Proceedings of the 30th Conference
on Decision and Control
Brighton, England • December 1991

MODELLING AND STATE FEEDBACK CONTROL OF NONHOLONOMIC MECHANICAL SYSTEMS

G. Campion†, B. d'Andrea-Novel‡, G. Bastin†

† Laboratoire d'Automatique,
Dynamique et Analyse des Systèmes
Université Catholique de Louvain
Place du Levant, 3,
B1348 Louvain la Neuve (Belgium)
E-mail : bastin@auto.ucl.ac.be

‡ Centre Automatique et Systèmes
Ecole Nationale Supérieure
des Mines de Paris
Rue Saint Honoré, 35,
F-77305 Fontainebleau, France
E-mail : dandrea@caia.ensmp.fr

ABSTRACT

The dynamics of nonholonomic mechanical systems are described by the classical Euler-Lagrange equations subjected to a set of non-integrable constraints. Non holonomic systems are strongly accessible whatever the structure of the constraints. They cannot be asymptotically stabilized by a smooth pure state feedback. However smooth state feedback control laws can be designed which guarantee the global marginal stability of the system with the convergence to zero of an output function whose dimension is the number of degrees of freedom.

1. INTRODUCTION

A mechanical system, whose configuration is completely described by a set of generalized coordinates, can be subjected to kinematic constraints (such as the pure rolling condition of a wheel on a plane), which are expressed by relations between the coordinates and their time derivatives. If these constraints are holonomic (that is integrable) it is possible to characterize the system configuration by a smaller number of coordinates (i.e. to use the constraints in order to eliminate the redundant coordinates) in such a way that the constraints are automatically satisfied in the new coordinates. Unfortunately, in case of non holonomic constraints, this elimination is not possible and the constraints have to be taken into account explicitly in the derivation of the dynamical equations. The present paper deals with control design of such systems, for which, due to the nonholonomic constraints, the standard control laws developped for holonomic mechanical systems (for instance robotic manipulators) are not applicable .

Motion planning and feedback control of non holonomic mechanical systems has been discussed in the literature through the special case of mobile wheeled robots (see e.g. [1] to [4]). In these papers however, the control is designed on the basis of a kinematic state-space model derived from the constraints, but not taking the internal dynamics of the system into account. The purpose of this paper is to derive a full dynamical description of such nonholonomic mechanical systems, including the constraints and the internal dynamics, and to show how a suitable change of coordinates allows to analyse globally the controllability and the state feedback stabilizability of the system. The feedback stabilizability of mechanical systems with constraints (holonomic or not) is also examined by Bloch and McClamroch [7]. However, they use another change of coordinates which is less efficient since it provides only local stability results and is not convenient for a controllability analysis.

The paper is organized as follows. The concept of non holonomic constraints for mechanical systems is introduced in Section 2 within the framework of the theory of nonlinear control systems. A general dynamical state-space model of non holonomic systems is then derived in Section 3, using the classical Euler-Lagrange formalism. By a suitable change of coordinates,this model can be partially linearized in such a way that the remaining nonlinearities only depend on the structure of the constraints. On this basis it is then shown, in Section 4, that non holonomic systems are strongly accessible whatever the structure of the constraints and, furthermore, small-time locally controllable from equilibrium configurations. Our main contribution is then presented in Section 5 where it is shown how to design smooth state feedback controllers which guarantee the global stability of the system and the convergence of the output function to zero.

2. NONHOLONOMIC CONSTRAINTS

We are concerned, in this paper, with mechanical systems whose configuration space is an n-dimensional simply connected manifold $\mathcal{M}$ and whose dynamics are described, in local coordinates, by the so-called Euler-Lagrange equations of motion. Usually, the local coordinates used for the description of these systems are termed "generalized coordinates" and denoted $q_1, q_2, \ldots, q_n$. Each configuration of the system is represented by the vector of these generalized coordinates and is denoted:

$$q \equiv \left[q_1, q_2, \cdots, q_n\right]^T$$

The configuration manifold $\mathcal{M}$, which is the set of all possible configurations, is represented in local coordinates by an open set $\Omega \subseteq \mathbb{R}^n$. The position of each material point of the system is a function of the generalized coordinates. A motion of the system is represented in the q coordinates by a smooth time function $q(t)$. The corresponding trajectory is a one-dimensional immersed submanifold of $\mathcal{M}$. The tangent vector at a point of the trajectory is then represented by the vector

$$\dot{q} \equiv [\dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n]^T$$

whose components are termed generalized velocities.

In many instances, the motion of mechanical systems is subjected to various constraints which are permanently satisfied during the motion and which take the form of algebraic relationships between the positions and the velocities of particular material points of the system. Two kinds of constraints can be distinguished: geometric constraints and kinematic constraints.

Geometric constraints. These constraints are represented by analytical relations between the generalized coordinates. When the system is subjected to m independent such constraints, m generalized coordinates can be eliminated and n-m generalized coordinates are sufficient to provide a full description of the configurations of the system.

Kinematic constraints. These constraints are represented by analytical relations between the generalized coordinates and velocities. In most applications, these relations are linear with respect to the generalized velocities and written as:

$$a_j^T(q)\dot{q} = 0 \qquad j = 1,\ldots,m \qquad (1)$$

where $a_1^T, a_2^T, \ldots, a_m^T$ are smooth n-dimensional covector fields on $\mathcal{M}$. In matrix form, the constraints (1) are written

$$A^T(q)\dot{q} = 0 \qquad (2)$$

where $A(q)$ is the $(n \times m)$ matrix made up of the vector functions $a_j(q)$ as follows:

$$A(q) \equiv \left[a_1(q), a_2(q), \ldots, a_m(q)\right] \qquad (3)$$

The m $(<n)$ constraints are said *independent* when this matrix has full rank for all q. Unlike geometric constraints, the kinematic constraints do not necessarily lead to the elimination of generalized coordinates from the system description. The elimination is possible only when the constraints are *holonomic* (that is: integrable). Our concern in this paper will precisely be to discuss the controllability and the feedback stabilization of mechanical systems with *nonholonomic* constraints.

Hence, without loss of generality, we can consider that all the redundant generalized coordinates associated to the geometric constraints have been eliminated and restrict our attention to mechanical systems subjected to m *independent* kinematic constraints only. These constraints are assumed to have the form (1).

We assume that the annihilator of the codistribution spanned by the covector fields $a_1^T, a_2^T, \ldots, a_m^T$, is an $(n$-$m)$-dimensional *smooth* nonsingular distribution Δ on $\mathcal{M}$. This distribution Δ is spanned by a set of $(n$-$m)$ smooth vector fields $s_1, s_2, \ldots, s_{n-m}$ which satisfy, in local coordinates, the following relations:

$$a_j^T(q)s_i(q) = 0 \qquad \forall q \in \Omega \qquad j = 1,m \quad i = 1,n-m \qquad (4)$$

Consider now the involutive closure of Δ, denoted Δ^*, and defined as the smallest involutive distribution containing Δ. Assume that this distribution is regular (that is has constant dimension on $\mathcal{M}$). Clearly: $n\text{-}m \leq dim(\Delta^*) \leq n$.

Let $(n - m^*)$ denote the dimension of Δ^*, with $m^* \leq m$. Then, as shown in [5], depending on the dimension of Δ^*, several situations may arise:

a) If $m^* = m$ (that is if Δ is involutive) the system is said to be *holonomic*. The configuration space can be characterized with $(n$-$m)$ coordinates only. The configuration space is thus an $(n$-$m)$-dimensional manifold.

b) If $m^* = 0$ (that is if $\dim(\Delta^*) = n$) the constraints are completely nonintegrable and the system is said to be *nonholonomic*. The characterization of the configuration space requires n coordinates.

c) If $0 < m^* < m$ it is possible to eliminate m^* coordinates. The configuration space is a manifold of dimension $n - m^*$.

Without loss of generality, we can thus assume that all the geometric and all the integrable kinematic constraints have been eliminated from the system description and restrict our attention to the situation b) that is to nonholonomic mechanical systems evolving in an n-dimensional configuration manifold and subjected to m independent nonintegrable constraints.

3. DYNAMICAL MODELLING.

Using the Lagrange formalism, the dynamics of a nonholonomic mechanical system are described by the following differential equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = A(q)\lambda + B(q)u \qquad (5)$$

with the following notations and definitions:

a) $L(q,\dot{q}) = T(q,\dot{q}) - W(q)$ is the Lagrangian of the system with $T(q,\dot{q})$ the kinetic energy and $W(q)$ the potential energy.

b) $B(q)u$ is the set of generalized forces applied to the system with $B(q)$ a $(n \times p)$ kinematic matrix and u the p-vector of external forces and torques applied to the system by the actuators.

c) $A(q)$ is the matrix associated with the nonholonomic constraints ; λ is the m-vector of Lagrange multipliers.

The kinetic energy $T(q,\dot{q})$ is defined as:

$$T(q, \dot{q}) = \frac{1}{2}\, \dot{q}^T M(q)\dot{q}$$

where $M(q)$ is the $(n \times n)$ definite positive symmetric inertia matrix. With these definitions, the model (5) is rewritten as follows:

$$M(q)\ddot{q} + f(q, \dot{q}) = A(q)\lambda + B(q)u \qquad (6)$$

with :

$$f(q, \dot{q}) \equiv \frac{dM(q)}{dt}\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}\left[\dot{q}^T M(q)\dot{q}\right] + \frac{\partial W(q)}{\partial q}$$

This equation (6), together with the constraint (2), provide a full description of the dynamics of the nonholonomic system.

Defining the full rank matrix $S(q)$ made up of the vector functions $s_i(q)$:

$$S(q) \equiv \left[s_1(q), s_2(q), \ldots, s_{n-m}(q)\right]$$

it results that :

$$S^T(q)A(q) = 0 \qquad \forall q \in \Omega \qquad (7)$$

Using this expression, we eliminate the Lagrange multipliers by premultiplying equation (6) by $S^T(q)$ to obtain:

$$S^T(q)\{M(q)\ddot{q} + f(q, \dot{q})\} = S^T(q)B(q)u \qquad (8)$$

A fundamental assumption which relies on the constructive structure of the nonholonomic mechanical system is as follows.

Assumption. The square $(n-m)x(n-m)$ matrix $S^T(q)B(q)$ has full rank for all q. ♦

This assumption is not restrictive since it just mean that the actuators have been set in such a way that the system is really (physically) controllable whith these actuators.

Moreover, the constraints (2) imply the existence of a vector time function $\eta(q, \dot{q})$ smooth in q and linear in $\dot{q}$ which satisfies the following equality along the trajectories of the system:

$$\dot{q} = S(q)\eta(q, \dot{q}) \qquad (9)$$

By differentiating (9) and from the model (8), it can the be shown after some algebraic manipulations (see [5]) that the dynamical model of a nonholonomic system is written in state space form as:

$$\dot{q} = S(q)\eta \qquad (10.a)$$

$$J(q)\dot{\eta} + g(q, \eta) = G(q)u \qquad (10.b)$$

with $dim\ q = n$ and $dim\ \eta = n\text{-}m$.

In equations (10.a-b), $J(q)$, $g(q,\eta)$ and $G(q)$ are computed as :

$$J(q) = S^T(q)M(q)S(q) \qquad (11.a)$$

$$g(q, \eta) = S^T(q)\left\{M(q)\frac{\partial}{\partial q}[S(q)\eta]\right\}S(q)\eta + S^T(q)f(q, S(q)\eta) \qquad (11.b)$$

$$G(q) = S^T(q)B(q) \qquad (11.c)$$

As a first step towards the analysis of the controllability and the design of a stabilizing controller for this system, we have the following property.

Lemma 1. The dynamical state-space model (10) is partially feedback linearizable with a control law $u(q, \eta)$ chosen such that:

$$G(q)u(q, \eta) = g(q, \eta) + J(q)v \qquad (12)$$

where v denotes an $(n\text{-}m)$ - dimensional external input. Indeed, with such a control law, the closed loop is written:

$$\dot{q} = S(q)\eta \qquad (13.a)$$

$$\dot{\eta} = v \qquad (13.b)$$

Thus it appears that the static state feedback (12) allows to reduce the system (10) to the simple form (13) whose structure only depends on the nonholonomic constraints.

Our concern is now to discuss the controllability properties of this model and the design of a second stabilizing state feedback loop $v(\eta,q)$. For this purpose, we first introduce some notations and properties of the model (13) that will be used in the next sections.

The model (13) can be written in compact form as follows :

$$\dot{x} = f(x) + \sum_{i=1}^{n-m} g_i v_i \qquad (14)$$

with :

$$x \equiv \begin{bmatrix} q \\ \eta \end{bmatrix}, \quad f(x) \equiv \begin{bmatrix} S(q)\eta \\ 0 \end{bmatrix}$$

and $g_1, g_2, \ldots, g_{n-m}$ as the columns of the matrix $\begin{bmatrix} 0 \\ I_{n-m} \end{bmatrix}$

For this model, the following properties can be readily established:

P1) $[f, g_i] = \begin{pmatrix} s_i(q) \\ 0 \end{pmatrix} \qquad i = 1, \ldots, n-m$

P2) $[g_i, g_j] = 0 \qquad \forall\, i, j \le n-m$

P3) $[g_i, [f, g_j]] = 0 \qquad \forall\, i, j \le n-m$

P4) $\big[[f, g_i], [f, g_j]\big] = \left[\begin{pmatrix} s_i \\ 0 \end{pmatrix}, \begin{pmatrix} s_j \\ 0 \end{pmatrix}\right] = \begin{pmatrix} [s_i, s_j]_{\mathcal{M}} \\ 0 \end{pmatrix} \quad \forall\, i, j \le n-m$

where $[\cdot\,,\,\cdot]_{\mathcal{M}}$ denotes the Lie bracket operation in the n-dimensional configuration manifold $\mathcal{M}$

4. CONTROLLABILITY.

It is well known that holonomic mechanical systems with n degrees of freedom and n actuators are completely controllable. This property is extended to nonholonomic systems as follows, whatever the structure of $S(q)$:

Theorem 1. A nonholonomic system evolving in an n-dimensional configuration manifold and subjected to m constraints is completely strongly accessible from any configuration q : the strong accessibility rank is $(2n\text{-}m)$ for all q.

Proof. The strong accessibility algebra of the model (14) contains the vector fields $g_1, g_2, \ldots, g_{n-m}$ and the involutive closure of the distribution spanned by the $n\text{-}m$ vector fields (see **P1**) :

$$[f, g_i] = \begin{pmatrix} s_i(q) \\ 0 \end{pmatrix} \qquad i = 1, \ldots, n-m$$

It results clearly from the nonholonomic nature of the constraints (see Section 2), that the dimension of this involutive closure is equal to $dim(\Delta^*) = n$. The theorem follows immediately. ♦

From the model (13) and in accordance with the physical reality, it follows that the system may be at rest at any configuration q and hence that any point with coordinates $(q, 0)$ in the state space may be an equilibrium point of the system.

Theorem 2. A nonholonomic system is small time locally controllable (STLC) from any equilibrium point $(q, 0)$.

Proof. In [9], it is shown that a dynamical system of the form (14) is STLC if the following conditions are satisfied.

C1. The system is strongly accessible.

C2. The vector field f is zero at the equilibrium.

C3 For each g_i separately, all the repeated Lie brackets of f and g_i, with an odd number of occurences of f and an even number of occurences of g_i, called *bad* Lie brackets hereafter, evaluated at the equilibrium point, can be expressed as a linear combination of brackets of lower order.

Conditions **C1** and **C2** are obviously satisfied here. Condition **C3** is also trivially satisfied because it can be checked, using properties **P2, P3, P4**, that the bad brackets either are identically zero when the number of occurences of f is less than the number of occurences of g_i, or are zero at the equilibrium in the opposite case. ♦

This theorem is an extension of the result presented by Bloch and Mc Clamroch([7]).

5. STATE FEEDBACK CONTROL.

In this section, we are concerned with the design of smooth state feedback stabilizing controls for the dynamical state space model (13). More precisely, we would like to stabilize the system at a particular configuration which may be taken, without loss of generality, as the origin of the generalized coordinates (i.e. $q = 0$). When a smooth state feedback control law $v(\eta, q)$, such that $v(0, 0) = 0$, is applied to the system (13), the closed loop dynamics :

$$\dot{q} = S(q)\eta \tag{16.a}$$

$$\dot{\eta} = v(\eta, q) \tag{16.b}$$

have the origin $(\eta, q) = (0, 0)$ as an equilibrium point.

It is a well known fact that holonomic systems are full state feedback linearizable and, therefore, that any equilibrium point can be made asymptotically stable by a smooth static state feedback. However, this property of holonomic systems does not extend to the nonholonomic case.

Theorem 3. The equilibrium point $(\eta, q) = (0, 0)$ of the closed loop (16) cannot be made asymptotically stable by a smooth static state feedback $v(\eta, q)$.

Proof. See reference [5].♦

Our purpose is then to investigate in which way the feedback stabilisability property of holonomic systems can be extended to the nonholonomic case. The key point of our argumentation is the following one. The feedback stabilizability property of holonomic systems can be rephrased as : *For an holonomic system with n degrees of freedom and n actuators, there exist an output vector function $y = h(q) = q$ and a static feedback control $v(q, \eta)$ such that the closed loop is stable (bounded state) and the output $y = q$ asymptotically converges to zero.*

In this section, we show that *exactly the same property* holds for nonholonomic systems with $(n-m)$ degrees of freedom and $(n-m)$ actuators (obviously, n is the dimension of the configuration space and m is the number of nonholonomic constraints). We first try to achieve this objective by feedback linearization.

Theorem 4. The largest feedback linearizable subsystem of the system (16) has dimension $2(n-m)$ with each controllability index equal to 2.

Proof. This result is readily established by a straightforward application of the algorithm of Marino [8].♦

An important consequence of this theorem is that there exists an output vector function :

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_{n-m} \end{pmatrix} = h(q) = \begin{pmatrix} h_1(q) \\ h_2(q) \\ \cdot \\ \cdot \\ h_{n-m}(q) \end{pmatrix} \tag{17}$$

which depends on the configuration state variable q only, but *not* on the state η, such that the largest linearizable subsystem is obtained by twice differentiating this output function as follows:

$$\dot{y} = \left[\frac{\partial h}{\partial q}(q)\right] S(q)\eta \tag{18.a}$$

$$\ddot{y} = \frac{\partial}{\partial q}\left\{\left[\frac{\partial h}{\partial q}(q)\right] S(q)\eta\right\} S(q)\eta + \left[\frac{\partial h}{\partial q}(q)\right] S(q)v \tag{18.b}$$

It follows from Theorem 4 that the matrix

$$P(q) = \left[\frac{\partial h}{\partial q}(q)\right] S(q)$$

is nonsingular (i.e invertible) for all q, so that the system (18) is clearly input/output feedback linearizable. This means that the system (16) (17) can be transformed by state feedback and diffeomorphism into a controllable linear subsystem of dimension $2(n-m)$ and a nonlinear subsystem of dimension m which does not affect the behaviour of the output $y = h(q)$. The required change of coordinates, which is easily checked to be a diffeomorphism, may be defined as follows :

$$z_1 = h(q), \quad z_2 = \left[\frac{\partial h}{\partial q}(q)\right] S(q)\eta, \quad z_3 = k(q) \tag{19}$$

where $k(q)$ is selected such that the transformation :

$$q \longrightarrow \begin{pmatrix} h(q) \\ k(q) \end{pmatrix}$$

is a diffeomorphism on $\mathbb{R}^n$.

In the new coordinates (19), the dynamics of the system are rewritten as :

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = b(z) + a(z)v \quad \dot{z}_3 = Q(z_1, z_3)z_2 \tag{20}$$

with $z = (z_1, z_2, z_3)$ and appropriate definitions of $a(z)$, $b(z)$ and $Q(z_1, z_3)$. It appears clearly that the equilibrium points of the system (20) are critical (that is some of the poles of the linearization of the system have zero real parts). This implies that a feedback linearizing control law is not guaranteed in general to give (even locally) a stable closed loop. One way to circumvent this difficulty would be to assume that the norm of the matrix $Q(z_1, z_3)$ satisfy some appropriate growth condition (see a related discussion in [10]). Instead, in the next theorem, we show that there is another family of output functions $y = h(q)$ for which there exists a nonlinear feedback control law that does *not* linearize the input/output behaviour but ensures the state boundedness of the closed loop and forces the convergence of the output to zero.

Theorem 5. For any diffeomorphic change of generalised coordinates $\phi(q)$ (with $\phi(0) = 0$), there exists a smooth static feedback control law $v(q, \eta)$ such that :

a) the state $q(t)$, $\eta(t)$ of the closed loop (16) is bounded for all t.

b) the $(n\text{-}m)$-dimensional output function :

$$h(q) = S^T(q)\left[\frac{\partial\phi}{\partial q}(q)\right]^T\phi(q) \qquad (21)$$

converges to zero.

Proof. The control design is a Lyapunov design based on the following candidate Lyapunov function:

$$V(q,\eta) = \frac{1}{2}\phi^T(q)\phi(q) + \frac{1}{2}(\eta + h(q))^T(\eta + h(q)) \qquad (22)$$

By selecting the following feedback control :

$$v(q,\eta) = -\left[\frac{\partial h}{\partial q}(q)\right]S(q)\eta - S^T(q)\left[\frac{\partial\phi}{\partial q}(q)\right]^T\phi(q) - \Lambda(\eta + h(q))$$

where Λ is an arbitrary Hurwitz matrix, the derivative of the Lyapunov function (22) reduces to :

$$\dot{V} = -h^T(q)h(q) - \frac{1}{2}(\eta + h(q))^T(\Lambda + \Lambda^T)(\eta + h(q))$$

This clearly achieves the proof. ♦

Comment. A variant of the Lyapunov design as presented in Theorem 5 can be obtained by introducing the matrix $J(q)$ in the Lyapunov function (22) as follows :

$$V(q,\eta) = \frac{1}{2}\phi^T(q)\phi(q) + \frac{1}{2}(\eta + h(q))^T J(q)(\eta + h(q))$$

and exploiting the fact that the term $g(q, \eta)$ in the model (10) can also be written under the form :

$$g(q,\eta) = S^T(q)\frac{\partial W}{\partial q}(q) + D(q,\eta)\eta$$

where the matrix $D(q, \eta)$ is such that the matrix :

$$\dot{J}(q) - 2D(q,\eta)$$

has the property of being skew symmetric (see e.g. [11]).

6. CONCLUSIONS.

Our main contribution in this paper has been to show that non holonomic systems: (i) are strongly accessible whatever the structure of the constraints; (ii) are small-time locally controllable from any equilibrium configuration; (iii) can be globally stabilized by a smooth state feedback with convergence to zero of an output function whose dimension is the number of degrees of freedom and which depends only on the configuration state q. An application of the foregoing theory to the control of mobile wheeled robots can be found in reference [6].

7. REFERENCES.

[1] Laumond J.P., "Feasible trajectories for mobile robots with kinematic and environment constraints", Int. Conf. on Intelligent Autonomous Systems, Amsterdam, 1986, pp .346,354.

[2] Barraquand J and J.C. Latombe, "On non holonomic mobile robots and optimal manoeuvring", Revue d'intelligence Artificielle, Vol 3-2, 1989, Ed. Hermes, pp. 77,103.

[3] Samson C. and K. Ait-Abderrahim, "Feedback control of a nonholonomic wheeled cart in cartesian space", Proc. IEEE Conf. Robotics and Automation, Sacramento, April 1991, pp. 1136 - 1141.

[4] Canudas de Wit C. and O.J. Sordalen, "Exponential Stabilization of mobile robots with nonholonomic constraints", Int. Report Lab. Automatique de Grenoble, 1991, Submitted for publication.

[5] Campion G., B. d'Andrea-Novel and G. Bastin, "Controllability and state feedback stabilisability of nonholonomic mechanical systems", in Advanced Robot Control, *Proc. Int. Workshop in Adaptive and Nonlinear Control: Issues in Robotics*, Grenoble, 1990, Canudas de Wit (Ed.), Springer Verlag, 1991.

[6] d'Andrea-Novel B., G.Bastin and G.Campion, "Modelling and state feedback control of mobile wheeled robots", Proc. IEEE Conference on Robotics and Automation, Sacramento, April 1991, pp. 1030 - 1035

[7] Bloch A.M. and N. H. McClamroch, "Controllability and stabilizability properties of a nonholonomic control system", Proc. 29th Conference on Decision and Control, Hawaii, pp.1312-1314, 1990.

[8] Marino R., "On the largest feedback linearizable subsystem", Systems and Control Letters 6 (1986), pp.345-351.

[9] Sussmann H.J., "A general theorem on local controllability", SIAM J. Control and Optimization, vol.25,n°1, (1987), pp. 158-194.

[10] Saberi A., P.V. Kokotovic and H.J. Sussmann, "Global stabilization of partially linear composite systems", SIAM Journal Control and Optimization, vol.28 (6), November 1990, pp. 1491 - 1503.

[11] Slotine J-J. and W. Li, "Applied nonlinear control", Prentice Hall, 1991.