

## **HYDROMAX : A Real-Time Application for River Flow Forecasting**

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# Hydromax : A Real-Time Application for River Flow Forecasting.

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## 1 Introduction

Hydromax is an application for river flow forecasting and flood alarms which provides in real-time short-term predictions of river flows based on rainfall and past river flow measurements, and long-term flood forecasting based on meteorological forecasts. The purpose of this brief paper is to give a general description of Hydromax and to demonstrate its performance with typical experimental examples and statistical assessments.

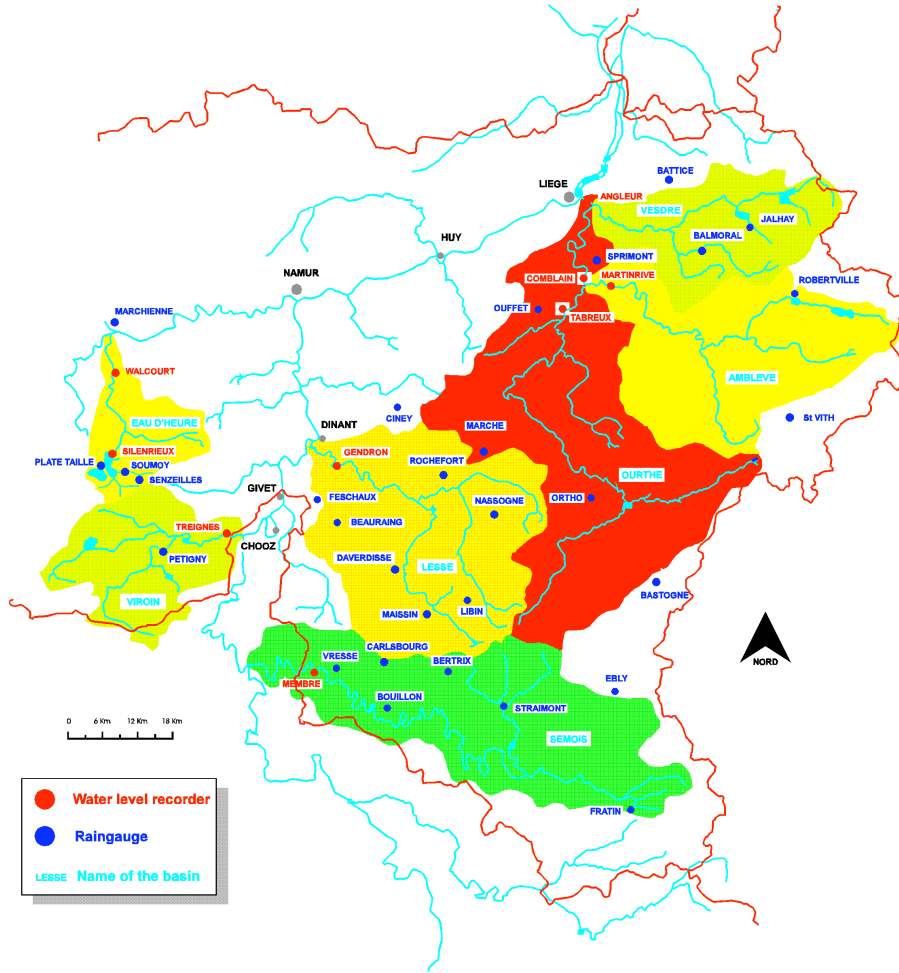
For each river basin, the forecasted river flows are produced by a mathematical model which involves four parts:

- 1) An optimal minimum variance interpolator which computes the mean areal rainfall on the watershed.
- 2) A non-linear conceptual production function which describes the water storage in the watershed and computes the effective rainfall from the mean areal rainfall.
- 3) A linear ARX transfer function which describes the superficial runoff of the net rainfall towards the watershed outlet and computes the short term river flow forecasting.
- 4) A simulation model which produces long term river flow forecasts from meteorological data.

The identification of the model is quite data saving because only rainfall and river flow measurements are required while a detailed physical description of the basin is not needed. Hydromax has been developed to be user friendly and to fulfill the real-time forecasting requirements. It is successfully in routine operation for more than five years in the Meuse river (Walloon region, Belgium) and its main tributaries.

## 2 Telemetry and data acquisition

To be operational, Hydromax must be connected to a reliable telemetry network and a data acquisition system accessible from the forecasting center and able to achieve frequent on-line field measurements of both rainfall depths in raingauges and water levels in rivers. In this paper, the Hydromax performance will be illustrated with data from the telemetry system of Sethy (Service d'études hydrologiques, Walloon Ministry of Public Works - Belgium). Hydromax uses about 60 stations scattered in the Meuse river basin as shown in Fig. 1.



**Fig. 1:** The telemetering network of Sethy in the Meuse river basin

The data are collected with a basic time-step ( $\Delta t = 1\text{h}$ ). Hourly rainfall and river flow measurements over a period of several years (including big floods) were thus available for the model development. Obviously, the basic time-step  $\Delta t$  must be much smaller than the mean concentration time of the considered river basins.

### 3 Estimation of the mean areal rainfall

The input of the model is the mean areal rainfall over the considered watershed. The possible spatial heterogeneity of the rainfall is thus not taken into account here. The point rainfall depth is denoted  $P(z)$  with  $z=(x,y) \in \mathbb{R}^2$ , the Cartesian coordinates. It is assumed to be a realization of a two-dimensional random field with constant mean and linear variogram. The rainfall measurements are available at  $n$  measurement stations and denoted:

$$P_1 = P(z_1), \quad P_2 = P(z_2), \quad P_n = P(z_n)$$

The average areal rainfall  $PB$  over a catchment area  $\Omega \in \mathbb{R}^2$  is then defined as:

$$PB = \frac{1}{|\Omega|} \int_{\Omega} P(z) dz$$

As is well known, an optimal (linear, unbiased, minimum variance) estimation of  $PB$  can be computed from the set of rainfall observation  $\{P_i, i=1, \dots, n\}$  as :

$$PB = \sum_{i=1}^n \lambda_i P_i$$

with the  $\lambda_i$  solutions of the so-called “kriging” system:

$$\sum_{i=1}^n \lambda_i e(z_i, z_j) + \mu = \frac{1}{|\Omega|} \int_{\Omega} e(z_i, z_j) dz \quad j = 1, \dots, n$$

$$\sum_{i=1}^n \lambda_i = 1$$

where  $\mu$  is a Lagrange multiplier and  $e(z_i, z_j)$  denotes the Euclidean distance between the points  $z_i$  and  $z_j$  in  $\mathbb{R}^2$ .

#### 4 Computation of the effective rainfall with the production function

The role of the production function is to transform the mean areal rainfall  $PB$  into an effective rainfall  $PN$  which is supposed to reach the basin outlet as surface runoff. The model describes the balance of water volumes during time intervals  $\Delta t$ . During each time interval the amount of precipitated water is decomposed as follows:

$$PB(t) = PN(t) + E_1(t) + W(t)$$

with  $t$  the discrete time index.  $E_1(t)$  represents the part of the rainfall  $PB(t)$  that directly evaporates during the current time interval.  $W(t)$  represents the amount of water that will not participate in the runoff but will be stored in the basin under various forms (vegetation interception, superficial depressions, soil moisture, etc ...). The storage of the water in the river basin is then represented by a linear reservoir with inflow  $W(t)$  described by the difference equation:

$$S(t) = S(t-1) + W(t) - E_2(t) - I(t)$$

where  $S(t)$  denotes the stock of water in the river basin,  $I(t)$  is the amount of water drained by percolation and  $E_2(t)$  is the part of stored water evapotranspiring during the current time interval. The percolation term  $I(t)$  is represented by a linear function of the available water stock:

$$I(t) = \alpha (S(t-1) + W(t))$$

with  $\alpha$  a specific percolation parameter. The evapotranspiration terms  $E_1(t)$  and  $E_2(t)$  are computed as:

$$E_1(t) = \min (PB(t), ETP(t))$$

$$E_2(t) = \max (0, \min (ETP(t) - PB(t), S(t-1) + W(t) - I(t)))$$

where  $ETP(t)$  represents an estimate of the seasonal potential evapotranspiration for the considered basin. It is furthermore assumed that there is a physical upper limit  $S_{max}$  of the amount of stored water  $S(t)$  in the river basin. The water storage  $W(t)$  is then expressed as a function of  $S(t)$  and  $PB(t)$  in order to:

- guarantee the condition  $0 \leq S(t) \leq S_{max} \quad \forall t$

- verify the hydrological principle that the effective rainfall  $PN(t)$  increases with both rainfall intensity  $PB(t)$  and soil moisture  $S(t)$ . The following function satisfies these requirements:

$$W(t) = [S_{\max} - S(t)] \left[ 1 - \exp \left( -\beta \frac{PB(t) - E_1(t)}{S_{\max} - S(t)} \right) \right]$$

with  $\beta$  a specific runoff coefficient.

The production function model then involves three parameters ( $\alpha$ ,  $\beta$ ,  $S_{\max}$ ) that have to be calibrated from experimental data for each considered river basin.

## 5 Computation of the short term river flow forecasting with a linear transfer function

At each time  $t$ , a forecasting  $\hat{Q}(t+h)$  is computed for the future time instant  $(t+h)$  (i.e. with a prediction horizon of  $h$  measurement time steps) as a linear combination of past river flow measurements and past effective rainfall values, with a linear regression model (ARX model) of the form:

$$\hat{Q}(t+h) = \sum_{i=1}^n a_i Q(t-(i-1)h) + \sum_{j=1}^m b_j PN(t-(j-1)h)$$

where  $Q(t-(i-1)h)$  denotes the riverflow measurements at the past time instants  $(t-(i-1)h)$  while  $PN(t-(j-1)h)$  represents the effective rainfall cumulated over  $h$  successive time steps and computed with the production function.

For each river basin, the values of the prediction horizon  $h$  and the coefficient  $a_i, b_j$  are determined from experimental data. To get accurate forecasts, the prediction horizon  $h$  must obviously be smaller than the natural response time of the river basin. As a rule of thumb, it is selected between the one fifth and the one third of the peak time of the unit hydrograph. The dimensions  $n$  and  $m$  of the regression terms in the model are selected using classical statistical tools of system identification theory (correlogram of prediction errors, Bayesian Information Criterion, etc ...) according to a parsimony principle. The parameters  $a_i$  and  $b_j$  are calibrated by linear regression.

## 6 Computation of long term river flow forecasts from meteorological data

The goal here is to compute river flow forecasts over prediction horizons that are significantly larger than the natural response time of the river basin. This obviously requires to anticipate the future rainfalls by meteorological informations. Such long-term river flow forecasts may be computed by iterating the short-term prediction model as follows:

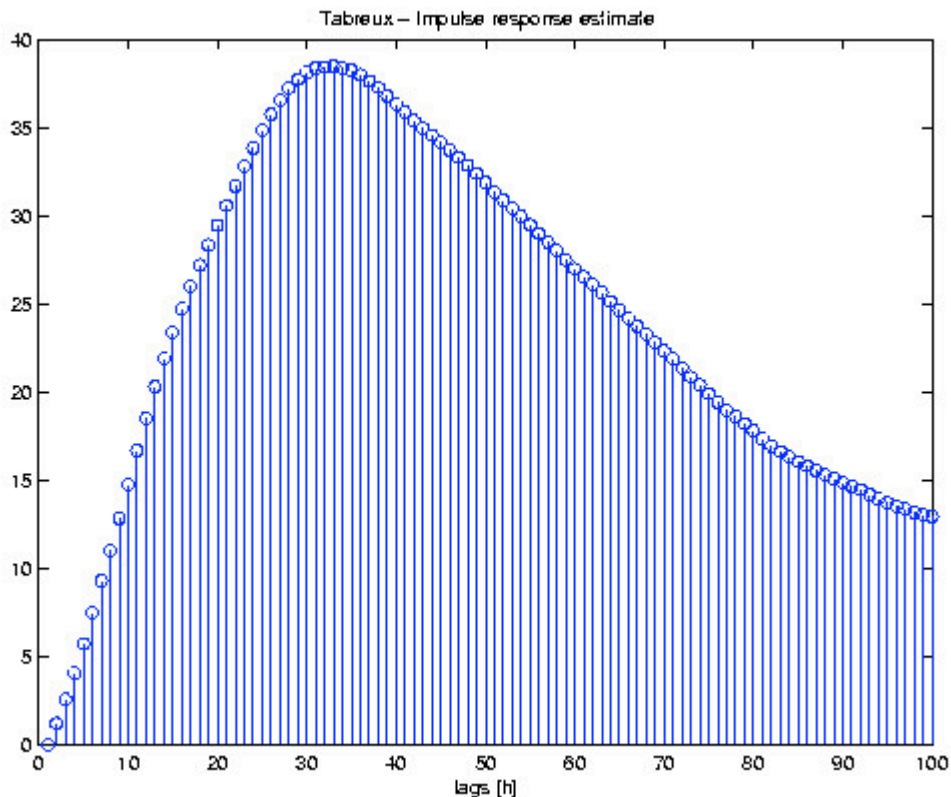
$$\begin{aligned} \hat{Q}(t+kh) &= \sum_{i=1}^{k-1} a_i \hat{Q}(t+(i-1)h) + \sum_{j=k}^n a_j Q(t+(j-1)h) \\ &+ \sum_{i=1}^{k-1} b_i \hat{PN}(t+(i-1)h) + \sum_{j=1}^m b_j PN(t+(j-1)h) \end{aligned}$$

where  $\hat{Q}(t+(i-1)h)$  represent successive iterated river flow forecasts and  $\hat{PN}(t+(i-1)h)$

effective rainfall forecasts to be provided by the user.

## 7 An example : the flood of January 1995

In this last section, the Hydromax performance is illustrated with a typical forecasting example in the Ourthe river basin. The outlet of the river basin is located at Tabreux (1607 Km<sup>2</sup>) and four raingauges are available (see map in Fig. 1). Hourly rainfall-riverflow data during 2 years (1992-1993) have been used to calibrate the model. The estimated model parameters are given in Table 1. The unit hydrograph of the river is also shown in Fig. 2. The selected predicted horizon is h=6 hours.



**Fig. 2 :** Tabreux – Unit hydrograph

**Table 1 :** Tabreux - Parameters of the model

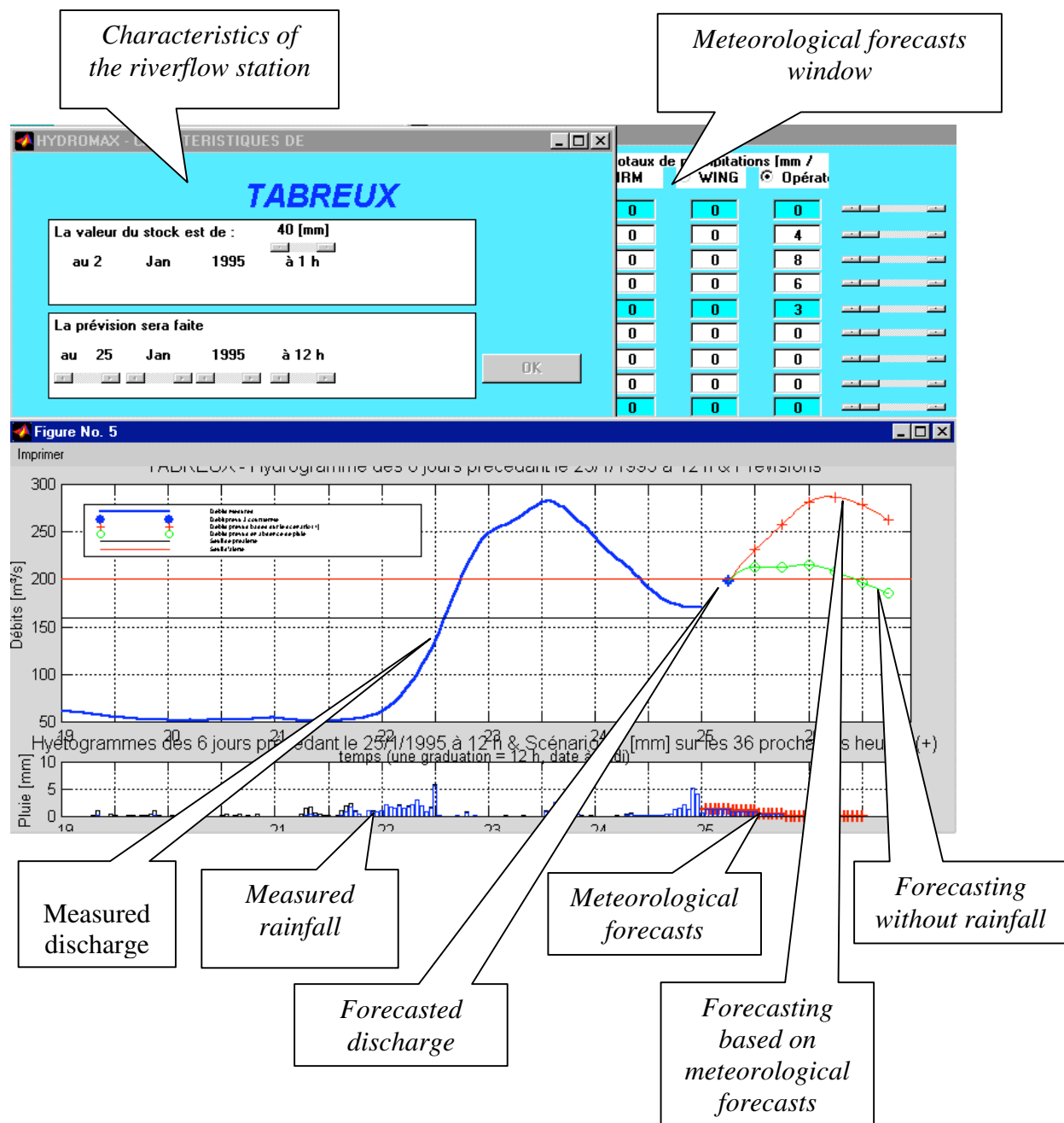
$\alpha$	$\beta$	$S_{max}$	$a_1$	$a_2$	$b_1$	$b_2$	$b_3$	$b_4$
0.00065	0.86	76	1.353	0.41	2.382	0.896	0.202	0.993

The predictive capability of the model is here illustrated with the big flood of January 1995 (which is not in data set for model construction). In Fig. 3, a typical example of on-line forecasting with Hydromax is shown. We can see that Hydromax computes a short term prediction for 18 p.m. of 198 m<sup>3</sup>/s (big blue dot on the figure), which is to be compared to the actual value of 200 m<sup>3</sup>/s. Hydromax also computes a long term prediction over an horizon of 42 hours (+ line) for the given scenario of future rainfalls and an “optimistic” prediction (o line) under the assumption that the rainfall will definitely stop.

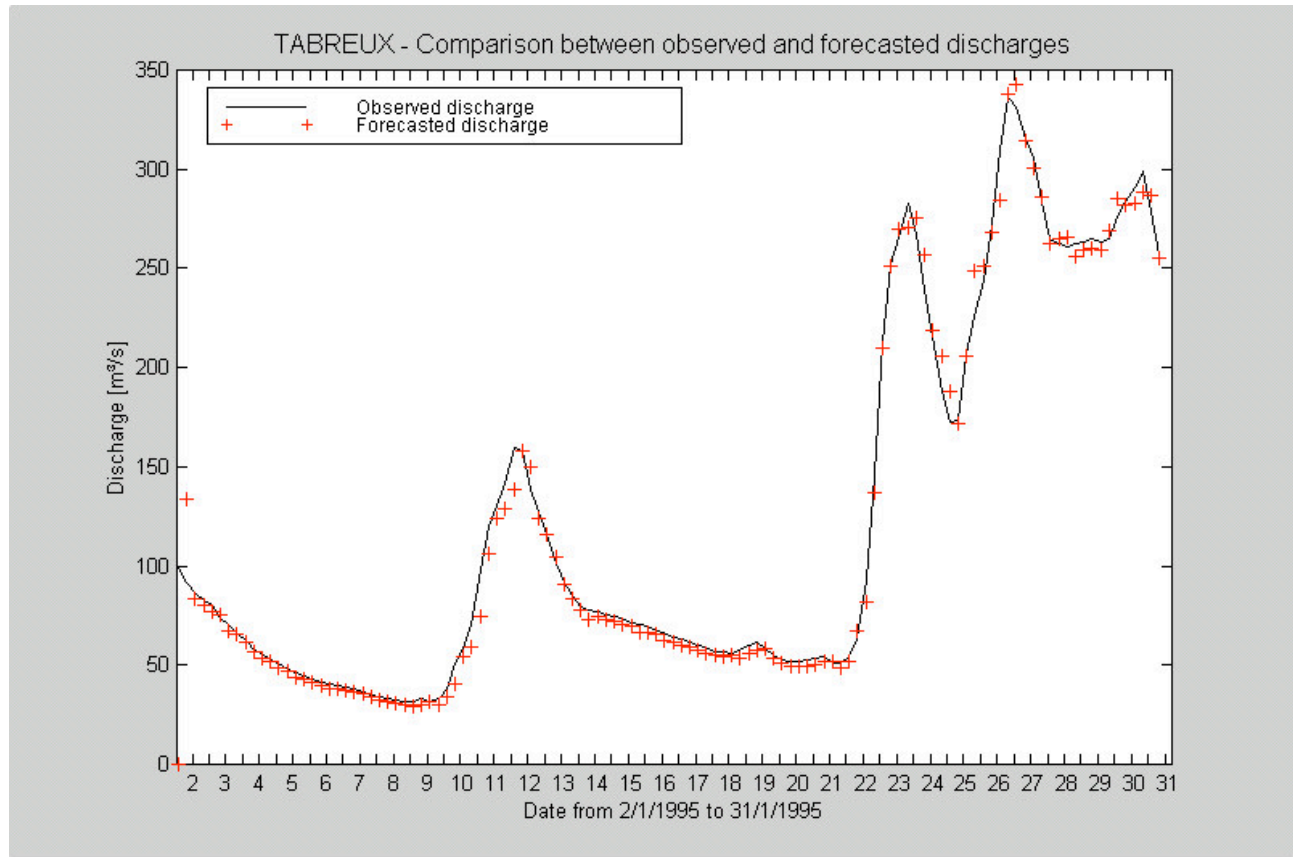
In Fig. 4, a comparison between the observed river flow discharges and the short-term predictions all along this big flood of January 1995 is presented. Finally the statistical accuracy of Hydromax at a level of 90 % is illustrated in Table 2.

**Table 2 :** Upper bound of the relative forecasting error at a level of 90 %

Horizon	6 hours ahead	12 hours ahead	18 hours ahead	24 hours ahead	30 hours ahead
$1 - \frac{\hat{Q}}{Q} \leq$	0.07	0.13	0.18	0.22	0.32



**Fig. 3:** Example of Hydromax windows : forecasting of Ourthe (a Meuse tributary) flow rate during the big flood of January 1995



**Fig. 4 :** Tabreux – Comparison between observed and forecasted discharges during the big flood of January 1995

## 8 Acknowledgements

Hydromax has been developed by the authors at Cesame (Center for Systems Engineering and Applied Mechanics, University of Louvain-la-Neuve). The kriging computation of the mean areal rainfall follows the lines of [1]. The structure of the short-term forecasting model was initially sketched in [2] and further investigated in the Ph.D. Thesis of B. Wéry [3]. The experimental data were provided by Sethy (Service d'études hydrologiques, Walloon Ministry of Public Works - Belgium) which is also gratefully acknowledged for its warm collaboration and financial support.



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