In the presented simulation results, the emphasis is on the progressive reallocation of capital and labour between the two sectors in order to reach sustainability.

1. The baseline system

We consider an economy where the aggregate production flow Y of the final goods is represented by a standard Cobb-Douglas function

$$Y = AK^{(1-\alpha)}L^{\alpha} \tag{1}$$

whith K the stock of physical capital for production, L the amount of labor used for production and A the productivity coefficient corresponding to the level of technical knowledge in the economy. The constant parameter $\alpha \in (0, 1)$ is the output elasticity of labour.

The dynamics of the capital stock K are represented by a standard balance equation

$$\frac{dK}{dt} = -\delta K + I \tag{2}$$

where I is the aggregate investment allocated to the production of the final goods. The constant parameter $\delta \in (0, 1)$ is the capital depreciation rate.

The model is completed with the equilibrium condition

$$Y = C + I + X - M \tag{3}$$

where C, X and M denote consumption, export and import flows respectively. For simplicity, we do not distinguish between the flows stemming from the private and public sectors.

In addition, we assume that the total labour L is varying over time according to the dynamics

$$\frac{dL}{dt} = \mu(t)L \tag{4}$$

where the specific evolution rate $\mu(t)$ is a time-varying exogenous variable.

The marginal product of capital is

$$r(t) = \frac{\partial Y(t)}{\partial K(t)} = (1-\alpha)A(t)\left(\frac{L(t)}{K(t)}\right)^{\alpha} = (1-\alpha)\frac{Y(t)}{K(t)} \tag{5}$$

and the marginal product of labour is

$$w(t) = \frac{\partial Y(t)}{\partial L(t)} = \alpha A(t)\left(\frac{K(t)}{L(t)}\right)^{1-\alpha} = \alpha\frac{Y(t)}{L(t)}. \tag{6}$$

We introduce the following notations for the specific growth rates of capital and output:

$$g_K(t) = \frac{1}{K}\frac{dK}{dt}, \quad g_Y(t) = \frac{1}{Y}\frac{dY}{dt}. \tag{7}$$

Then, from (1)-(2)-(3)-(5)-(7), we have:

$$Y(t) - I(t) = C(t) + X(t) - M(t) = K(t)\left(\frac{r(t)}{1-\alpha} - (g_K(t) + \delta)\right) \tag{8}$$

and

$$\frac{1}{r}\frac{dr}{dt} = (g_Y(t) - g_K(t)) = \frac{1}{A}\frac{dA}{dt} - \alpha\Big(g_K(t) - \mu(t)\Big). \tag{9}$$

A balanced path is defined as the special case where the marginal product of capital (and consequently the output-capital ratio) are constant:

$$r(t) = (1-\alpha)\frac{Y(t)}{K(t)} = \text{ constant } \quad \Longrightarrow \quad g_K(t) = g_Y(t) = g(t) \;\; \forall t. \tag{10}$$

Along a balanced path, the evolution rates of technical level A and marginal product of labour w are:

$$\frac{1}{A}\frac{dA}{dt} = \alpha\Big(g(t) - \mu(t)\Big) = \gamma(t) \qquad \frac{1}{w}\frac{dw}{dt} = \frac{\gamma(t)}{\alpha}. \tag{11}$$

2. Identification of a benchmark numerical model

The setting of a numerical simulation model requires to select parameter values and to specify initial conditions. The year is taken as the time unit. Numerical values of the parameters α (output elasticity of labour) and δ (capital depreciation rate) in the intervals

$$\alpha \in [0.50, 0.80], \qquad \delta \in [0.05, 0.11],$$

are widely accepted as relevant in the literature. In footnote, we give a set of references[1] where such values are proposed and, in certain cases, validated from empirical data. We will use the central value of each interval in our simulations: $\alpha = 0.65$, $\delta = 0.08$.

In order to get simulation results having a realistic flavour, we set up a benchmark model which is initialized with orders of magnitude corresponding to the state of OECD economy during the period 1998 - 2008. The evolution of GDP, consumption, employment, imports and exports during this period are shown in Fig.1-2-3-4. Employment is taken as the measurement of labour L.

These empirical data show small economic fluctuations around an exponential path (represented by the red curves fitted on the data) which is assumed to be a balanced path. From the empirical data of Fig.1, the following least-squares estimate is computed:

$$\frac{1}{Y}\frac{dY}{dt} = g \simeq 0.0281. \tag{12}$$

From the data of Fig.3, we have

$$\frac{1}{L}\frac{dL}{dt} = \mu \simeq 0.011 \tag{13}$$

and therefore

$$\frac{1}{A}\frac{dA}{dt} = \alpha(g - \mu) = \gamma \simeq 0.0091. \tag{14}$$

It is assumed that imports equal exports $X(t) = M(t)\ \forall t$ along the balanced path, see Fig.4. The initial time (t = 0) for the numerical simulations of the benchmark economy is the year 2000. On the balanced path represented in Fig.2-3-4, we have directly the initial values

$$Y(0) = 31.60, \quad C(0) = 24.96, \quad L(0) = 495, \quad X(0) = M(0) = 6.8.$$

Therefore

$$K(0) = \frac{Y(0) - C(0)}{g + \delta} = 63.24 \text{ and } r = (1 - \alpha)\frac{Y(0)}{K(0)} = 0.174.$$

From (5), we derive

$$A(0) = \frac{r}{1 - \alpha}\left(\frac{K(0)}{L(0)}\right)^{\alpha} = 0.13 \tag{15}$$

The initial values are collected in Table 1 where the corresponding units are also given.

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[1] [Blanchard and Galli, 2006], [Bodart et al., 2006], [Boucekhine and Ruiz-Tamarit, 2008], [Bréchet et al., 2011], [Gaitan and Roe, 2012], [Gapen and Cosimano, 2005], [King et al., 1988], [Lucas, 1988], [Victor and Rosenbluth, 2007].

Variable		Value	Units
Capital	K(0)	64.21	T US dollars
Labour	L(0)	495	millions of people
Output rate	Y(0)	31.60	T US dollars/year
Consumption	C(0)	24.96	T US dollars/year
Technical level	A(0)	0.13	$(\text{T US \$})^{\alpha}/(\text{million people})^{\alpha} \times$year
CO_2 Emissions	E(0)	12.55	GT CO_2/year
CO_2 Intensity	h(0)	0.4	kg CO_2/US $
Capital rental rate	r	0.1716	1/year

Table 1: Initial conditions of the balanced path for the year 2000

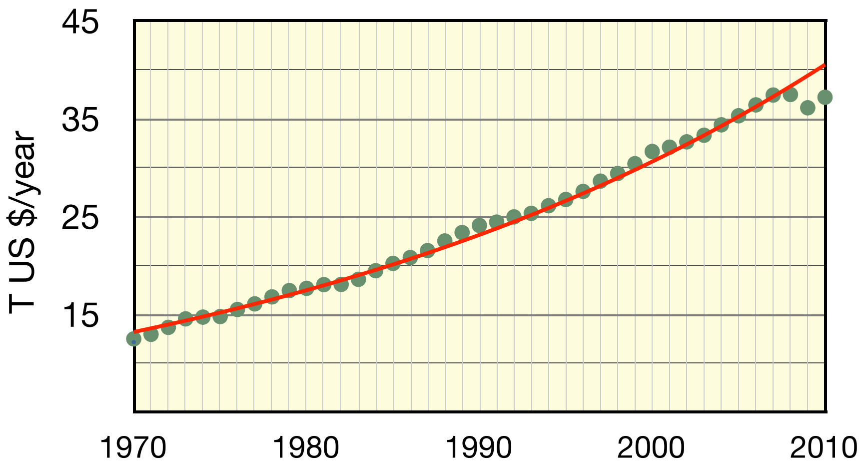

Fig.1: GDP in OECD from 1970 to 2010 (constant prices 2005). The green dots are empirical data from stat.oecd.org. The red curve represents a superimposed LS estimate of the balanced path.

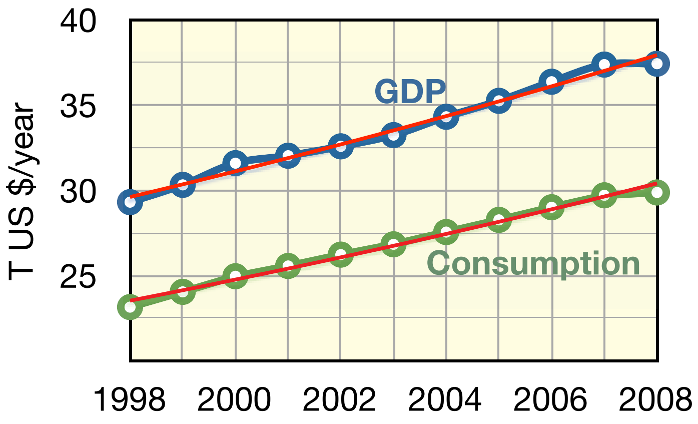

Fig.2: GDP and Consumption in OECD from 1998 to 2010 (constant prices 2005). The dots are annual empirical data from stat.oecd.org. The red curves represent superimposed LS estimates of the balanced path.

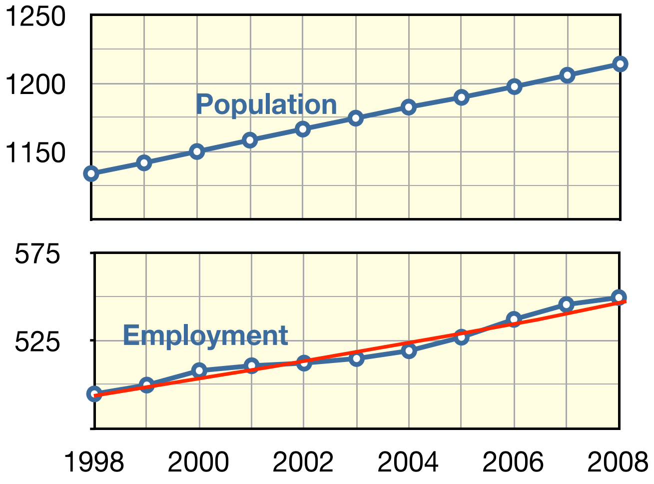

Fig.3: Population and Employment in OECD from 1998 to 2008 (millions of people). The dots are annual empirical data from stat.oecd.org. The red curves represent superimposed LS estimates of the balanced path.

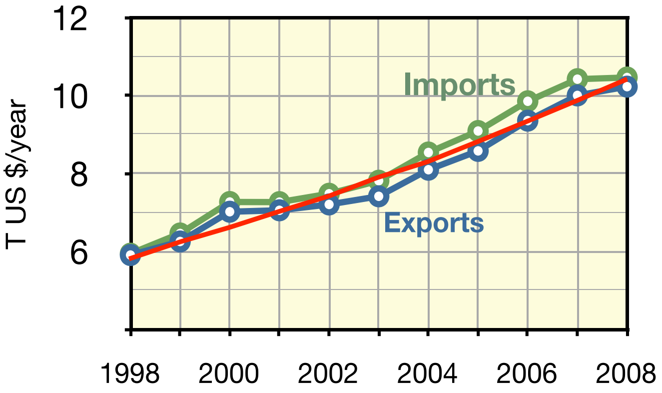

Fig.4: Imports and Exports in OECD from 1998 to 2010 (constant prices 2005). The dots are annual empirical data from stat.oecd.org. The red curve represents the superimposed LS estimate of the balanced path.

3. Carbon dioxide dynamics

As in [Nordhaus, 2008], we assume that CO_2 emissions are representative of total GHG emissions. The flux balance equation for atmospheric CO_2 is written:

$$\frac{d}{dt}\Delta_C = \kappa_0\Big(E_w - q(\Delta_C)\Big) \text{ with } \Delta_C = [CO_2] - [CO_2]_p, \tag{16}$$

where $[CO_2]$ is the average concentration of atmospheric CO_2, $[CO_2]_p$ is the natural pre-industrial atmospheric CO_2 concentration, E_w is the flow of CO_2 emissions into the atmosphere from world human economic activities, $q(\Delta_{CO_2})$ is a monotone increasing function representing the natural planet absorption rate of CO_2 and κ_0 is a constant coefficient.

The CO_2 emission rates for OECD and non-OECD countries during the period 1970-2008 are shown in Fig.5. In non-OECD countries CO_2 emissions are steadily increasing proportionally

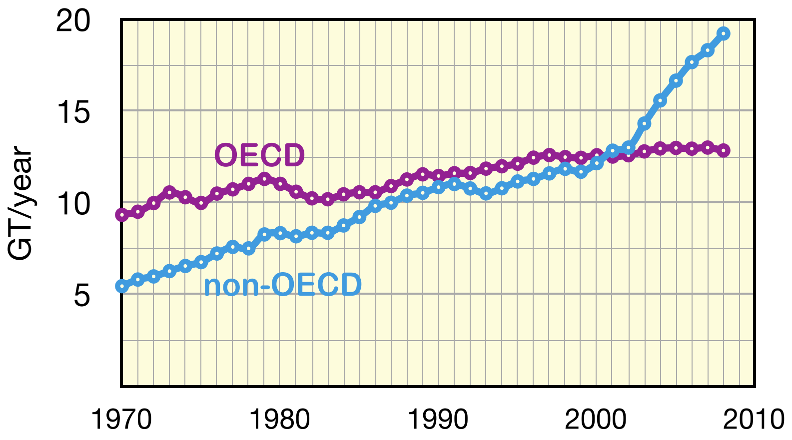

Fig.5: CO_2 emissions (Data from World Bank Development Indicators.)

to GDP. In contrast, the increase of CO_2 emissions is much slower in OECD countries and even almost zero over the last ten years. Assuming that the CO_2 emissions are related to the economic production, there is no loss of generality in writing

$$E(t) = h(t)Y(t) \tag{17}$$

where $E(t)$ is the CO_2 emission flow and $h(t)$ is the carbon intensity of the economic production $Y(t)$. The OECD empirical data for $h(t)$ are shown in Fig.6 and the following exponentially decreasing function can be fitted on the data:

$$h(t) = h(0)e^{-\varepsilon t} \text{ kg } CO_2/US\$ \text{ with } \varepsilon \simeq 0.0208. \tag{18}$$

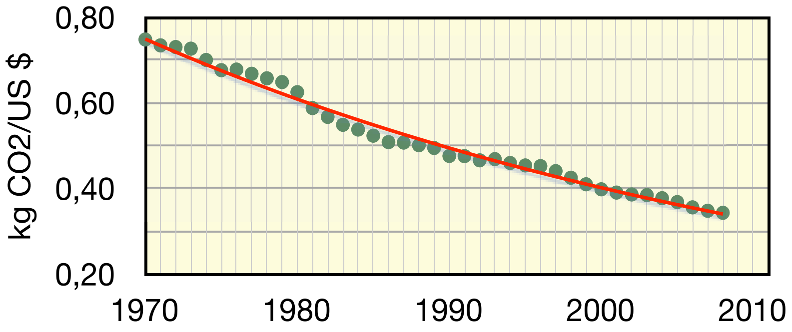

Fig.6: CO_2 intensity in OECD countries computed with data from Fig.1 and Fig.5. (The exponential function $h(t)$ is represented by the red line.)

By differentiating equation (17), we obtain:

$$\frac{dE}{dt} = \frac{dh}{dt}Y + h\frac{dY}{dt} = \left(\frac{1}{Y}\frac{dY}{dt} + \frac{1}{h}\frac{dh}{dt}\right)E = (g_Y(t) - \varepsilon)\,E. \tag{19}$$

An important point here is obviously that $\varepsilon \simeq 0.0208 < g_Y \simeq 0.0281$ which means that the efficiency of CO_2 abatement is not sufficient to compensate for GDP growth: the decoupling between growth and greenhouse gas emissions is relative but not absolute ([Jackson, 2009, p.53]).

The famous Keeling's curve is shown in Fig.7. It represents the accumulation of atmospheric CO_2 during the last 50 years. It is generally accepted that the net CO_2 inflow rate in the atmosphere is about 60% of the total emissions. Assuming a linear CO_2 absorption function

$$q(\Delta_C) = \kappa_1 \Big([CO_2] - [CO_2]_p \Big) \tag{20}$$

with $[CO_2]_p$ = 280 ppm, and using the data of Fig.5 and Fig.7, we can estimate the parameter values $\kappa_0 \simeq 0.17\,\mathrm{ppm/GT}$ and $\kappa_1 \simeq 0.18\,\mathrm{GT/ppm \times year}$.

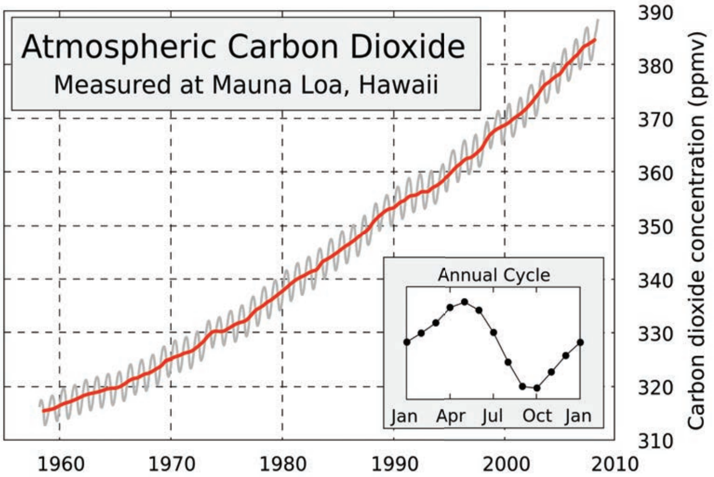

Fig.7: The Keeling's Curve of atmospheric CO_2 concentrations measured at the Mauna Loa Observatory. Source: Wikipedia.

4. First simulation : Business as Usual

In this first simulation, we assume that the economy continues to follow the balanced path that we have identified above. The balanced path is a solution of the following set of state space equations:

$$\frac{dL}{dt} = \mu(t)L, \tag{21}$$

$$\frac{dA}{dt} = \gamma(t)A, \tag{22}$$

$$\frac{dK}{dt} = \left(\frac{\gamma(t)}{\alpha} + \mu(t) \right) K, \tag{23}$$

$$\frac{dE}{dt} = \left(\frac{\gamma(t)}{\alpha} + \mu(t) - \varepsilon \right) E \tag{24}$$

and

$$Y = AK^{(1-\alpha)}L^{\alpha}, \tag{25}$$

$$I = \left(\frac{\gamma(t)}{\alpha} + \delta + \mu(t) \right) K, \tag{26}$$

$$C = Y - I, \tag{27}$$

$$r = \frac{(1-\alpha)Y}{K}. \tag{28}$$

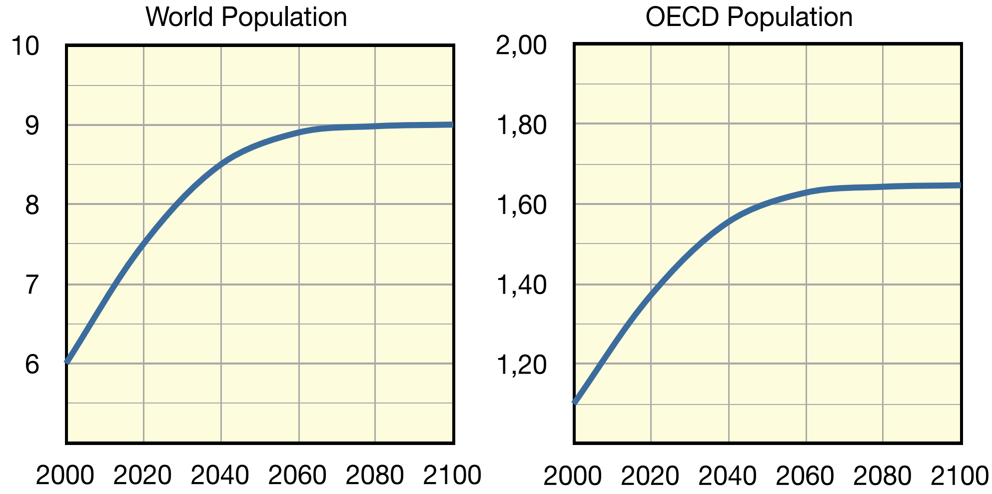

Fig.8: Evolution of the population from 2000 to 2100 (milliards of people) in the benchmark model.

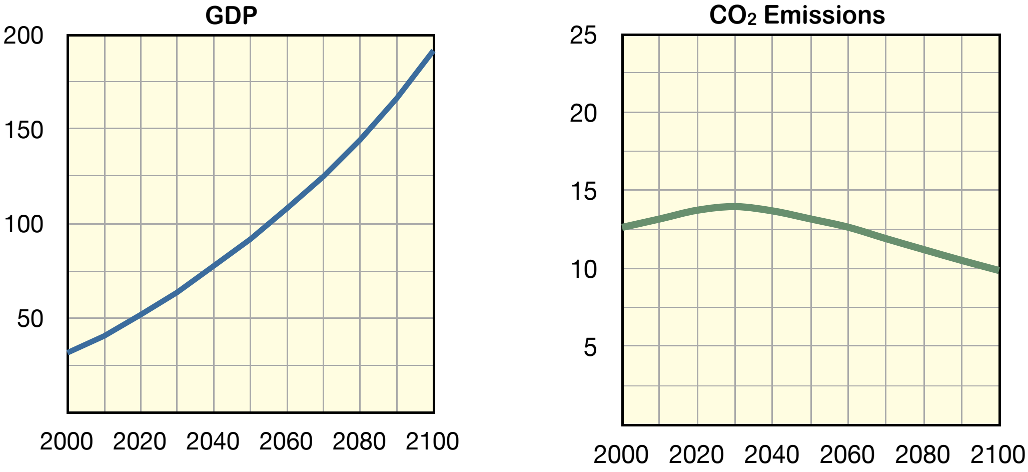

Fig.9: Business as usual. Left: GDP (T US $/year); Right: CO_2 emissions (GT/year).

In this model, the population growth rate $\mu(t)$ and the technology growth rate $\gamma(t)$ are exogenous variables. For the population dynamics we adopt the medium prediction of the United Nations (see [UN, 2004]) such that the population increases until about 2050 and then stabilizes for a while as shown in Fig.8. The exogenous specific growth rate $\mu(t)$ is computed accordingly. The employment is supposed to be a constant fraction of the population. For the technology, we assume a constant growth rate $\gamma = 0.0091$ as computed in Section 3. The model is initialized in 2000 with the values of Table 1. The model equations are encoded in Matlab-Simulink.

The results of the simulation experiment are illustrated in Fig.9. As it can be expected, the economy keeps growing exponentially and does not significantly reduce the level of CO_2 emissions. There is a slight reduction during the second half of the century which is due to the conjugate effects of population stabilization and carbon intensity decrease. But, at the end of the century, the CO_2 emission per capita is about 6T/year. Extended to the whole planet, such an emission rate per capita would make the CO_2 atmospheric concentration reaching unsupportable values in 2100 (over 800 ppm, see e.g.[Nordhaus, 2010]) whereas we know that the supportable limit is generally considered to be at most 450 ppm (see e.g. IPCC reports).

5. Green Growth

Despite the capitalist propension to efficiency and despite a significant decrease of carbon intensity (50% since 1970), it can clearly be suspected from the results of the previous section that the current economic trend will not succeed in reaching a sustainable economy. Vigorous new public policies are most probably needed to modify this trend in the desired direction. In this section, we investigate a so-called "green growth" public policy. For this purpose we extend the model by introducing the additional assumption that a share of the total investment is funded by the government and explicitly allocated to the development of "Novel Green Technical Knowledge". These innovations are pure public goods that are both non-rival and non-excludable. In other words they are freely made available to all producers in order to further reduce the greenhouse gas emissions.

Therefore, we now consider an economy with two sectors:

1) A conventional sector with production function

$$Y_{cs} = AK_{cs}^{(1-\alpha)}L_{cs}^{\alpha}, \tag{29}$$

which is endowed with the dynamics, the parameter values and the initial conditions of the benchmark model of the previous section.

2) A "green technology" sector that produces the public green technical knowledge denoted H. The production flow of H is represented by a Cobb-Douglas production function:

$$\frac{dH}{dt} = Y_{gs} = AK_{gs}^{(1-\alpha)}L_{gs}^{\alpha}. \tag{30}$$

where K_{gs} and L_{gs} are the physical capital and the labor allocated to the public research in green technical knowledge. The dynamics of the capital stock K_{gs} is represented by the equation

$$\frac{dK_{gs}}{dt} = -\delta K_{gs} + I_{gs}, \tag{31}$$

where I_{gs} denotes the green investment.

For simplicity, we assume that the two sectors have identical production functions, but this could be relaxed to some degree. The two sectors are aggregated by defining the total capital $K = K_{cs} + K_{gs}$, the total investment $I = I_{cs} + I_{gs}$ and the total output $Y = Y_{cs} + Y_{gs}$. It is then readily checked that:

$$\frac{dK}{dt} = -\delta K + I. \tag{32}$$

We consider equilibrium economic paths with competitive factor markets. This implies that, along the economic path, the marginal product of capital is identical in the two sectors:

$$r(t) = \frac{\partial Y_{cs}(t)}{\partial K_{cs}(t)} = \frac{\partial Y_{gs}(t)}{\partial K_{gs}(t)} \quad \forall t. \tag{33}$$

Using equations (29)-(30), we have

$$r(t) = (1-\alpha)A(t)\left(\frac{L_{cs}(t)}{K_{cs}(t)}\right)^{\alpha} = (1-\alpha)\frac{Y_{cs}(t)}{K_{cs}(t)} \tag{34}$$

$$= (1-\alpha)A(t)\left(\frac{L_{gs}(t)}{K_{gs}(t)}\right)^{\alpha} = (1-\alpha)\frac{Y_{gs}(t)}{K_{gs}(t)}. \tag{35}$$

$$= (1-\alpha)A(t)\left(\frac{L(t)}{K(t)}\right)^{\alpha} = (1-\alpha)\frac{Y(t)}{K(t)} \tag{36}$$

These equations mean that, along the path, the product-capital and the labour-capital ratios are identical in the two sectors. This also implies that the total output obeys a global Cobb-Douglas function

$$Y = Y_{cs} + Y_{gs} = AK^{(1-\alpha)}L^{\alpha} \tag{37}$$

and therefore that the structure of the economy is not modified with respect to business as usual. But the nature of the production is different since the representative output Y is now partly composed of the public green knowledge Y_{gs} (in addition to the current private green technologies that are already incorporated in the conventional production).

Let us now turn to the issue of the sustainable transition. Concerning greenhouse gas, we assume that the objective of the transition to a sustainable economy is to guarantee a **constant** CO_2 **atmospheric concentration at the level of 450 ppm with equitable emissions all over the planet.** This can be achieved with steady-state total world emissions:

$$E_w^* = \kappa_1 \times \Delta[CO_2] = 0.18 \times (450 - 280) = 30.6 \text{ GT/Year}.$$

As it can be observed from the data of Fig.5, this value is almost equal to the present level of world emissions (in 2008), with about 42% for OECD and 58% for the rest of the world. Therefore, the sustainable challenge is not to decrease the global emissions with respect to the present situation. The goal is rather to maintain the emission level at its present value while ensuring progressively a fair distribution with the same emissions per capita everywhere in the world. Obviously this implies strongly reducing the OECD emissions while still allowing for a moderate increase in non-OECD countries. Since the ratio of OECD to world population is 0.183, the target for OECD emissions in 2100 must be (at most)

$$E^* = 0.183 \times E_w^* = 0.183 \times 30.6 = 5.61 \text{ GT/Year}.$$

In order to achieve this goal, the model of CO_2 emissions is extended to incorporate the effect of green technologies as follows:

$$E = h(t)Y(t)e^{-\eta H(t)}.$$

With this model we thus now assume that E is not only linearly increasing with final output production as above but also exponentially decreasing with the level of public green technical knowledge H. The parameter η is an elasticity coefficient. The function $h(t)$ is given by expression (18) and represents the current private decrease of CO_2 intensity.

As above, a balanced path is defined as the special case where $r(t)$ is constant. Along a balanced path, the dynamics that connect the public green investment to the CO_2 emissions are then given by the two equations

$$\frac{dE}{dt} = E\left(\frac{\gamma}{\alpha} + \mu(t) - \varepsilon - \eta\frac{r}{1-\alpha}K_{gs}\right) \tag{38}$$

$$\frac{dK_{gs}}{dt} = -\delta K_{gs} + I_{gs}. \tag{39}$$

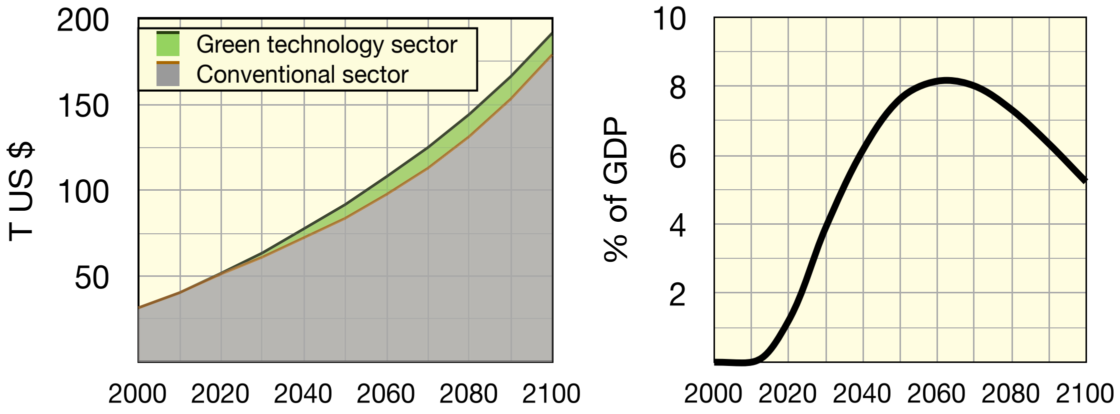

Fig.10: Green growth in the OECD benchmark model. Left: GDP; Right: Public cost of green growth as a percentage of GDP.

Equation (38) is a modification of (19) which accounts for the influence of H. Equation (39) is identical to (31). Obviously the elasticity η is a key parameter in this model since it determines how much can be achieved in CO_2 abatement per unit of time with a given investment. The answer to this question has given rise to an abundant literature but is still, nevertheless, a widely open question. Depending of the assumptions, the estimates of the cost of achieving 50% reduction in CO_2 emissions in 2050 span a very wide range, from 1% to 8% of GDP. In our simulation, we set $\eta = 0.002$ which provides a cost in this range. All the other constant parameters needed for the simulation have been given previously. The model is initialized in 2000 with the values of Table 1 for the conventional sector and with zero initial conditions for the green technology sector. In order to achieve the goal of CO_2 abatement, an endogenous feedback investment policy is applied to the system from 2014. The public green investment I_{gs} is simply assumed to change proportionally to the excess of CO_2 emissions with respect to the target E^*:

$$\frac{dI_{gs}}{dt} = \theta\,(E - E^*). \tag{40}$$

The constant parameter θ is adjusted by trial and error at the value $\theta = 0.003$.

The result of the simulation experiment is illustrated in Fig.10. It must be clearly understood that, in this result, the conventional sector involves the "usual" technical progress towards CO_2 abatement at the rate ε which is not sufficient to reach sustainability. In addition, the green public sector produces supplementary free public innovations that are used to further accelerate CO_2 abatement in order to reach the sustainable target.

In order to estimate the impact of this policy on the planet atmospheric CO_2 concentration, we also need to have a scenario for CO_2 emissions in non-OECD countries. The future effective evolution of CO_2 emissions in non-OECD countries depends on many factors such as the international trade, the extent of exported emissions [Davis and Caldeira, 2010] or the efficiency of international negociations (Kyoto, Copenhagen, Doha ...). In any case, the highest admissible projection of sustainable CO_2 emissions for non-OECD is given in Fig.11, because higher emissions would definitely lead to an excess of atmospheric CO_2 with respect to the target of 450 ppm. The corresponding evolution of emissions per capita in both subregions is shown in Fig.12. By integrating equation (16) with the total CO_2 emissions for E_w, we then get the CO_2 atmospheric evolution depicted in Fig.13. From this figure, we see that, under the assumptions of the simulation, the investment policy in public green technologies is effectively able to stabilize the CO_2 concentration at the set point of 450 ppm which is reached after 60 years approximately.

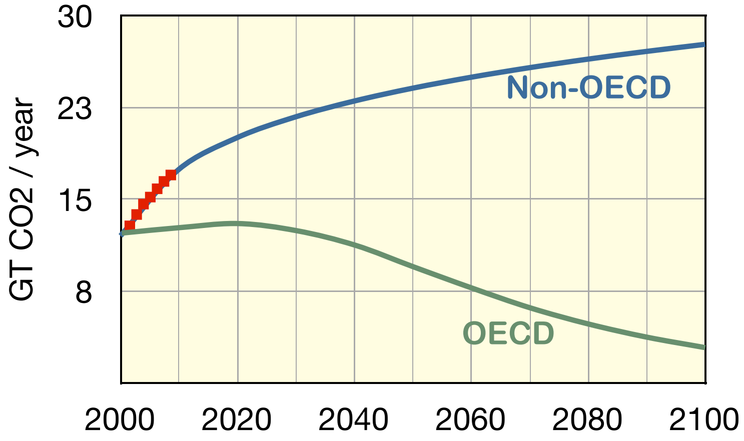

Fig.11: CO_2 emissions for the period 2000-2100: simulation result for the OECD benchmark and highest admissible projection for non-OECD countries. The red dots are empirical data.

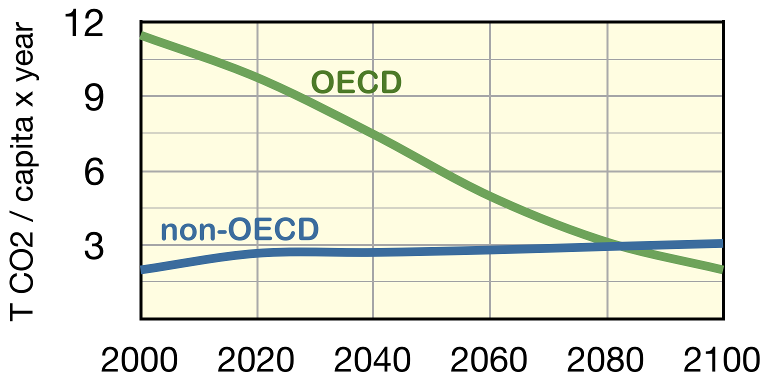

Fig.12: CO_2 emissions per capita for the period 2000-2100

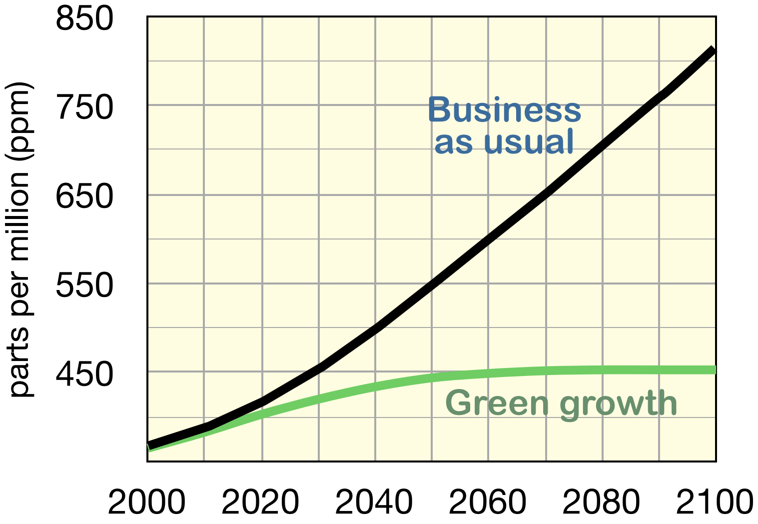

Fig.13: Atmospheric CO_2 concentration

There are however many major objections that can be invoked against the feasibility of green growth. A very fundamental objection is that green growth relies essentially on a blind faith into the technological progress. Indeed it seems as well reasonable to believe that the required massive technological breakthrough is in fact out of reach. For this reason, a sound principle is to consider also alternatives like the low-growth strategy defended for instance by Tim Jackson [Jackson, 2009], [Victor, 2012] and The Club of Rome [Meadows et al., 2004].

6. Low Growth

The principle of a low growth public policy is to foster a structural shift of the economy composition towards activities which have low (or even zero) carbon intensity. Such activities are by nature labour intensive and far less subject to productivity growth (see e.g. [Jackson and Victor, 2011]). The simplest case is to consider an economy with two sectors:

1) a conventional sector with production function

$$Y_{cs} = AK_{cs}^{(1-\alpha)}L_{cs}^{\alpha}, \tag{41}$$

which is endowed with the dynamics, the parameter values and the initial conditions of the benchmark model of Section 4.

2) a "transition" sector of activities having zero carbon intensity and a constant labour productivity, with a production function

$$Y_{ts} = BK_{ts}^{(1-\beta)}L_{ts}^{\beta}, \tag{42}$$

where the productivity B is constant. We assume also that $\beta > \alpha$ which implies that the transition sector is more labour intensive than the conventional sector.

Let us take the conventional output as numeraire and denote by π the relative price of the transition sector output. Along an economic equilibrium path, the factor markets are competitive and therefore the marginal products of capital and labour are equal in the two sectors:

$$r(t) = \frac{\partial Y_{cs}(t)}{\partial K_{cs}(t)} = \pi(t)\frac{\partial Y_{ts}(t)}{\partial K_{ts}(t)} \quad \forall t \tag{43}$$

$$w(t) = \frac{\partial Y_{cs}(t)}{\partial L_{cs}(t)} = \pi(t)\frac{\partial Y_{ts}(t)}{\partial L_{ts}(t)} \quad \forall t. \tag{44}$$

Using equations (41)-(42), we have

$$r(t) = (1-\alpha)\frac{Y_{cs}(t)}{K_{cs}(t)} = (1-\beta)\pi(t)\frac{Y_{ts}(t)}{K_{ts}(t)}, \tag{45}$$

$$w(t) = \alpha\frac{Y_{cs}(t)}{L_{cs}(t)} = \beta\pi(t)\frac{Y_{ts}(t)}{L_{ts}(t)}. \tag{46}$$

In this economy, the CO_2 emissions are proportional to Y_{cs} only:

$$E(t) = h(t)Y_{cs}(t) \tag{47}$$

with the carbon intensity function $h(t)$ given by (18). The strategy for the transition to a sustainable economy is a sectorial to activities with low or zero CO_2 emissions in order to reach the

2100 target $E^* = 5.61$ GT/Year. Hereafter we present a simulation of a low-growth scenario that produces, along time, the same CO_2 emissions as the green growth scenario of the previous section. Therefore the emission profile $E(t)$ of Fig.11 which has been computed in the green growth scenario is a reference which is used, in the simulation, as an exogenous driving variable to compute $Y_{cs}(t)$ from equation (47). As in the previous sections, a balanced path is defined as the special case where $r(t)$ is constant. Along a balanced path, we have:

$$K_{cs} = (\frac{1-\alpha}{r})Y_{cs}, \quad L_{cs}^{\alpha} = (\frac{1}{A})(\frac{r}{1-\alpha})K_{cs}^{\alpha}, \quad L_{ts} = L - L_{cs}, \tag{48}$$

$$K_{ts} = (\frac{\alpha}{1-\alpha})(\frac{1-\beta}{\beta})(\frac{L_{ts}}{L_{cs}})K_{cs}, \quad Y_{ts} = BK_{ts}^{1-\beta}L_{ts}^{\beta}, \quad \pi = (\frac{\alpha}{\beta})(\frac{L_{ts}}{L_{cs}})(\frac{Y_{cs}}{Y_{ts}}). \tag{49}$$

The constant productivity in the transition sector is set to $B = 0.13$ which is the initial productivity $A(0)$ in the conventional sector. The labour elasticity in the transition sector is set to $\beta = 0.85$, i.e. 30% higher than in the conventional sector. All the other constant parameters needed for the simulation have been given previously.

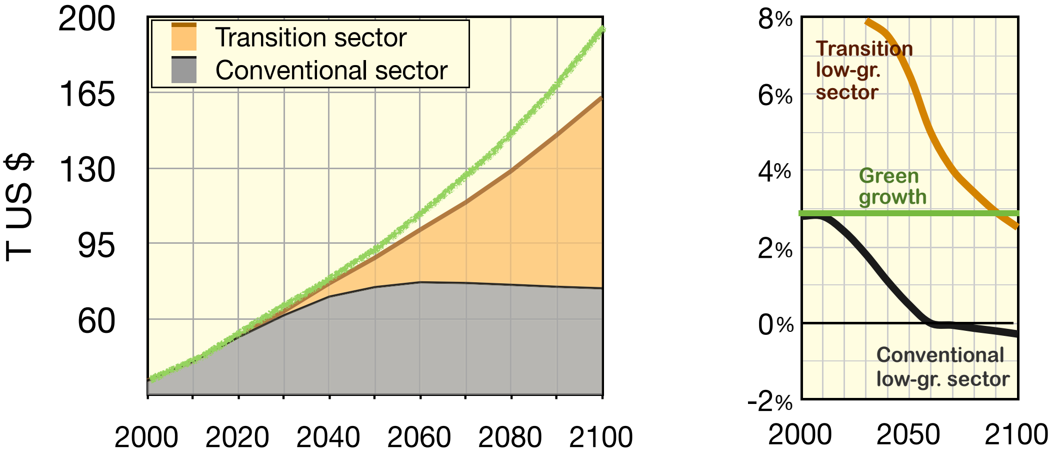

Fig.14: Low growth in the OECD benchmark model. Left: GDP at market prices (The green line is a reminder of the GDP exponential growth in the green growth scenario). Right: Growth rates in the two sectors for the low growth scenario, compared to the green growth scenario.

The model is initialized in 2000 with the values of Table 1 for the conventional sector and with zero initial conditions for the transition sector. The new low-growth policy is activated in 2014. The government strategy to foster the transition is to tax the conventional production (using e.g. carbon taxes) and subsidize the transition sector in order to equalize the market price between the two sectors. The result of the simulation experiment is illustrated in Fig.14 and Fig.15. Naturally, in this case, a balanced economy means that capital and output vary at the same rate within each sector, but at different rates between the sectors because of the reallocation of labor and capital as illustrated in these figures. As expected, in this scenario, the economic growth in the conventional sector is drastically reduced (with even a small de-growth from 2060) and only partially compensated by the expansion of the transition sector. This results in a global economic growth slowned down as compared to the green growth scenario. Fig.15 illustrates quantitatively the labour reallocation which is needed.

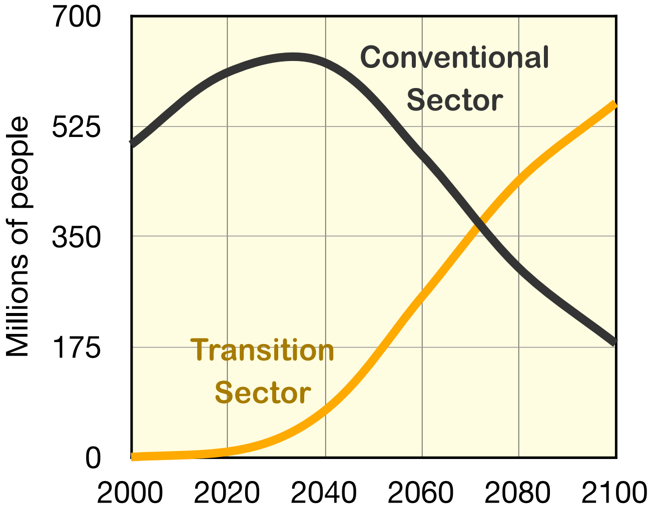

Fig.15: Employment in the two sectors for the low growth scenario.

7. Conclusions

This paper has given a modest contribution to the modelling of the transition to a sustainable economy. The focus has been on accelerating the green technological change in the green growth option or shifting the structure of the economy towards zero carbon emission activities in the low growth option. With the green growth option, the economic trajectory reaches the IPCC objective of 450ppm within about sixty years with a specific public development of massive additional green technologies representing a cost up to 8% of GDP. With the low growth option, it is possible to achieve the same objective, within the same time horizon, without blind faith in technologies, by systematically subsidising a transition to low carbon and low capital intensive activities, leading to a sectoral shift from the conventional sector (from 100% to 45% of GDP) to the transition sector (from 0% to 55% of GDP). Obviously, by running linear combinations of these two extremes, all intermediate trajectories are possible.

The model, as it has been set up in this paper, represents a very narrow and limited perspective regarding the transition to sustainability. Many relevant aspects of the impact of global warming on the economy are ignored and the structure of the economy itself has been extremely simplified. Important related issues such as social inequalities or international finance unreliability are not addressed. However, we hope that our parcimonious modelling contributes to highlight some of the fundamental challenges in terms of economic policy. Moreover as we have mentioned in the introduction, the model can be easily extended to include more subregions and economic subsectors or explicit fiscal policies. One important issue which has been omitted relies on the modelling of the mechanisms that underly the public policy and their impact on the economy dynamics. This issue will be dealt with in an extended version of this paper.

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