STATE FEEDBACK TRAFFIC CONTROL OF THE TRANSIENT BEHAVIOUR OF UNDERGROUND PUBLIC TRANSPORTATION SYSTEMS

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Abstract. This paper proposes two methods of traffic regulation of public underground transportation systems during transients, for example during the modification of the intervals between the trains. Both methods are based on optimal control theory for linear systems with quadratic criterion. In the first approach the system is controlled with respect to a new nominal time schedule to be generated accordingly to the new situation while the second method realizes interval control without reference to a nominal schedule. The properties of both methods are analyzed and simulation results are discussed and compared.

Keywords. Traffic Control; Underground Railway Systems; Linear Quadratic Optimal Control.

1. INTRODUCTION

An underground public transportation system is known to have an intrinsically unstable behaviour. Consider, for instance, a delayed train arriving at a station. Because of this delay the time interval since the last train departure is increased and more passengers have to get on the vehicle, with a resulting increasing delay. In order to restore a disturbed traffic to the nominal situation control is therefore necessary. For man operated systems as well as for fully automatised systems the control actions consist of instructions (speed during the stations, waiting time modification) elaborated by the centralized controller on the basis of the available information (i.e. the situation of the other trains). Several constraints must of course be satisfied, such as speed limits, miminum waiting times at the stations or the other security rules.

On the basis of a linear model introduced by Sasama and Okhawa [1], we have proposed in [2] and [3] a particularly attractive state-space formulation making possible the implementation of an optimal control minimizing a quadratic performance index. The properties of this policy as well as simulation results can be found in these papers but only in steady state nominal situations, i.e. in case of constant nominal time intervals between trains, in correspondance with a given nominal time schedule.

On the other hand as the passengers' flow at the station varies significantly from one hour to the other (consider for instance the rush hours of the morning or the evening) the nominal time interval during the successive trains, and therefore the number of trains exploited simultaneously on the line, have to be modified from time to time, in order to be adapted to the passengers' affluence at the stations : several transient periods have therefore to be considered, corresponding to these frequency modifications. Without control these transients lead to a generalized unstable behaviour of the system. The purpose of the present paper is to propose two different methods for the traffic control during these transients. In the first one a new nominal time schedule is generated, corresponding to the new frequency, and the system is controlled during the transient in order to converge to the new schedule. This control policy appears therefore as the extension to the transient case of the control proposed in [2] and [3] for the steady-state situation. In the second method no reference at all is made to a new nominal schedule and the system is controlled in order to impose to the time intervals between successive trains to be equal to the desired steady state value. This original control, based on a new state space formulation, realizes time interval control without reference to a nominal schedule.

In section 2 we describe briefly the linear model of the traffic dynamics. In section 3 and 4 we detail the two state-space representations, as well as the corresponding control policies, for the two proposed methods, respectively with or without reference to a nominal schedule. In section 5 we present a case study of frequency modification and we give simulation results relatively to the proposed methods.

2. THE MATHEMATICAL MODEL FOR TRAFFIC DYNAMICS

We now briefly summarize the mathematical model for traffic dynamics proposed by Sasama and Okhawa [1].
Consider I trains (upper index i = 1, ..., I) on a line with K + 1 stations (lower index k = 0, ..., K). The departure time t^i_{k+1} of the i-th train from station (k+1) can be expressed as

$$t^i_{k+1} = t^i_k + r^i_k + s^i_{k+1} + w^i_k \qquad (1)$$

where r^i_k is the running time for i-th train from the k-th station to (k+1)th station
s^i_{k+1} is the staying time of train i at station (k+1)

and w^i_k is a disturbance term affecting train i between the stations k and (k+1)

The running time r^i_k can be modelled further as

$$r^i_k = R_k + u1^i_k \tag{2}$$

where R_k is the nominal running time from k to $k+1$, and $u1^i_k$ is the portion of the control action applied to train i, between stations k and (k+1), in order to increase ($u^i_k > 0$) or decrease ($u^i_k < 0$) the running time.

The staying time s^i_{k+1} depends on the time interval between the departure of the preceding train and the arrival of the present train. This dependance is modelled linearly by

$$s^i_{k+1} = a^i_{k+1}(t^i_{k+1} - s^i_{k+1} - t^{i-1}_{k+1}) + D_{k+1} + u2^i_k \tag{3}$$

where D_{k+1} is the minimal staying time at a station and $u2^i_k$ is the portion of the control action applied to the i-th train at station k. Equation (3) constitutes the basic assumption of this traffic dynamic model : the staying time increases linearly with the number of passengers getting on the train, and therefore to the time elapsed since the departure of the last train.
In these relations, D_k, R_k and the coefficients a^i_k are characteristic parameters of the line to be estimated from statistical data (see ref.[1]).

Defining $c^i_k = \frac{a^i_k}{1+a^i_k}$ and $b^i_k = \frac{1}{1+a^i_k}$

equation (1) can be rewritten as

$$t^i_{k+1} = t^i_k + R_k + c^i_{k+1} (t^i_{k+1} - t^{i-1}_{k+1}) + b^i_{k+1} D_{k+1} + w^i_k + u^i_k \tag{4}$$

where $u^i_k = u1^i_k + b^i_k u2^i_k$ represents the global control action applied to the i-th train between its departures from stations k and (k+1).
Equation (4) characterizes the transfer of the i-th train from station k to the station (k+1).

3. TRAFFIC CONTROL WITH REFERENCE TO A NOMINAL SCHEDULE

3.1. The "time schedule" equation

Accordingly to equation (4) a nominal time schedule, representing the evolution of the system without disturbances and control (i.e. $u^i_k = w_k = 0$) can be defined by

$$T^i_{k+1} = T^i_k + R_k + c^i_{k+1} (T^i_{k+1} - T^{i-1}_{k+1}) + b^i_{k+1} D_{k+1} \tag{5}$$

Defining x^i_k as the deviation of the actual departure time t^i_k from the nominal value T^i_k, i.e.

$$x^i_k = t^i_k - T^i_k \tag{6}$$

we obtain the following basic transfer equation :

$$(1-c^i_{k+1})\, x^i_{k+1} + c^i_{k+1}\, x^{i-1}_{k+1} = x^i_k + u^i_k + w^i_k \tag{7}$$

3.2. The "real time model" (RTM)

Several state space representations corresponding to eq.(7) can be found in [1]. In the original representation referred to as the "Real time Model" in [2] and [3], the state vector, the control and the disturbance vectors are defined respectively as

$$X_j = \begin{bmatrix} x_1^{j-1} \\ x_2^{j-2} \\ \cdot \\ \cdot \\ \cdot \\ x_K^{j-K} \end{bmatrix}, \quad U_j = \begin{bmatrix} u_0 \\ u_1^{j-1} \\ \cdot \\ \cdot \\ \cdot \\ u_{K-1}^{j-K+1} \end{bmatrix},$$

$$V_j = \begin{bmatrix} w_0^{\,j} \\ w^{j-1} \\ \cdot \\ \cdot \\ \cdot \\ w_{K-1}^{j-K+1} \end{bmatrix} \tag{8a}$$

This definition of the state space vector is based on the structure of the basic transfer equation (7) : it can be observed that the deviation x^i_{k+1} depends on control and perturbation terms, but on two deviations (x_{k+1} and x_k) characterized by the same value for the sum of their upper and lower indices. It is therefore natural to define as the state vector, with index j, the set of the K deviations x^i_k with $i + k = j$. The set of the transfer equations (7) for all the trains and all the stations can then be rewritten under vectorial form

$$X_{j+1} = A X_j + B U_j + B V_j \tag{9}$$

where A and B are two (KxK) matrices

$$A = \begin{bmatrix} \frac{-c_1}{1-c_1} & \cdot & \cdot & \cdot & 0 \\ \frac{1}{1-c_2} & \frac{-c_2}{1-c_2} & & & \cdot \\ & \ddots & \ddots & & \cdot \\ 0 & \cdots & \frac{1}{1-c_K} & & \frac{-c_K}{1-c_K} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{1-c_1} & \cdot & \cdot & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & \cdot & \cdot & \frac{1}{1-c_K} \end{bmatrix}$$

On the other hand, as the set of t^i_k corresponding to the components of X_j are nearly simultaneous, the index j can be interpreted as the time index for a discrete time difference equations system.

For a loop line (where the same trains are operated) the definitions of the state and control vectors, as well as the dynamical matrices have to be adapted. Consider, for instance, a loop line with K stations, where I trains (I<K) are operated. At a given station the sequence of trains is periodic, from circuit to circuit :

1, 2, ..., I-1, I, 1, 2, ..., I-1, I, 1, 2, ...

For train number 1, the preceding train has index I. To take into account this periodicity we define an augmented state vector (dimension K) X_j as

$$X_j = \begin{bmatrix} x^I_{j-k} \\ \vdots \\ x^I_{j-I-1} \\ x^I_{j-I} \\ \hline x^I_{j-I+1} \\ \vdots \\ x^I_{j-1} \end{bmatrix} \quad \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} (K-I) \text{ components} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} I \text{ components} \end{matrix} \qquad (8b)$$

, where the lower index (station index) is defined modulo K.

The A_j and B_j matrices become

$$A_j = \begin{bmatrix} 0 & 1 & 0 & & & \cdot & \cdot & \cdot & & 0 \\ 0 & 0 & 1 & 0 & & \cdot & \cdot & \cdot & & 0 \\ & & \ddots & \ddots & & & & & & \\ 0 & \cdot & \cdot & 0 & 1 & 0 & - & - & - & 0 \\ 0 & - & - & (1+a_1) & -a_1 & 0 & - & - & - & 0 \\ 0 & - & - & 0 & 1+a_2 & -a_2 & 0 & - & - & 0 \\ & & & & & \ddots & \ddots & & & \\ 0 & & - & - & - & 0 & 1+a_{k-1} & -a_{k-1} & & \\ -a_K & & - & - & - & 0 & 0 & 1+a_k & & \end{bmatrix}$$

and

$$B_j = \begin{bmatrix} 0 & & & & 0 \\ & - & - & - & \\ 0 & & & & 0 \\ 1+a_1 & - & - & - & 0 \\ & \ddots & & & \\ 0 & - & - & - & 1+a_k \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} K-1 \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} I \end{matrix}$$

3.3. Traffic Regulation

The proposed control is based on linear quadratic optimal control theory. The performance criterion has to take into account the two regulation objectives : regularity with respect to the nominal schedule and regularity of the time interval between successive trains. We select the following quadratic criterion penalizing the deviations with respect to the nominal schedule as well as the interval deviations :

$$J_1 = \frac{1}{2} \sum_{i,k} [p_k (x^i_k)^2 + q_k (x^i_k - x^{i-1}_k)^2 + u^i_k]^2 \qquad (10a)$$

or, equivalently,

$$J_1 = \frac{1}{2} \sum_j [X^T_{j+1} P X_{j+1} + (X_{j+1} - X_j)^T Q (X_{j+1} - X_j) + U^T_j U_j] \qquad (10b)$$

where P and Q are two diagonal matrices :

$P = \mathrm{diag}\,(p_1, p_2, \ldots p_K)$ and $Q = \mathrm{diag}\,(q_1, q_2, \ldots, q_K)$

The optimal control for the linear quadratic problem defined by (9) and (10) is known to be a state-feedback control, which takes a particularly simple form, when the problem is restricted to a one step optimization problem (i.e. the sum in (10b) is restricted to only one term) :

$$u^i_k = g_{k+1} x^i_k + f_{k+1} x^{i-1}_{k+1} \qquad (11)$$

The expressions of g_{k+1} and f_{k+1} as well as a discussion of the stability properties of the closed-loop systems can be found in [2].

4. TRAFFIC CONTROL WITHOUT REFERENCE TO A NOMINAL SCHEDULE

4.1. The "interval equation"

Consider the equations describing respectively the transfer of the trains (i-1) and i from the k-th station to the (k+1)th station :

$$t^i_{k+1} = t^i_k + R_k + c_{k+1}(t^i_{k+1} - t^{i-1}_{k+1}) + b_k D_k + u^i_k + w^i_k \qquad (12a)$$

$$t^{i-1}_{k+1} = t^{i-1}_k + R_k + c_{k+1} (t^{i-1}_{k+1} - t^{i-2}_{k+1}) + b_k D_k + u^{i-1}_k + w^{i-1}_k \qquad (12b)$$

Defining $\delta t^i_k = t^i_k - t^{i-1}_k$, i.e. the time interval between (i-1) and i at station k, (13a)

$$\delta u^i_k = u^i_k - u^{i-1}_k, \qquad (13b)$$

and $$\delta w^i_k = w^i_k - w^{i-1}_k, \qquad (13c)$$

we obtain from (12a) and (12b)

$$(1-c_{k+1})\, \delta t^i_{k+1} + c_{k+1}\, \delta t^{i-1}_{k+1} = \delta t^i_k + \delta u^i_k + \delta w^i_k \qquad (14)$$

In this formulation we have neglected the variation of C_k, at a given station between two successive trains.

It must be noticed that the structure of eq.(14) is the same as for the basic "time schedule" transfer equation (7) but it has been derived without any reference to a nominal time schedule. This equation describes the evolution of the time intervals between successive trains and will be referred to as the "interval equation".

4.2. State Space Representation

Taking benefit from this similarity of structure, we define, as for the "time schedule" approach a real-time model :

$$\Delta t_{j+1} = A\, \Delta t_j + B\, \Delta U_j + B\, \Delta W_j \qquad (15)$$

where the state vector Δt_j, the control and the disturbance vectors are defined similarly to (8)

A and B have the same structure as the matrices A and B characterizing the "time schedule" RT Model, for loop lines as well as for open lines. Using (13b) the control vector can be related to ΔU_j by

$$U_{j+1} = L\,U_j + \Delta U_j \qquad (17)$$

where L is a (IxI) matrix

$$L = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ & & \ddots & \ddots & \\ 0 & - & - & 0 & 1 \\ 1 & - & - & - & 0 \end{bmatrix}$$

Equations (15) and (17) constitute the complete description of the system, without reference to a nominal schedule.

4.3. Traffic Regulation

a) As we now consider a time interval model the only control objective is to ensure interval regularity, i.e. to impose to the time interval to be as close as possible to the desired time interval, ΔT. We define therefore a new state vector whose components are the differences between the intervals δt^1_k and ΔT, i.e.

$Y_j = \Delta t_j - I_K \Delta T$ where I_K is the (KxK) unity matrix

Equation (15) is then rewritten as

$$Y_{j+1} = (A - I_K) Y_j + B\,\Delta U_j \qquad (18)$$

For the linear system (17)-(18) we introduce a quadratic performance index, as in (10)

$$J_2 = \frac{1}{2} \sum_j [Y^T_{j+1} P\, Y_{j+1} + U^T_{j+1} Q\, U_{j+1}] \qquad (19)$$

b) By restriction to a one step optimization problem, the optimal ΔU_j is linear in Y_j and U_j

$$\Delta U_j = K_1 Y_j + K_2 U_j \qquad (20)$$

For open lines the eigenvalues of the closed-loop system are inside the unit circle, ensuring the convergence to the steady state situation characterized by Y = 0 and U = 0.

For loop lines, all the eigenvalues are inside the unit circle, except one, which is equal to one. The closed-loop system converges therefore to a stable situation characterized by steady state Y* and U*. If ΔT is chosen equal to the natural period of the system (i.e. the period assuring the periodicity without control action) this steady state solution corresponds to Y* = 0 (i.e. all the intervals are equal to Δt) and U* = 0 (i.e. the control actions are zero). The natural period, ΔT^*, of the loop lines with I trains is characterized by

$$I \Delta T^* = \sum_{j=1}^{K} R_j + \sum_{j=1}^{K} D_j + \left(\sum_{j=1}^{K} c_j\right) \Delta T^* \qquad (21)$$

5. CONTROL OF THE TRANSIENTS - CASE STUDY

In [2] and [3] simulation results relative to steady-state disturbed situations are given and the efficiency of the proposed control, corresponding to the "time schedule" approach is pointed out. In this section we are interested in the traffic control during transients of the systems, namely during frequency modifications.

5.1. Statement of the problem

We consider a loop line with 30 stations. The system parameters are chosen as follows

* the c_k are equal and constant (c_k = 0.02)
* the minimal staying time in station, D, is constant and the same for each station (D = 15 sec)
* the standard running times between stations are assumed to be known (of the order of 60-100 sec).

We impose several constraints :

* the security requirements of a real line are implemented in the simulation program (traffic lights)
* the control actions are bounded and the relative variation of the running times and the staying times cannot exceed 10% of their nominal values.

Assume that the frequency has to be modified : we consider a first nominal natural steady-state schedule with constant time intervals of 300 sec and we want to increase these intervals up to 400 sec. According to the first nominal schedule 9 trains are operated on the line. In order to achieve the new frequency only 7 trains have to be operated so two trains are suppressed. This maneuver occurs at a given "taking-off" station. The choice of the suppressed train is arbitrary and has of course an influence on the system performance.

5.2. Traffic regulation with reference to a nominal schedule

a) The new nominal schedule

A new nominal steady-state schedule corresponding to the modified time interval (400 sec) has first to be generated and the selection of the suppressed trains to be made. Before starting the operations the conditions of implementation of this new schedule are defined as follows : the new schedule is in application for a given train at its first stop at the "taking off" station occuring after the "taking off" hour and we consider, in addition, that for the first train for which the new steady-state schedule is in application the initial deviation is zero, i.e. this train is on schedule. In addition the standard staying times (D_k) are modified in such a way that 400 sec becomes the natural period for the loop line with 7 trains, accordingly to equation (21).

This rule is clearly arbitrary and other new schedule implementation policies can be applied. In our simulations the suppressed trains are the 6th one and the 8th one stopping at the "taking off" station after the "taking off" hour.

b) Simulation results

The simulation results are summarized, for each case, in two diagrams giving respectively the deviations of the trains at the taking off station, with respect to the new nominal schedule, and the time intervals between trains starting from this station (the new nominal value is 400 sec). In order to show clearly the evolution from these characteristics from one circuit to the next one, vertical dotted lines separate the different circuits of the 7 remaining trains.

Case 1 : Free system (no control) Fig. 1 and 2
Consider the departures of the 7 remaining trains after the taking off hour. For the first five trains the deviations are negative with increasing absolute values (these trains are in advance with respect to the new schedule) and the intervals are 300 sec. For the next train, following the first suppressed trains, the delay is -200 sec, with an interval of 600 sec, and for the last train, following the second suppressed trains, the deviation is zero, with an interval of 600 sec. This situation at the first crossing over the taking off station is the initial condition for the next circuit, and it can be seen that without regulation the deviations as well as the differences of intervals increase from circuit to circuit.

Several regulation policies are implemented, corresponding to different choices of the weighting coefficients p and q characterizing the performance index (10).

Case 2 p = 0.1 and q = 0.1 (fig. 3 and 4)

Case 3 p = 0.0 and q = 1.0 (fig. 5 and 6)

In these two situations it can be seen that the deviations with respect to the new nominal schedule decrease from circuit to circuit, and that the intervals converge to the new nominal value (400 sec), and at the third crossing over the taking-off station the new steady-state situation is nearly reached. By comparing the results of case 2 and case 3 it can be seen that if only interval regularity is desired (p = 0.0, i.e. no penalization of the deviations) the deviations converge to a constant value which is different from zero.

Case 4

The new nominal schedule is slightly different : we consider that the first train crossing over the taking off station has a delay of 400 sec (translation of the new nominal schedule). The weighting coefficients are p = 0.1, q = 0.1. See fig. 7 and 8. The transient is different for the deviation evolution but the new steady state situation is reached after 3 circuits, as for the first nominal schedule.

5.3. Traffic regulation without reference to a nominal schedule

As seen in 5.2 the implementation of the new nominal schedule is arbitrary, and the choice of a particular policy can influence the transient behaviour of the system. If we avoid the reference to the nominal schedule, as it is done in the interval equation approach, this difficulty does not more exist.
We give now simulations results corresponding to the implementation of the control policy (20), independent of a nominal schedule. The suppressed trains are the same as in 5.2 : the 6th and the 8th trains crossing over the taking-off station, after the taking-off hour. The control action is applied to each train from its crossing over this station.

Case 5. Time interval approach - p=1.0, q=1.0.
As 400 sec is the natural period of the line with 7 trains the system converges to the stable situation with ΔT = 400 sec and steady state control actions equal to zero. Fig. 9 gives the evolution of the time intervals : in less than two circuits the new steady state situation is reached. The transient behaviour is much better than for cases 2 and 4 and comparable to case 3. In fact for case 3, as p = 0.0, the control tends to minimize the differences of deviations between successive trains, i.e.

$$x_k^i - x_k^{i-1} = (T_k^i - T_k^{i-1}) - (t_k^i - t_k^{i-1}) = \Delta T - \Delta t_k^i$$

The criterion J_1 with p=0 and q=1.0 is therefore equivalent to J_2 with p=1.0 and q=1.0. The advantage of the time interval approach is that it does not need the generation of a new steady-state schedule.

6. CONCLUSIONS

1) It is shown that the optimal control method proposed previously for the traffic regulation in steady-state situations can easily be extended to the transients, by generation of a new nominal time schedule. This approach is particularly usefull when the system has to reach a new specified steady-state behaviour for which a nominal schedule can be defined.

2) We developed in this paper our new approach for the modelisation and the traffic regulation without reference to a nominal time schedule. This method is particularly usefull when a nominal schedule cannot be generated and when therefore only interval regulation is desired.

3) Both methods have been tested for a particular transient : the modification of the time interval between successive trains. Other transients can be regulated. Assume, for instance, that, due to a technical reason, a given station cannot be operated, and that the line has therefore to be decomposed into two sublines without connection, to be exploited independently. In this case a nominal schedule cannot be defined easily and the interval regulation without reference to a nominal schedule appears to be the best regulation policy.

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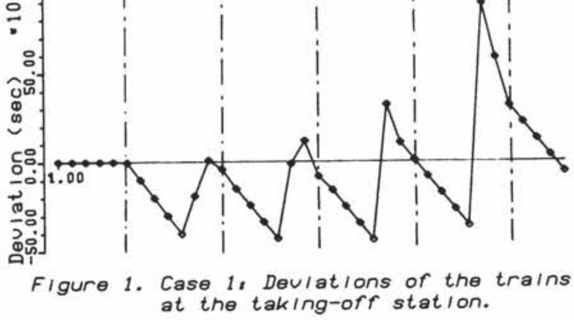

Figure 1. Case 1: Deviations of the trains at the taking-off station.

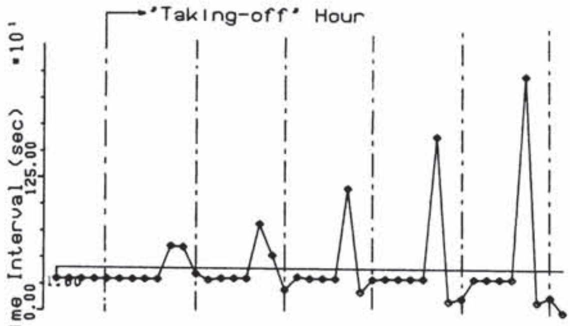

Figure 2. Case 1: Time interval between successive trains.

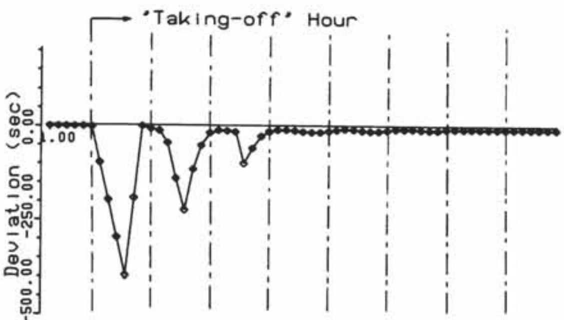

Figure 3. Case 2: Deviations of the trains at the taking-off station.

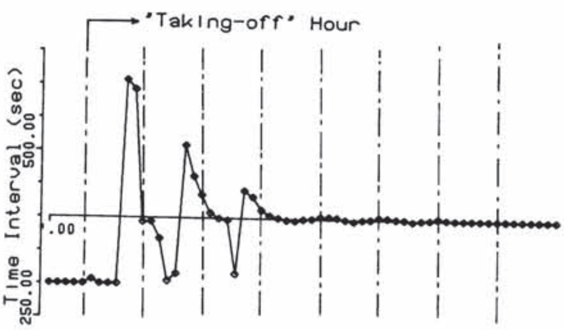

Figure 4. Case 2: Time interval between successive trains.

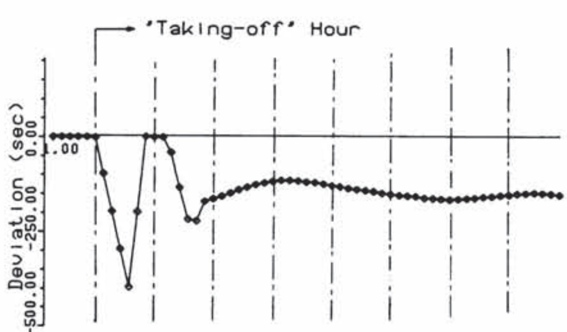

Figure 5. Case 3: Deviations of the trains at the taking-off station.

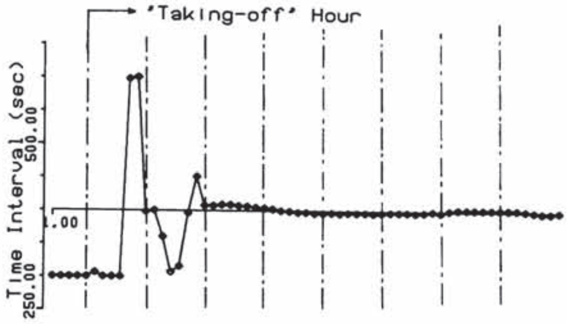

Figure 6. Case 3: Time interval between successive trains.

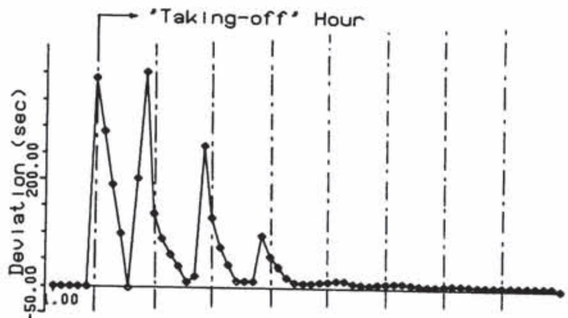

Figure 7. Case 4: Deviations of the trains at the taking-off station.

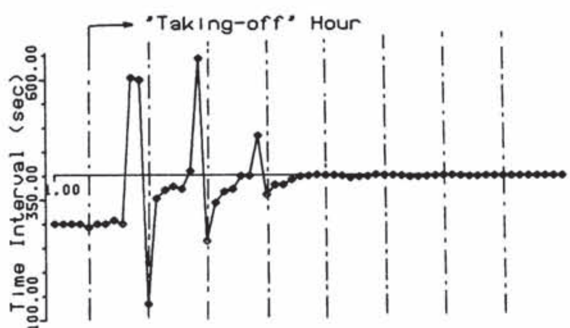

Figure 8. Case 4: Time interval between successive trains.

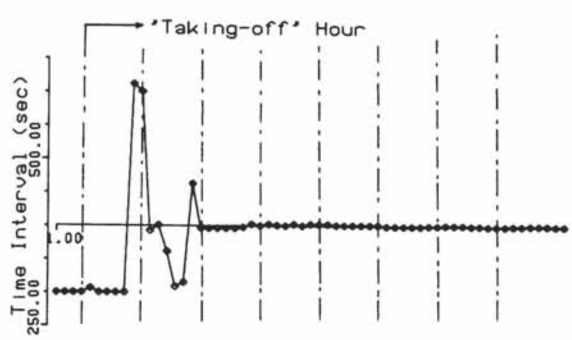

Figure 9. Case 5: Time interval between successive trains.